

Optics and Modern Physics

CHAPTER 6

Atomic Physics

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6

Atomic Physics

INTRODUCTION

The idea that matter is made up of discrete units is a very old idea, appearing in many ancient cultures such as Greece and India. The term “atom” comes from the Greek word for indivisible, because it was once thought that atoms were the smallest form of matter in the universe and could not be divided. Over the past two centuries, physicists and chemists developed the concept of an atom. In the nineteenth century, though still unseen, atoms were used to explain a large variety of natural phenomena. Eventually, atoms were found to have structure and to be divisible, though not by means of ordinary physical and chemical processes. Today, we can produce images of individual atoms by using a scanning tunneling electron microscope. Thus, atoms which were once only a philosopher’s dream, have become a physical reality.

In this chapter, we will learn the atomic models of the past two centuries and the Bohr model of the atom in early 1900s. In fact, the atom as we understand it today is based on quantum mechanics which is radically different from Newton’s classical mechanics.

ATOMIC STRUCTURE

DALTON’S ATOMIC THEORY

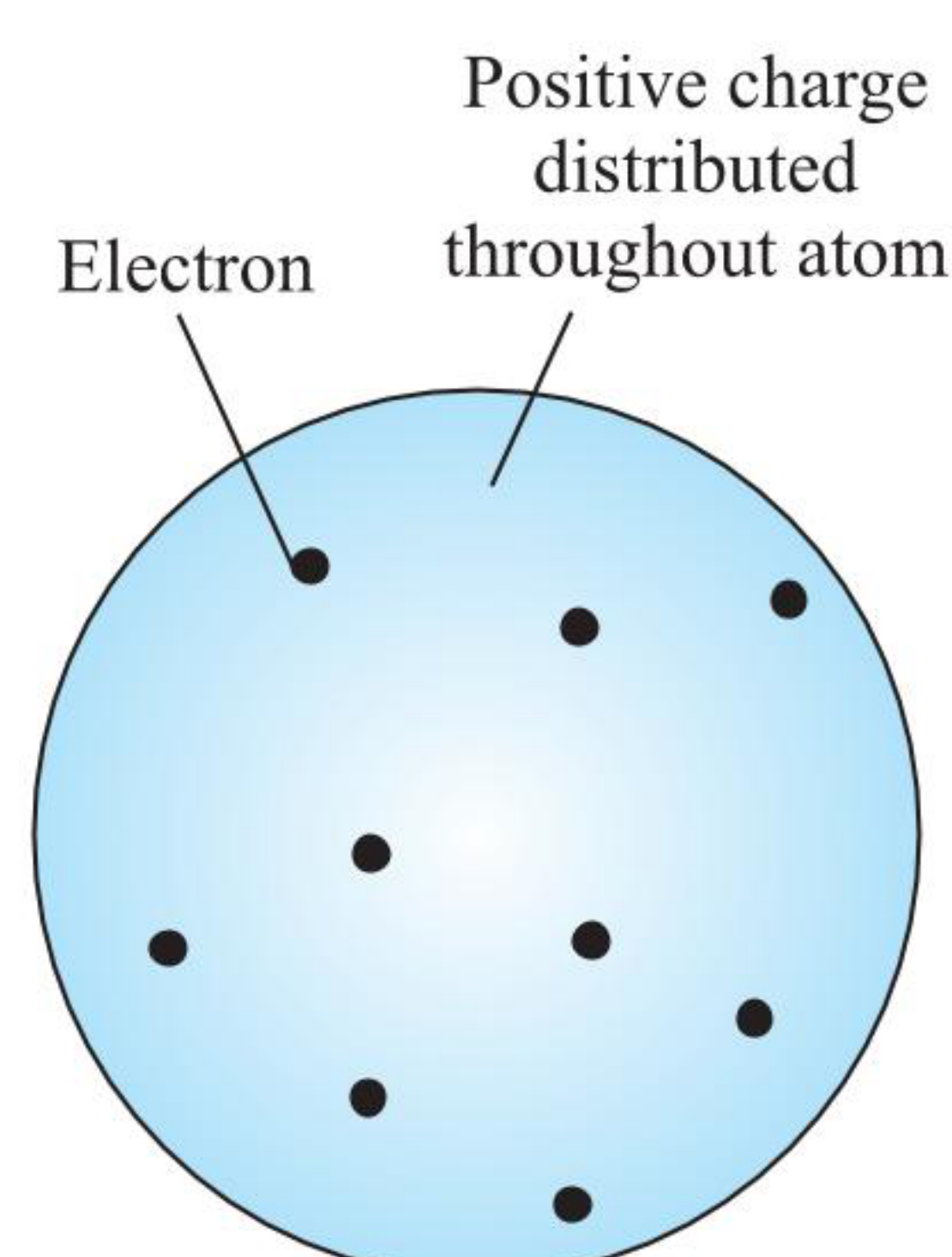
John Dalton, in 1808, put forward his theory, according to which

- (i) Every material is composed of minute particles known as atom. Atom is indivisible i.e. it cannot be subdivided. It can neither be created nor be destroyed.
- (ii) All atoms of same element are identical physically as well as chemically, whereas atoms of different elements are different in properties.
- (iii) The atom is stable and electrically neutral.

Nobody seriously questioned Dalton’s theory until about the beginning of the twentieth century.

THOMSON’S ATOMIC MODEL

In 1897, the English physicist Joseph J. Thomson suggested a model that describes the atom as a region in which positive charge is spread out in space with electrons embedded throughout the region, much like the seeds in a watermelon or raisins in thick pudding. The atom as a whole would then be electrically neutral. In Thomson’s view

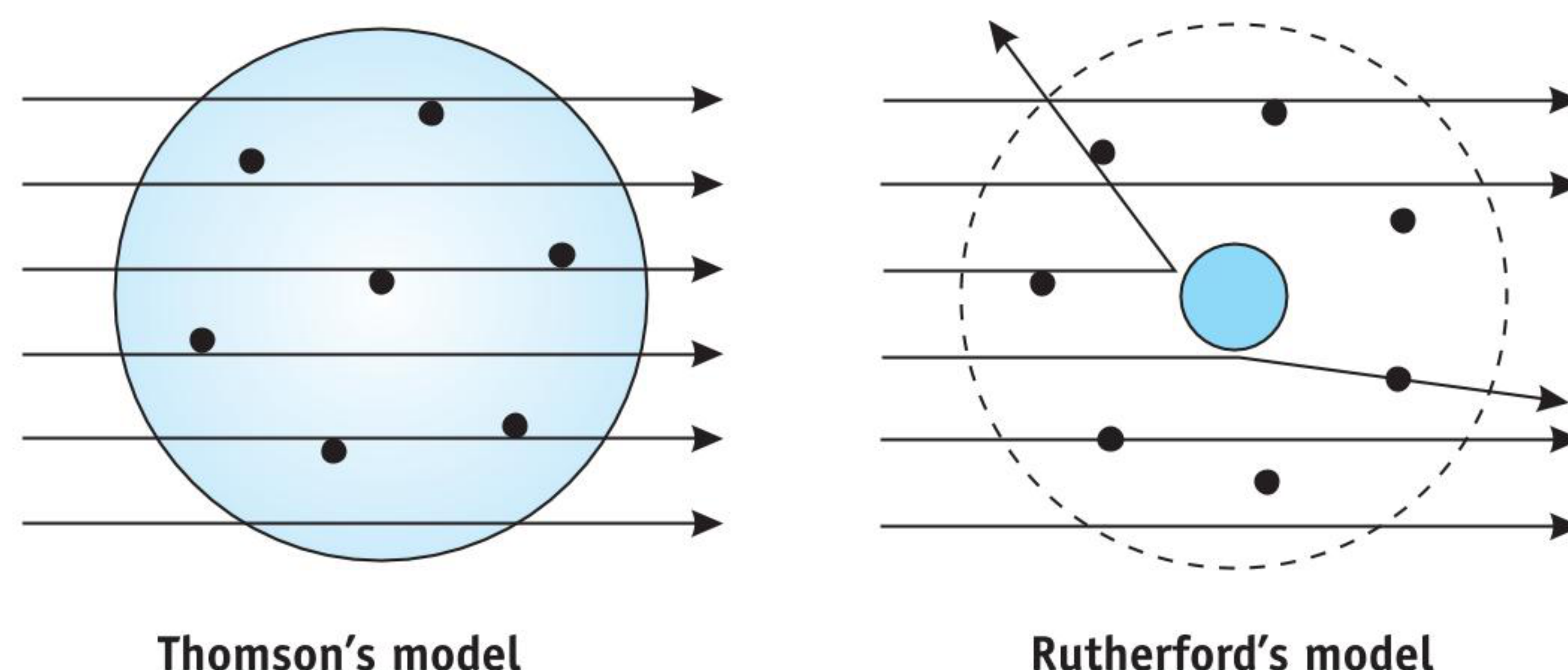


1. Atom is a positively charged sphere of radius of the order of 10^{-10} m.
2. There was no nucleus at the centre of an atom. Instead, the positive charge (and also mass) was assumed to be spread throughout the atom, forming a kind of “paste” or “pudding.” The negative electrons were assumed to be suspended like “plums.”
3. The number of electrons is such that their total negative charge is equal to the total positive charge. Hence, atom is electrically neutral.

The “plum-pudding” model was discredited in 1911 when the New Zealand physicist Ernest Rutherford (1871–1937) published the experimental results that this model could not explain the atom.

RUTHERFORD’S ATOMIC MODEL

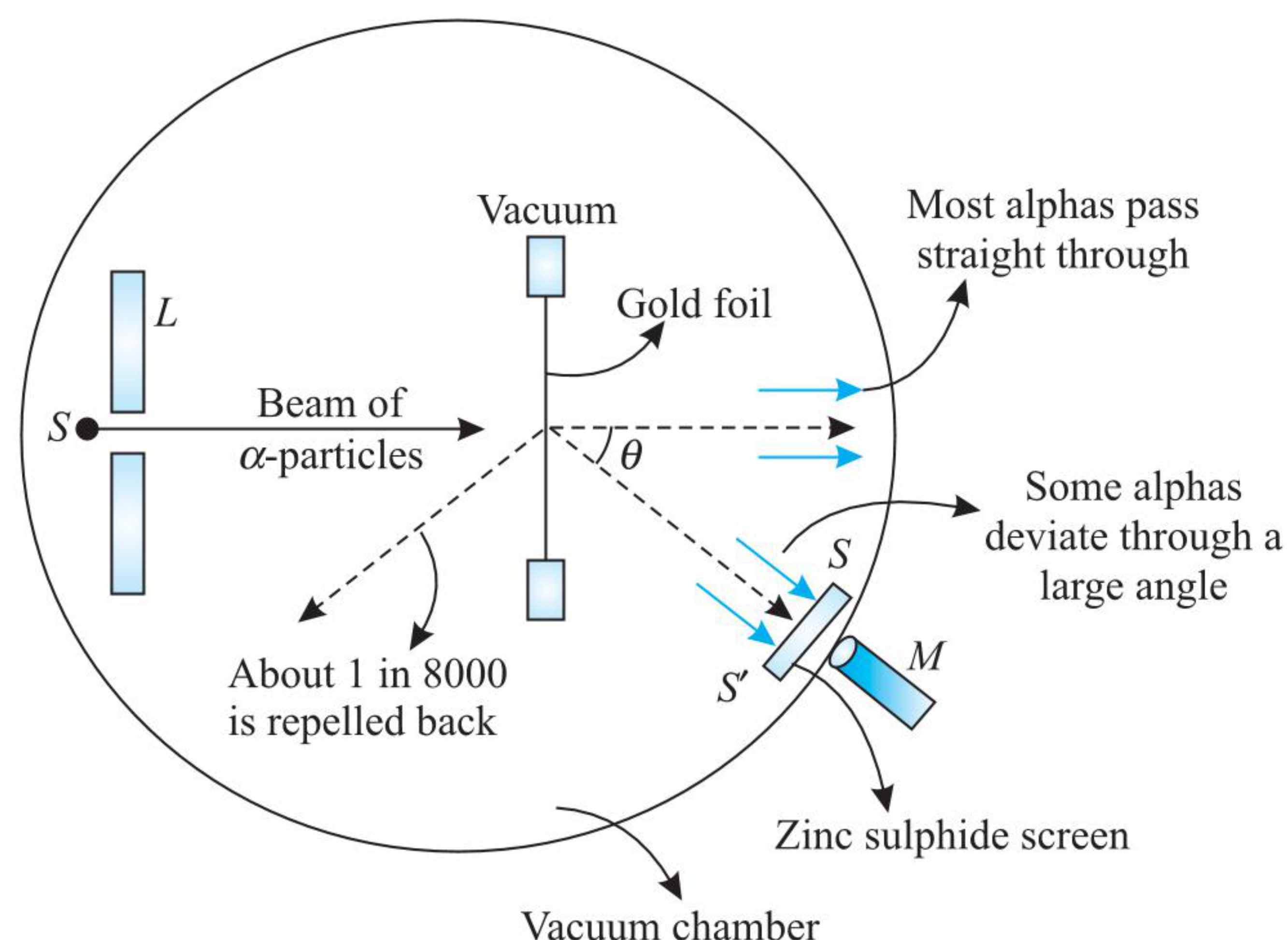
Rutherford and his co-workers directed a beam of alpha particles at a thin metal foil made of gold. Alpha particles are positively charged particles (the nuclei of helium atoms, although this was not recognized at the time) emitted by some radioactive materials. If the “plum-pudding” model were correct, the α -particles would be expected to pass nearly straight through the foil. After all, there is nothing in this model to deflect the relatively massive α -particles, since the electrons have a comparatively small mass and the positive charge is spread out in a diluted “pudding.” Using a zinc sulphide screen, which flashed briefly when struck by an α -particle, Rutherford and co-workers were able to determine that not all the α -particles passed straight through the foil. Instead, some particles were deflected at large angles, even backward.



Experimental Arrangement

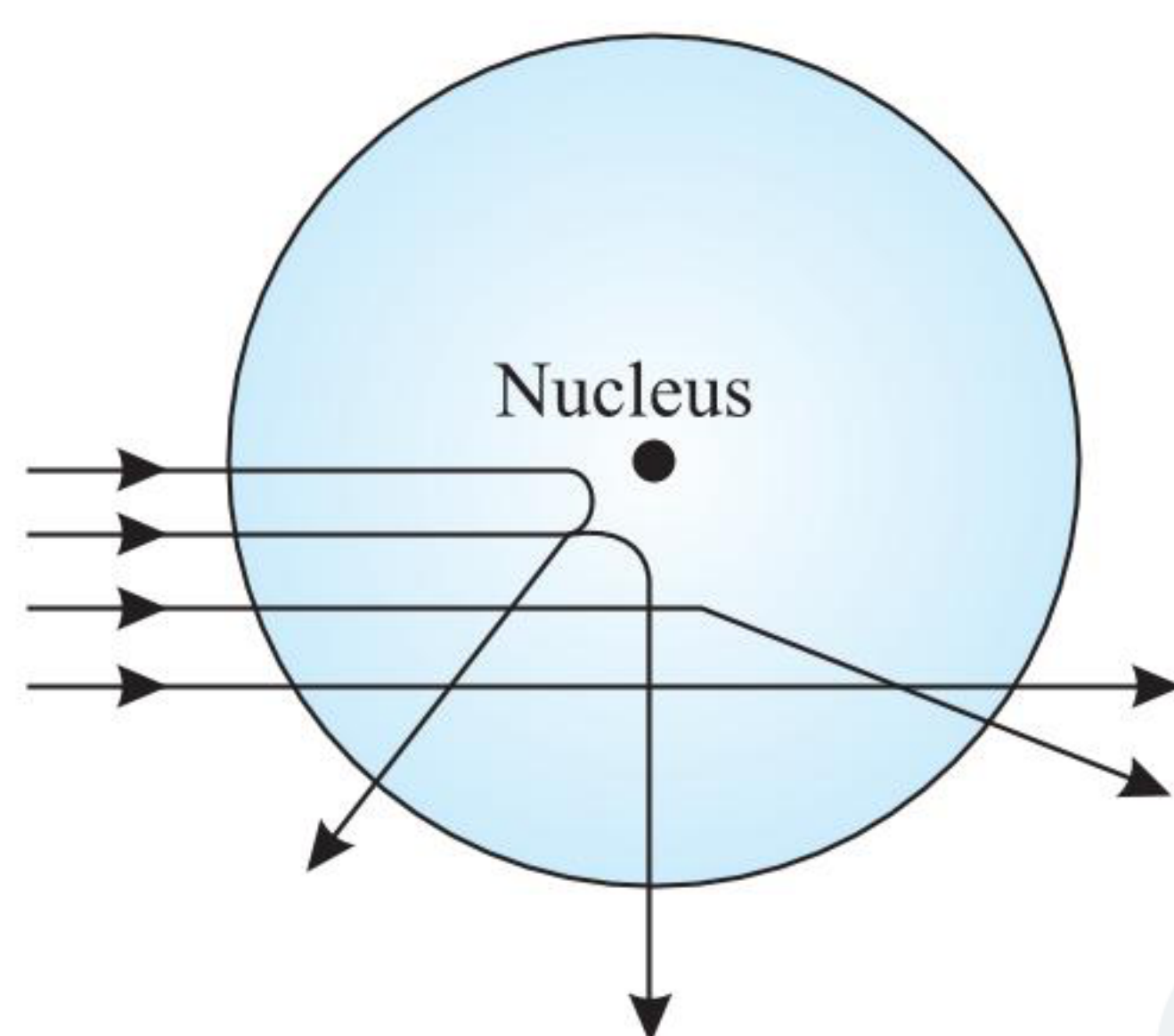
The experimental arrangement is shown in figure. α -particles are emitted by some radioactive material (polonium), kept inside a thick lead box. A very fine beam of α -particles passes through a small hole in the lead screen. This well collimated beam is then allowed to fall on a thin gold foil. While passing through the gold foil, α -particles are scattered through different angles.

A zinc sulphide screen was placed out the other side of the gold foil. This screen was movable, so as to receive the α -particles, scattered from the gold foil at angles varying from 0 to 180° . When an α -particle strikes the screen, it produces a flash of light and it is observed by the microscope.



Observations

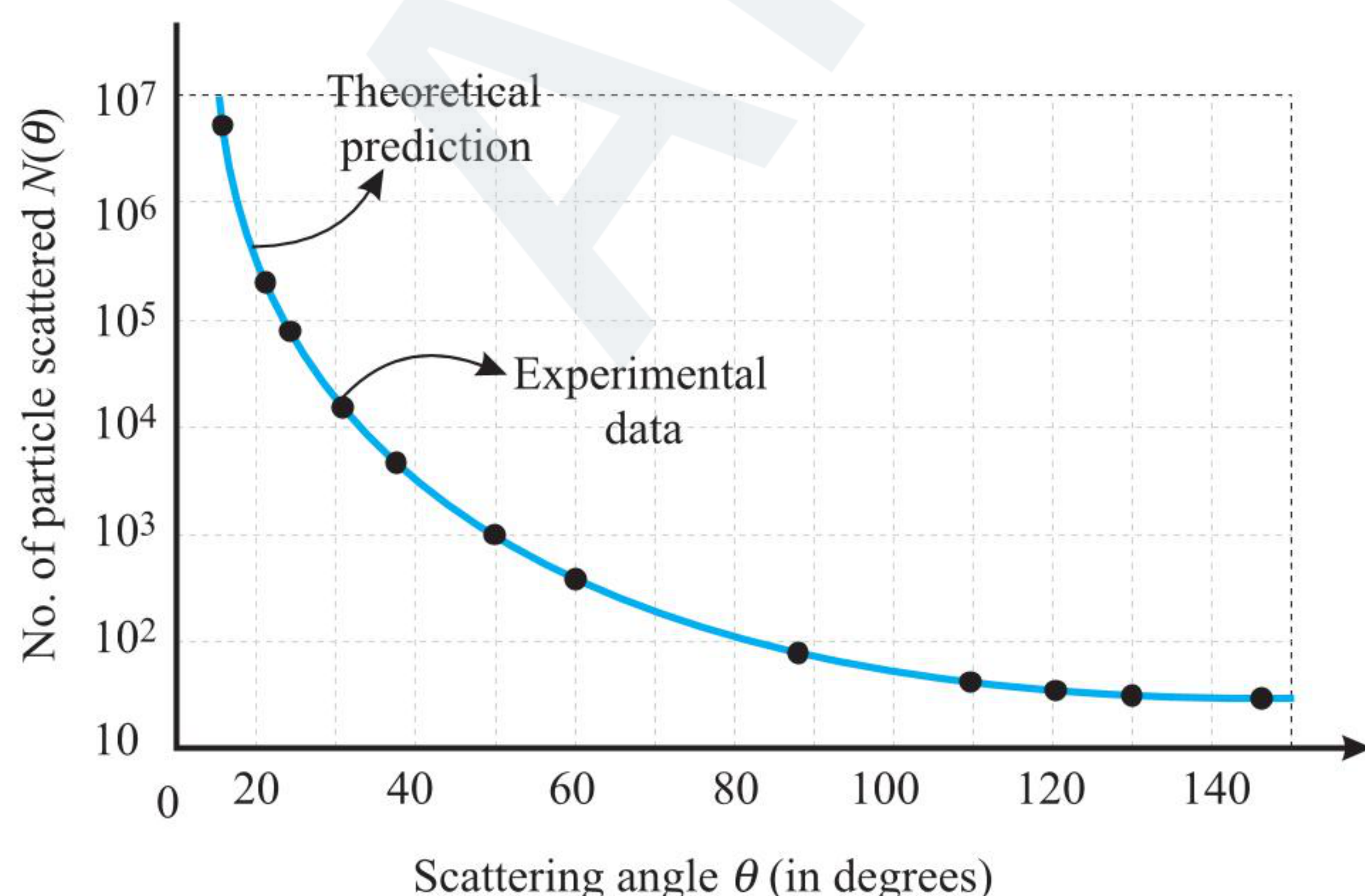
The beam of α -particles is scattered in various directions. While most of the α -particles passed straight through the foil, some of them bounced off at small angles, whereas a few underwent deflections through very large angles.



According to **Rutherford scattering formula**,

$$N(\theta) \propto \frac{1}{\sin^4(\theta/2)}$$

where $N(\theta)$ is the number of α -particles per unit area that reach the screen at a scattering angle of θ . A graph between $N(\theta)$ and (θ) is shown in the figure. The experimental findings follow this curve as only 0.14 percent of the incident α -particles are scattered by more than 1° .



The above graph reveals the following facts:

1. Most of the α -particles pass straight through the gold foil or suffer only small deflections.
2. A few α -particles, about 1 in 8,000, get deflected through 90° or more.
3. Occasionally, an α -particle gets rebounded from the gold foil, suffering a deflection of nearly 180° .

Conclusions: Since α -particles are relatively heavy and have high initial speeds, strong electrical forces are required to deflect them through large angles. This led Rutherford to postulate that:

1. The whole positive charge of the atom and almost its entire mass is concentrated in a small region of the atom. Rutherford named this region as the **nucleus**.
2. The nucleus is surrounded by electrons. As the atom on the whole is neutral, the total negative charge of the electrons is equal to the positive charge on the nucleus.
3. Using high speed α -particles, the nuclear diameter has been found to be of the order of 10^{-14} m.
4. For the stability of the atom, Rutherford assumed that the electrons are revolving at high speeds around the nucleus in closed circular orbits so that the force of attraction between the nucleus and the electrons is balanced by the centrifugal force acting on the electrons.

If the electrons in an atom were stationary, these would fall into the nucleus due to the electrostatic force of attraction between the electrons and the nucleus.

5. The existence of sufficient empty space within the atom explains why most of the α -particles go undeflected. Small angle of scattering is accounted for by the fact that the nucleus occupies only a fraction of the total volume of the atom.

Thus, the shortcomings of Thomson's model were removed with the discovery of nucleus.

ILLUSTRATION 6.1

The number of particles scattered at 60° is 100 per minute in an α -particle scattering experiment, using gold foil. Calculate the number of particles per minute scattered at 90° angle.

Sol. Number of α -particles scattered at an angle θ ,

$$N \propto \frac{1}{\sin^4(\theta/2)} \text{ i.e., } N = \frac{K}{\sin^4(\theta/2)},$$

where K is a proportionality constant

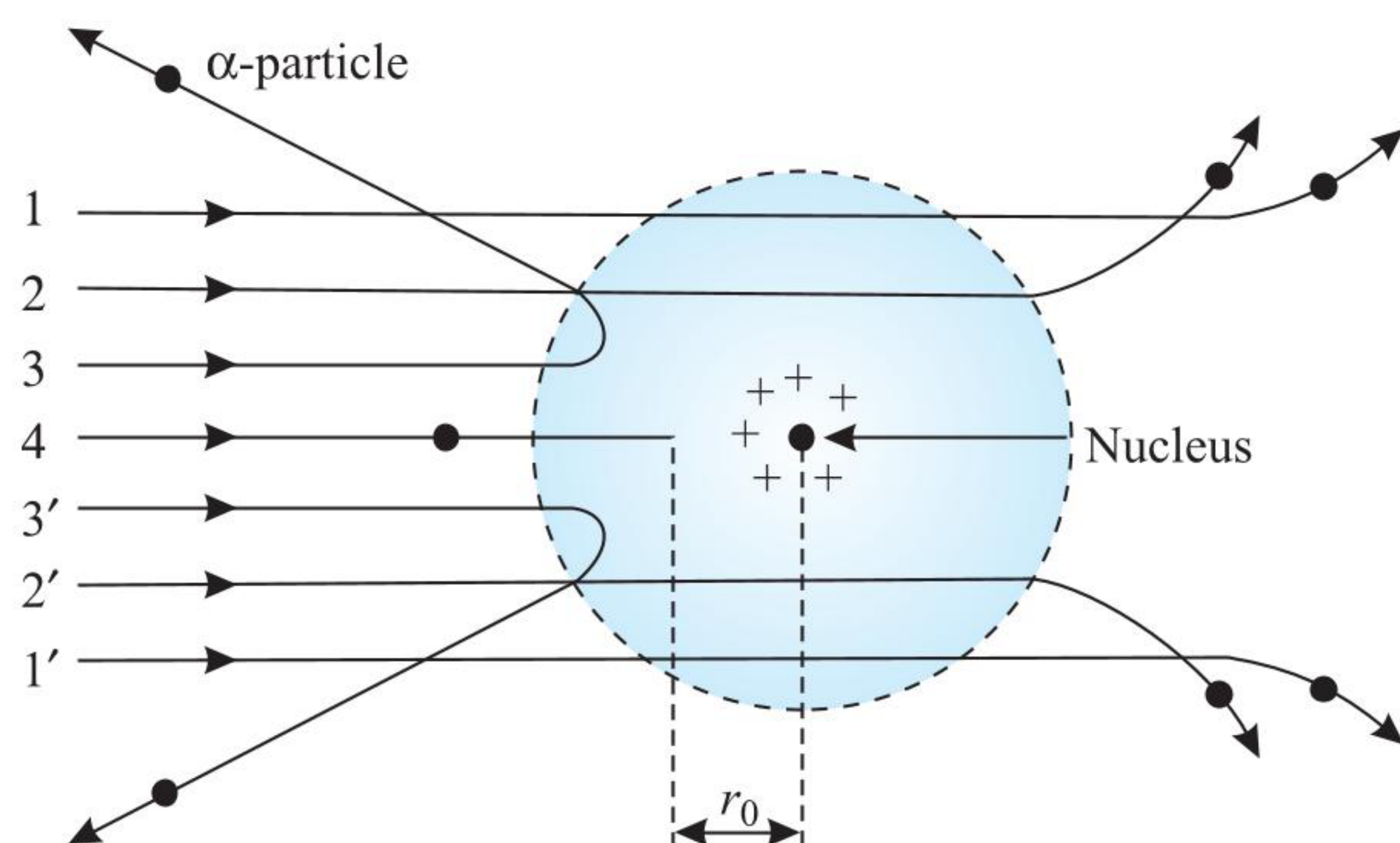
$$\therefore \frac{N_{90^\circ}}{N_{60^\circ}} = \frac{\sin^4(60^\circ/2)}{\sin^4(90^\circ/2)}$$

$$\begin{aligned} \text{or } N_{90^\circ} &= \frac{\sin^4 30^\circ}{\sin^4 45^\circ} \times N_{60^\circ} = \left[\frac{1/2}{1/\sqrt{2}} \right]^4 \times 100 \\ &= \frac{100}{4} = 25 \text{ particle min}^{-1} \end{aligned}$$

Distance of Closest Approach: Estimation of Nuclear Size

The scattering of the α -particles is due to the Coulombic repulsion between the positively charged α -particles and the nuclei. In the

figure α -particles like 1 or 1', passing through the atom at large distance from the nucleus, experiences small repulsion and passes almost undeflected. But the α -particles like (2, 2'), (3, 3') which pass closer to the nucleus, experience large repulsive forces and hence scatter through large angles. Very rarely, an α -particle like 4 travels head-on towards the nucleus. At a certain distance r_0 from the nucleus, the strong repulsive force slows down the α -particle, which is finally stopped and then repelled back along its original path, i.e., it is scattered through an angle of 180° . The distance r_0 is called the **distance of closest approach**. At this distance r_0 , the entire initial kinetic energy of the α -particle gets converted into electrostatic potential energy.



Let an α -particle (of charge $+2e$) is moving with velocity v is directed towards a nucleus (of charge $+Ze$), here Z is the atomic number of foil atoms.

At the distance (r_0) of nearest approach, the kinetic energy of the alpha particle is completely converted into the potential energy of the system.

Electrostatic PE of α -particle and nucleus at distance r_0 ,

$$U = k \frac{q_1 q_2}{r_0} = k \frac{2e \cdot Ze}{r_0} = k \frac{2Ze^2}{r_0}$$

By conservation of energy, $K_\alpha = U$

$$K_\alpha = k \frac{2Ze^2}{r_0}$$

$$\text{or } r_0 = k \frac{2Ze^2}{K_\alpha} \quad \dots(i)$$

$$\text{where } k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2\text{C}^{-2}$$

It is observed that in case of gold nucleus ($Z = 79$), head-on collision takes place when the kinetic energy of the α -particle is 5.5 MeV, i.e., when

$$K = 5.5 \text{ MeV} = 5.5(1.6 \times 10^{-13} \text{ J}) = 8.8 \times 10^{-13} \text{ J}$$

$$\begin{aligned} \text{Clearly, from Eq. (i), } r_0 &= (9 \times 10^9) \times \frac{2(79)(1.6 \times 10^{-19})^2}{8.8 \times 10^{-13}} \\ &= 4.13 \times 10^{-14} \text{ m} = 41.3 \text{ fm} \end{aligned}$$

This distance is considerably larger than the sum of the radii of the gold nucleus and the α -particle. The radius of α nucleus is of the order of a fermi, where 1 fermi (fm) = 10^{-15} m.

Impact Parameter

The trajectory traced by an α -particle depends upon the impact parameter of collision.

Impact parameter is defined as the perpendicular distance of the velocity vector of α -particle from the centre of the nucleus (when α -particle is far away from the atom).

It is denoted by b as shown in figure.

Angle of scattering (θ) is defined as the angle between the direction of approach and the direction of recede of the α -particle. It is denoted by θ . Rutherford calculated that

$$\cot(\theta/2) = \frac{2b}{r_0} \quad \dots(i)$$

$$\text{where } r_0 = k_e \frac{2Ze^2}{K}$$

$$\text{Thus, } \cot(\theta/2) = \frac{2b}{k_e \frac{2Ze^2}{K}} = \frac{4\pi\epsilon_0 K}{Ze^2} b$$

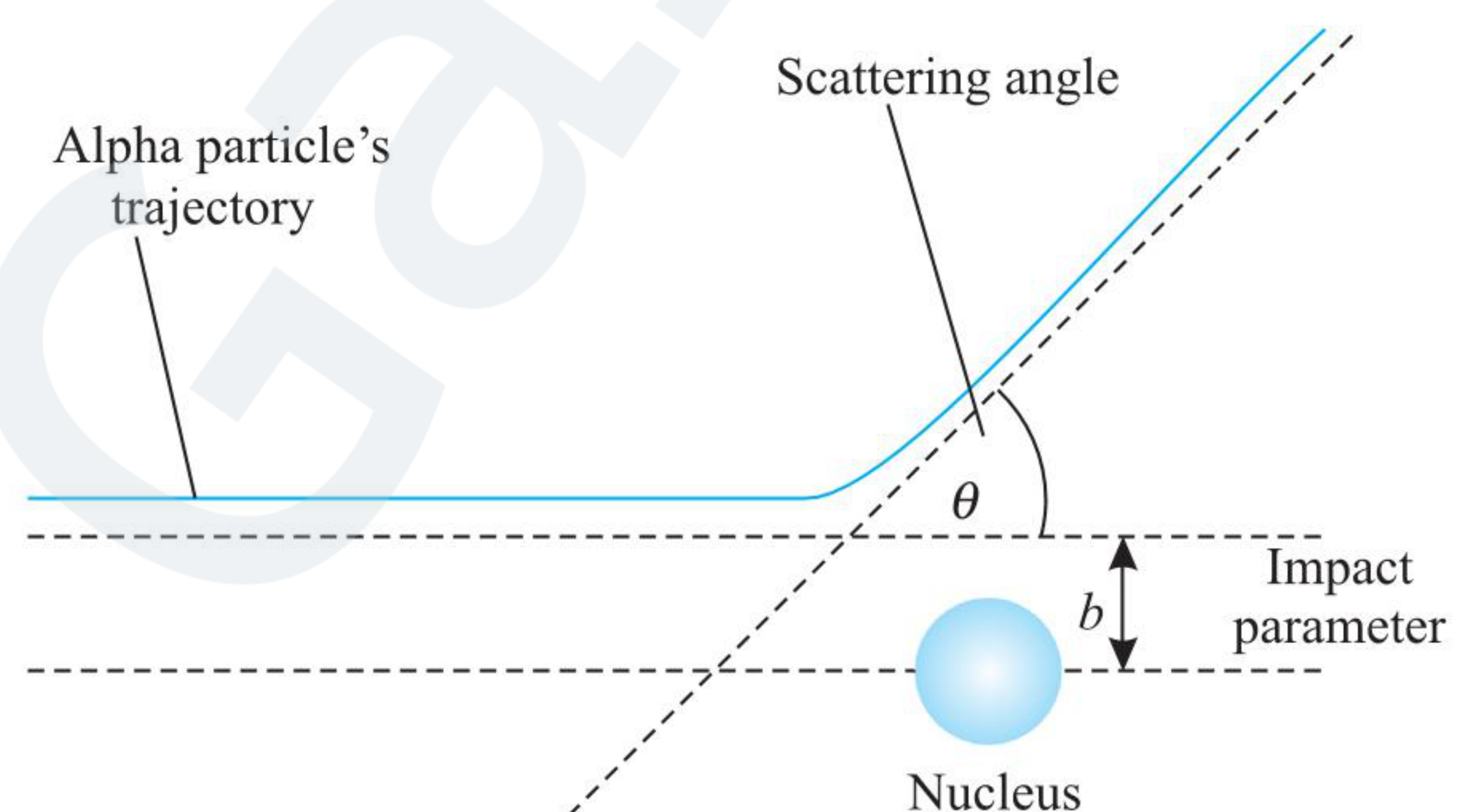


ILLUSTRATION 6.2

In a Geiger-Marsden experiment, what is the distance of closest approach to the nucleus of a 7.7 MeV α -particle before it comes momentarily to rest and reverses its direction?

Sol. Here $K = 7.7 \text{ MeV} = 7.7 \times 1.6 \times 10^{-13} \text{ J}$, $Z(\text{Au}) = 79$

At the distance of closed approach of an α -particle,

$$K = \frac{1}{2}mv^2 = \frac{1}{4\pi\epsilon_0} \frac{Ze \cdot 2e}{r_0} \quad [q_1 = Ze, q_2 = 2e]$$

$$\begin{aligned} \therefore r_0 &= \frac{1}{4\pi\epsilon_0} \frac{2Ze^2}{K} \\ &= \frac{9 \times 10^9 \times 2 \times 79 \times (1.6 \times 10^{-19})^2}{7.7 \times 1.6 \times 10^{-13}} \text{ m} \\ &= 29.5 \times 10^{-15} \text{ m} \approx 30 \text{ fm} \quad [1 \text{ fm} = 10^{-15} \text{ m}] \end{aligned}$$

The radius of gold nucleus is less than 30 fm. In fact, the actual radius of gold nucleus 6 fm.

ILLUSTRATION 6.3

An α -particle after passing through a potential difference of $2 \times 10^6 \text{ V}$ falls on a silver foil. The atomic number of silver is 47. Calculate (i) the kinetic energy of the α -particle at the time of falling on the foil, (ii) the kinetic energy of the α -particle at a distance of $5 \times 10^{-14} \text{ m}$ from the nucleus and (iii) the shortest distance from the nucleus of silver to which the α -particle reaches.

Sol.

- (i) Charge on
- α
- particle,
- $q = 2e$
- ,

$$V = 2 \times 10^6 \text{ V}$$

KE of α -particle,

$$K = qV = 2 \times 1.6 \times 10^{-19} \times 2 \times 10^6 = 6.4 \times 10^{-13} \text{ J}$$

- (ii) Charge on silver nucleus =
- $Ze = 47e$

PE of the α -particle at a distance of $5 \times 10^{-14} \text{ m}$ from the silver nucleus

$$\begin{aligned} U &= \frac{1}{4\pi\epsilon_0} \cdot \frac{47e \times 2e}{5 \times 10^{-14}} \\ &= \frac{9 \times 10^9 \times 94 \times (1.6 \times 10^{-19})^2}{5 \times 10^{-14}} = 4.3 \times 10^{-13} \text{ J} \end{aligned}$$

So, $4.3 \times 10^{-13} \text{ J}$ of KE gets converted into PE $\therefore$ KE of the α -particle at a distance of $5 \times 10^{-14} \text{ m}$ from the silver nucleus = $6.4 \times 10^{-13} - 4.3 \times 10^{-13} = 2.1 \times 10^{-13} \text{ J}$

- (iii) Distance of closest approach,

$$\begin{aligned} r_0 &= \frac{2kZe^2}{E} = \frac{2 \times 9 \times 10^9 \times 47 \times (1.6 \times 10^{-19})^2}{6.4 \times 10^{-13}} \\ &= 3.4 \times 10^{-14} \text{ m} \end{aligned}$$

ILLUSTRATION 6.4

For scattering by an 'inverse-square' field (such as that produced by a charged nucleus in Rutherford's model) the relation between impact parameter b and the scattering angle θ is given by

$$b = \frac{Ze^2 \cot \theta / 2}{4\pi\epsilon_0 \left(\frac{1}{2}mv^2 \right)}$$

- (a) What is the scattering angle for $b = 0$?
- (b) For a given impact parameter b , does the angle of deflection increase or decrease with increase in energy?
- (c) What is the impact parameter at which the scattering angle is 90° for $Z = 79$ and initial energy equal to 10 MeV?
- (d) Why is it that the mass of the nucleus does not enter the formula above but its charge does?
- (e) For a given energy of the projectile, does the scattering angle increase or decrease with decrease in impact parameter?

Sol. In Rutherford's model, the impact parameter is given by

$$b = \frac{Ze^2 \cot \theta / 2}{4\pi\epsilon_0 \left(\frac{1}{2}mv^2 \right)}$$

- (a) Given
- $b = 0$

$$\therefore \frac{Ze^2 \cot \theta / 2}{4\pi\epsilon_0 \left(\frac{1}{2}mv^2 \right)} = 0$$

$$\text{or } \cot \frac{\theta}{2} = 0 \quad [\because \text{All other quantities are finite}]$$

$$\frac{\theta}{2} = 90^\circ$$

$$\text{or } \theta = 180^\circ$$

which is the value expected physically for a head-on collision.

- (b) For a given value of
- b
- ,
- $\frac{Ze^2 \cot \theta / 2}{4\pi\epsilon_0 \left(\frac{1}{2}mv^2 \right)} = \text{constant}$

$\therefore$ As the energy $\left(\frac{1}{2}mv^2 \right)$ increases, the value of $\cot \frac{\theta}{2}$ increases and hence the value of scattering angle θ decreases, as expected.

- (c) Given
- $\theta = 90^\circ$
- ,
- $Z = 79$
- ,
- $e = 1.6 \times 10^{-19} \text{ C}$

$$E = \frac{1}{2}mv^2 = 10 \text{ MeV}$$

$$= 10 \times 10^6 \times 1.6 \times 10^{-19} \text{ J}$$

$$= 1.6 \times 10^{-12} \text{ J}$$

$$\therefore b = \frac{Ze^2 \cot \theta / 2}{4\pi\epsilon_0 \left(\frac{1}{2}mv^2 \right)}$$

$$= \frac{9 \times 10^9 \times 79 \times (1.6 \times 10^{-19})^2 \cot 45^\circ}{1.6 \times 10^{-12}} \text{ m}$$

$$= 9 \times 79 \times 1.6 \times 10^{-11} \text{ m}$$

$$= 11376 \times 10^{-11} \text{ m} \simeq 1.1 \times 10^{-10} \text{ m}$$

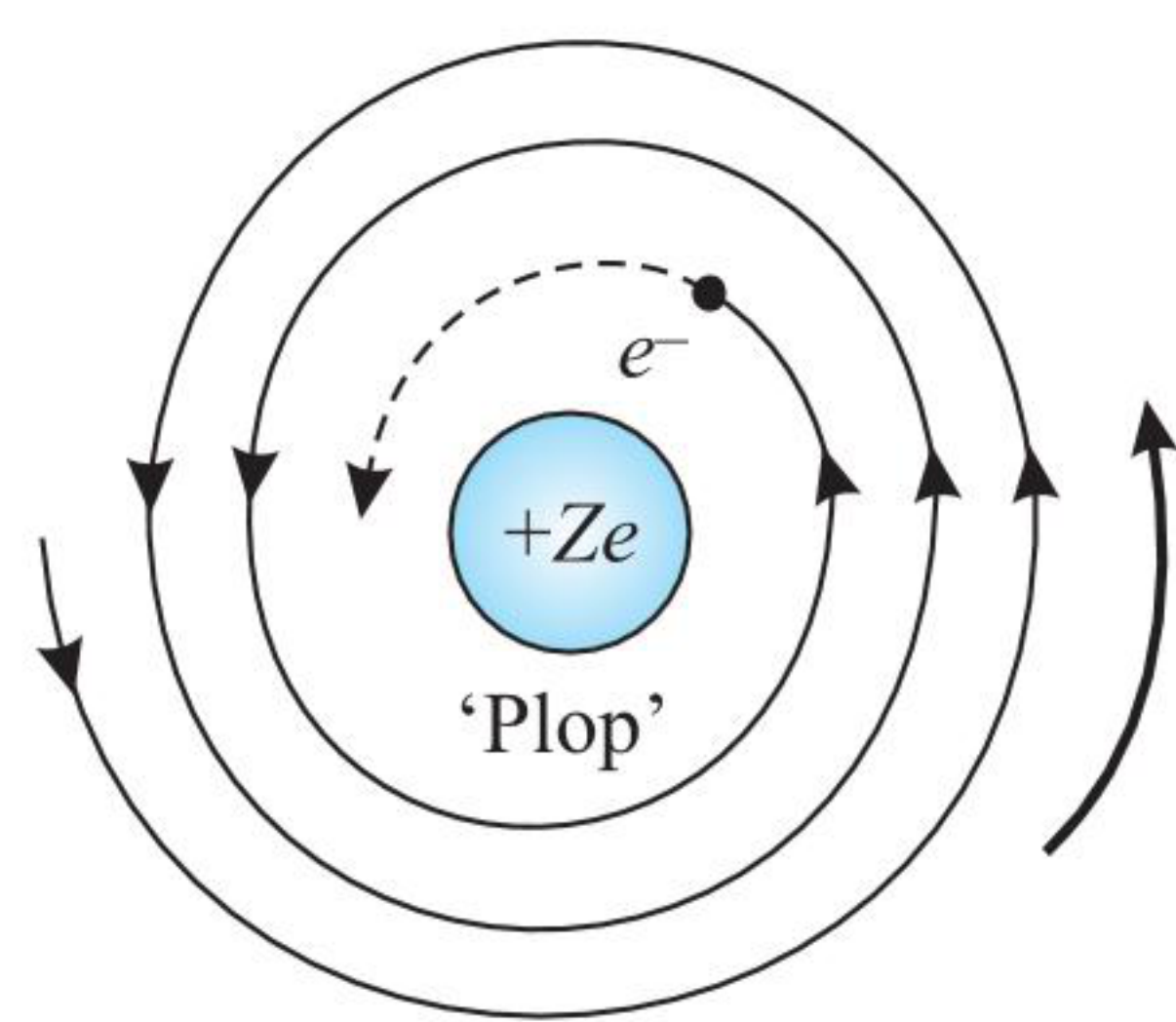
- (d) It is the charge on the nucleus which provides the electrostatic field and due to which scattering of α -particles occurs. If $Z = 0$, then from given formula we have, $\theta = 0^\circ$. This means that scattering does not occur when nucleus carries no charge. Mass of nucleus does not appear in the expression for b , because recoil of the nucleus is being ignored, i.e., the nucleus is assumed to be at rest during its interaction with the α -particle.

- (e) For a given energy $\left(\frac{1}{2}mv^2 \right)$ of the projectile, the decrease in impact parameter b implies a decrease in the value of $\cot \theta/2$ and hence an increase in the scattering angle θ .

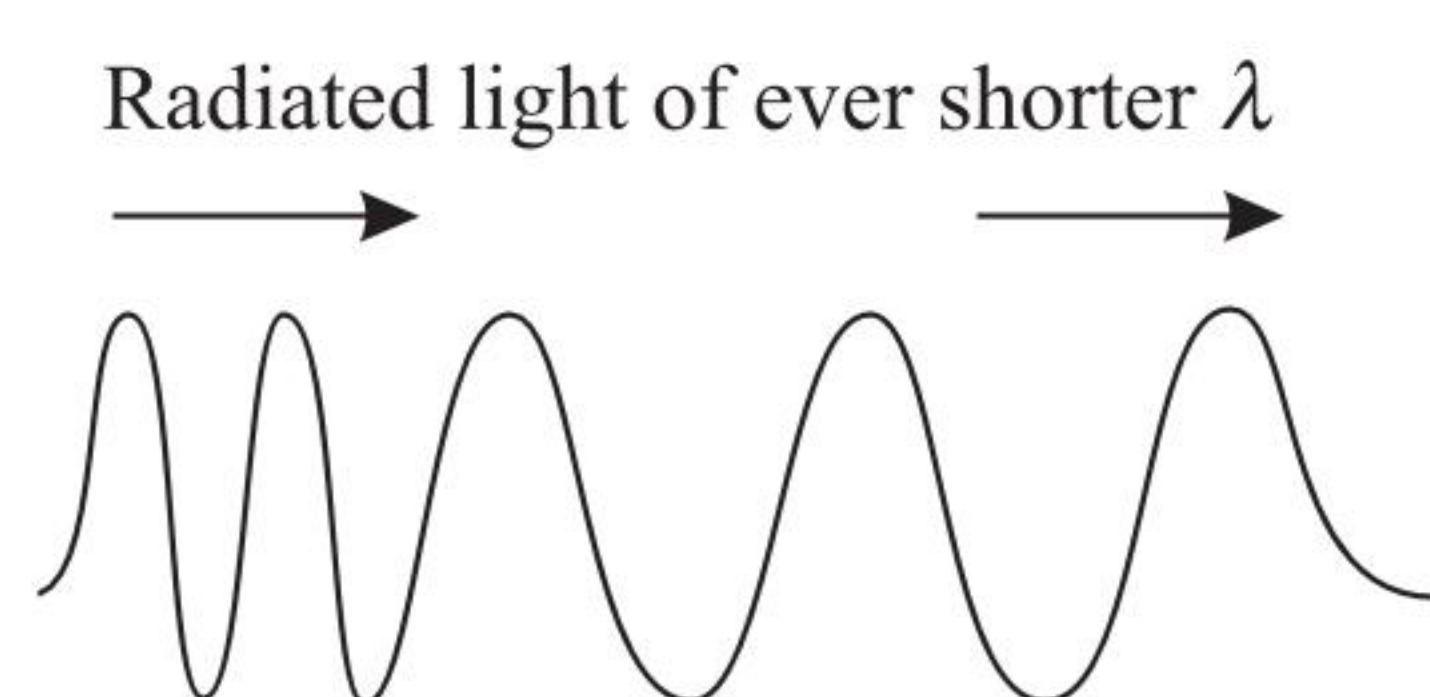
Drawbacks of Rutherford's Atomic Model

The Rutherford's atomic model in spite of strong experimental support, has the following drawbacks.

- (i) According to electromagnetic theory, an accelerated charged particle always radiates energy. The negatively charged electron revolving around the nucleus possesses centripetal acceleration and would lose its energy continuously. The radius of its orbit, therefore, would go on decreasing and it would finally spiral into the nucleus, resulting in an ultimate catastrophic collapse of the atom (called "plop") as shown in figure. But in practice, atoms do not collapse.



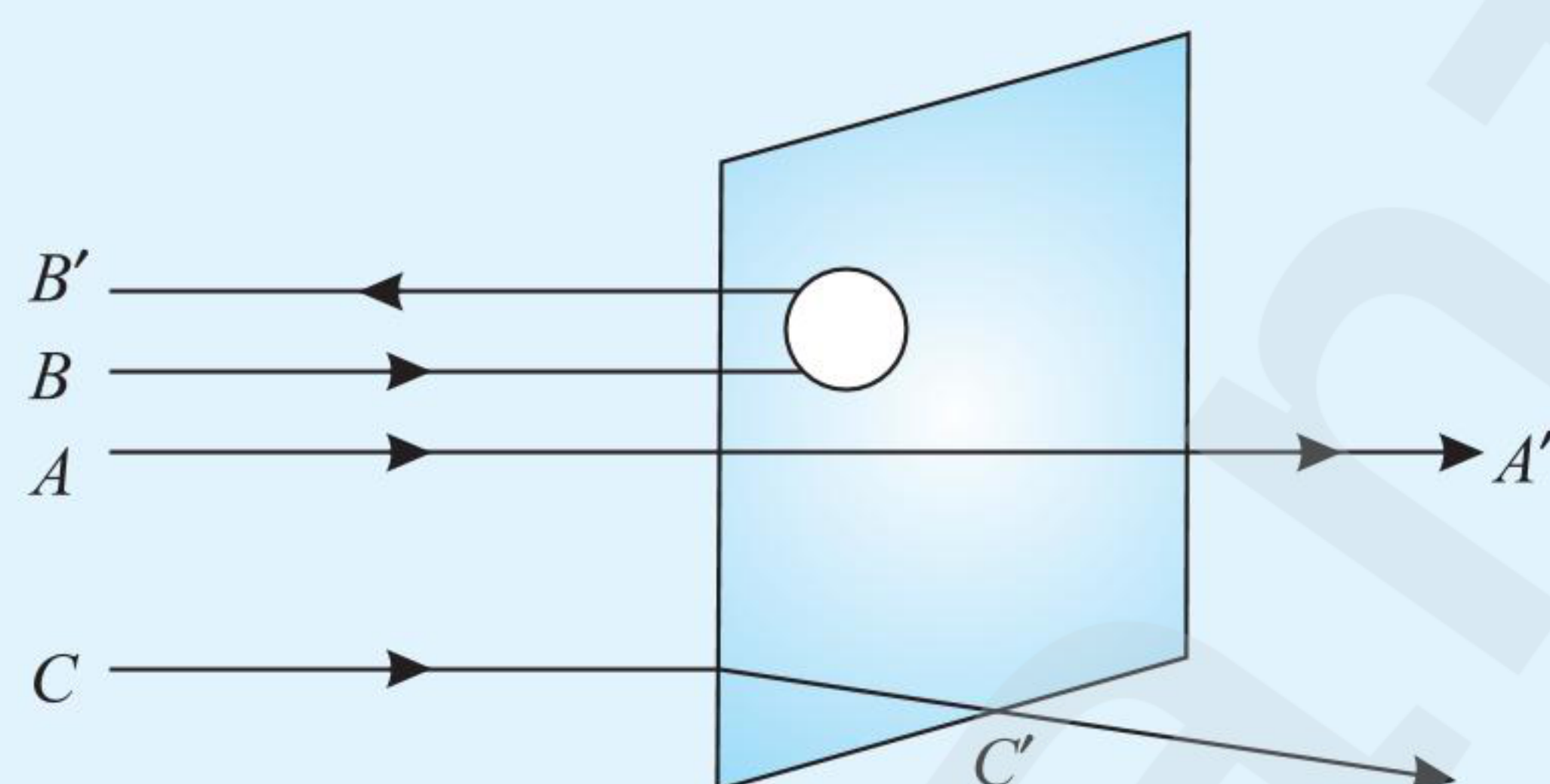
- (ii) As an electron can revolve in any orbit, according to em theory, it must emit continuous radiations of all wavelengths, i.e., (frequencies) as shown in the figure. Actually, the elements are found to emit spectral lines of definite frequencies and not all of them.



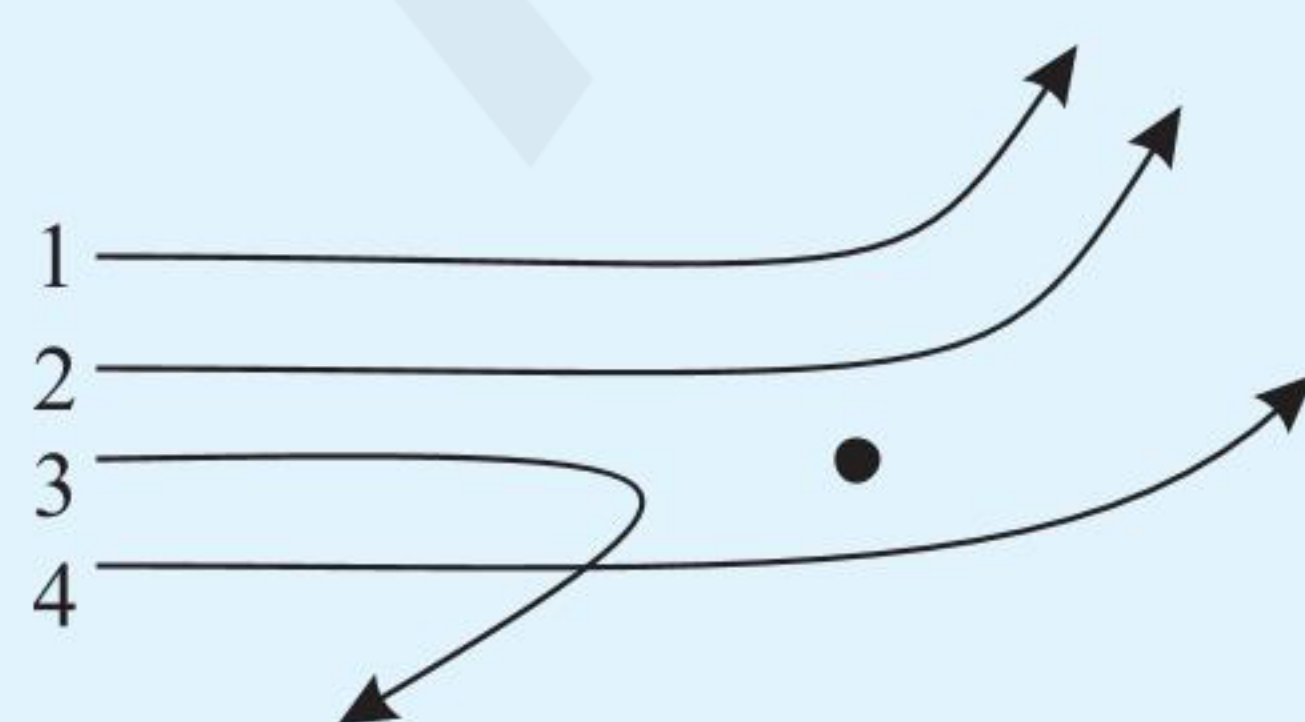
To account for the drawbacks of Rutherford atom model, Bohr suggested another model which has come to be known as Bohr's atomic model.

CONCEPT APPLICATION EXERCISE 6.1

1. A beam of fast moving alpha particles were directed towards a thin film of gold. The parts A' , B' and C' of the transmitted and reflected beams corresponding to the incident parts A , B and C of the beam, are shown in the adjoining diagram. The number of alpha particles in



- (a) B' will be minimum and in C' maximum
 (b) A' will be maximum and in B' minimum
 (c) A' will be minimum and in B' maximum
 (d) C' will be minimum and in B' maximum
2. The diagram shows the path of four α -particles of the same energy being scattered by the nucleus of an atom simultaneously. Which of these are/is not physically possible?



- (a) 3 and 4
 (b) 2 and 3
 (c) 1 and 4
 (d) 4 only

3. If scattering particles are 56 for 90° angle then this will be at 60° angle
 (a) 224 (b) 256
 (c) 98 (d) 108
4. An α -particle of 5 MeV energy strikes with a nucleus of uranium at stationary at an scattering angle of 180° . The nearest distance up to which α -particle reaches the nucleus will be of the order of
 (a) $1 \text{ \AA}$ (b) 10^{-10} cm
 (c) 10^{-12} cm (d) 10^{-15} cm
5. What is the distance of closest approach when a 5.0 MeV proton approaches a gold nucleus?
6. In a head-on collision between an α -particle and a gold nucleus, the distance of closest approach is 41.3 fermi. Calculate the energy of the particle. (1 fermi = 10^{-15} m).

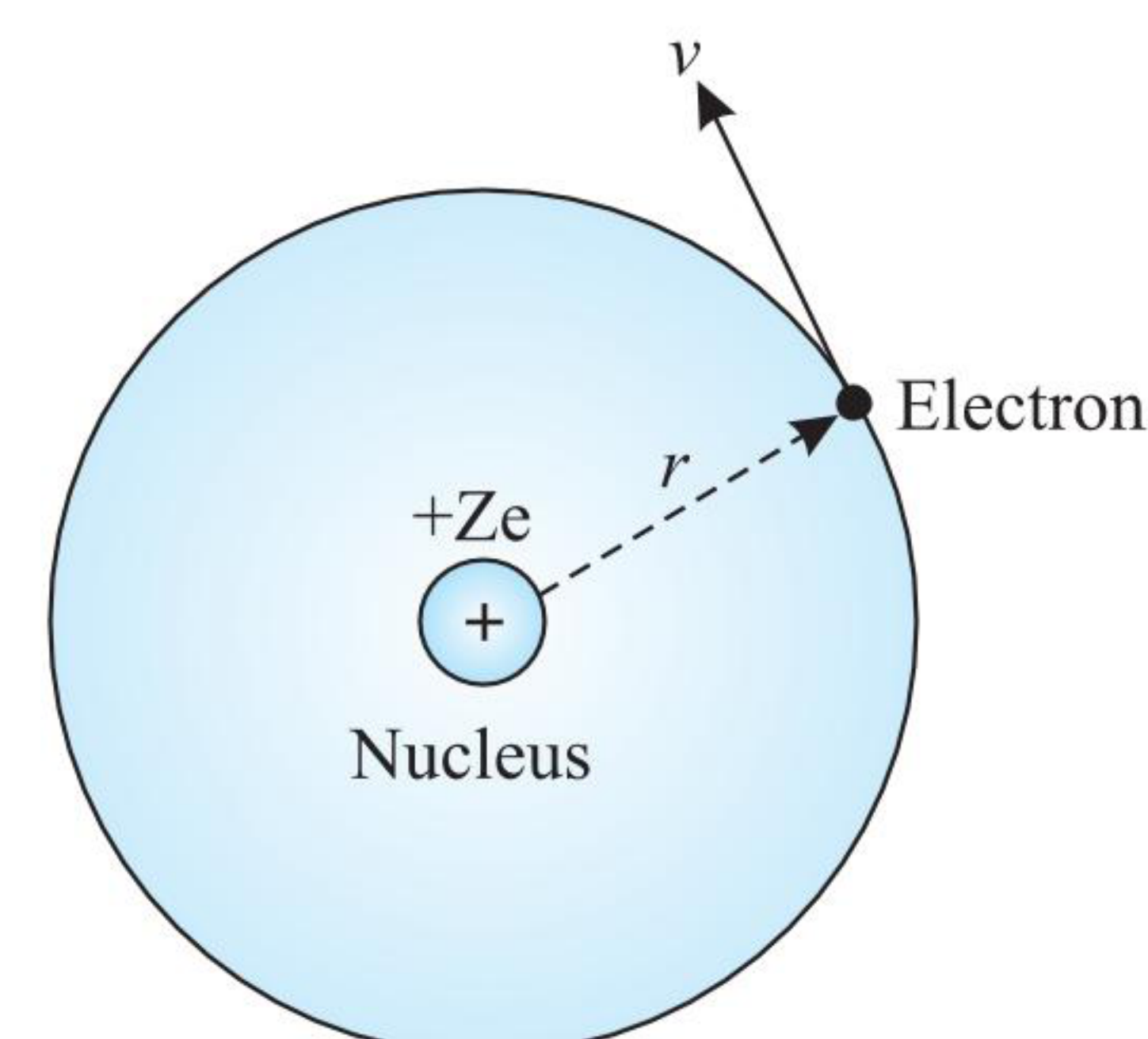
ANSWERS

1. (b) 2. (d) 3. (a) 4. (c)
 5. $2.3 \times 10^{-14} \text{ m}$ 6. 5.51 MeV

BOHR MODEL OF THE HYDROGEN ATOM

In 1913, Niels Bohr explained the hydrogen atom spectrum by applying the quantum theory of radiation to Rutherford's atomic model. Bohr's theory begins with Rutherford's picture of an atom as a nucleus surrounded by electrons moving in circular orbits, the necessary centripetal force to electrons is provided by the electrostatic force of attraction between nucleus and electrons. In analyzing this picture, Bohr made a number of assumptions in order to combine the new quantum ideas of Planck and Einstein with the traditional description of a particle in uniform circular motion. Thus, with the introduction of quantum concept, the entire picture of atom became complete. Bohr's theory is based on the following three postulates.

Postulate 1: There is a positively charged nucleus at the centre of the atom around which the electron revolves in circular orbits. The necessary centripetal force is provided by the Coulomb's force of attraction exerted by the positively charged nucleus on the negatively charged electron.



$$\text{i.e. } \frac{1}{4\pi\epsilon_0} \frac{(Ze)e}{r^2} = \frac{mv^2}{r} \quad \dots(i)$$

ϵ_0 = Absolute permittivity of free space
 $= 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$
 m = Mass of electron

v = Velocity (linear) of electron

r = Radius of the orbit in which electron is revolving.

Z = Atomic number of hydrogen like atom. For hydrogen $Z=1$.

Postulate 2: The electron moves in certain discrete orbits, called the stationary orbits, for which the total angular momentum of the moving electron is an integral multiple of $h/2\pi$, h being the Planck's constant.

The magnitude of the angular momentum is $L = mvr$ for a particle with mass m moving with speed v in a circle of radius r . So, according to Bohr's postulate,

$$mvr = \frac{nh}{2\pi} \quad (n = 1, 2, 3, \dots) \quad \dots(ii)$$

where $n = 1, 2, 3, \dots$ and is called the **principal quantum number**.

Equation (ii) is known as the **quantum condition** and it limits the number of permissible orbits.

Adopting Planck's idea of quantized energy levels, Bohr hypothesized that in a hydrogen atom there can be only certain values of the total energy (electron kinetic energy plus potential energy) for the electrons. These allowed energy levels correspond to different orbits for the electron as it moves around the nucleus, the larger orbits being associated with larger total energies. In addition, Bohr assumed that an electron in one of these orbits does not radiate electromagnetic wave. For this reason, the orbits are called stationary orbits or stationary states. Bohr recognized that radiation less orbits violated the laws of physics. But the assumption of such orbits was necessary because the traditional laws indicated that an electron radiates electromagnetic waves as it accelerates around a circular path, and the loss of the energy carried by the waves would lead to the collapse of the orbit.

Postulate 3: Electrons in outer orbits have greater energy than those in inner orbits. The orbiting electron emits energy when it jumps from an outer orbit (higher energy states, E_i) to an inner orbit (lower energy states E_f), it radiates energy as a single photon of frequency f .

$$E_i - E_f = hf \quad \dots(iii)$$

Here a question arises, how do electrons get into the higher energy orbits? They get there by picking up energy when atoms collide, which happens more often when a gas is heated, or by acquiring energy when a high voltage is applied to a gas.

Radius of Orbit

We have

$$\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{(Ze)(e)}{r^2} \quad \dots(i)$$

$$\text{and } mvr = \frac{nh}{2\pi} \quad \dots(ii)$$

$$\text{From Eq. (ii), } v = \frac{nh}{2\pi mr}$$

Substituting the value of v in Eq. (i), we get

$$r_n = \frac{n^2 h^2 \epsilon_0}{\pi m e^2 Z}$$

$$\text{or } r_n = (0.53) \frac{n^2}{Z} \text{ \AA}$$

$$\text{So, for H-like atoms, } r_n \propto \frac{n^2}{Z}.$$

Stationary orbits are not equally spaced.

Velocity of Electron in n th Orbit

$$\text{Since } v = \frac{nh}{2\pi mr} \quad \text{and} \quad r = \frac{n^2 h^2 \epsilon_0}{\pi m e^2 Z}$$

Put the value of r to get

$$v = \frac{Ze^2}{2\epsilon_0 nh} \quad \text{or} \quad v = \left(\frac{e^2}{2\epsilon_0 ch} \right) \left(\frac{cZ}{n} \right)$$

$$\text{or } v = \alpha \left(\frac{cZ}{n} \right)$$

where $\alpha = e^2/2h\epsilon_0 c$ is Sommerfeld's fine structure constant (a pure number) whose value is $1/137$. Therefore,

$$v = \frac{1}{137} \left(\frac{cZ}{n} \right)$$

i.e., velocity of electron in Bohr's first orbit of hydrogen ($Z=1$) is $c/137$, in second orbit is $c/274$, and so on.

Orbital Frequency of Electron

$$f = \frac{v}{2\pi r}$$

$$\text{Put the value of } v \text{ and } r \text{ to get: } f = \frac{mZ^2 e^4}{4\epsilon_0^2 n^3 h^3} \Rightarrow f \propto \frac{Z^2}{n^3}$$

ILLUSTRATION 6.5

The electron in a hydrogen atom makes a transition $n_1 \rightarrow n_2$, where n_1 and n_2 are the principal quantum numbers of the two energy states. Assume Bohr's model to be valid. The time period of the electron in the initial state is eight times that in the final state. What are the possible values of n_1 and n_2 ?

Sol. Here, $Z=1$. We can derive the expression for time period of electron in n th orbit as

$$T_n = \frac{1}{f_n} \propto n^3$$

$$\text{Hence } \frac{T_1}{T_2} = \frac{n_1^3}{n_2^3}. \text{ As } T_1 = 8T_2, \text{ therefore we get}$$

$$8 = \left(\frac{n_1}{n_2} \right)^3 \Rightarrow n_1 = 2n_2$$

Thus, the possible values of n_1 and n_2 are $n_1 = 2, n_2 = 1$; $n_1 = 4, n_2 = 2$; $n_1 = 6, n_2 = 3$; and so on.

ILLUSTRATION 6.6

How many times does the electron go round the first Bohr orbit of hydrogen atoms in 1 s?

Sol. Here, we have to find the frequency which is given as

$$f = \frac{mZ^2 e^4}{4\epsilon_0^2 n^3 h^3}$$

$$= \frac{9.1 \times 10^{-31} \times 1^2 \times (1.6 \times 10^{-19})^4}{4 \times (8.85 \times 10^{-12})^2 \times 1^3 \times (6.6 \times 10^{-34})^3} = 6.62 \times 10^{15} \text{ Hz}$$

ILLUSTRATION 6.7

What is the angular momentum of an electron in Bohr's hydrogen atom whose energy is -3.4 eV?

Sol. First, we need to identify the quantum number of the energy level.

$$\text{As } E = -\frac{13.6}{n^2}, \text{ we have } -3.4 = -\frac{13.6}{n^2} \quad \text{or} \quad n = 2$$

$$\text{The angular momentum quantization gives, } L = mvr = \frac{nh}{2\pi}$$

$$\text{Substituting } n = 2, \text{ we get } L = \frac{2h}{2\pi} = \frac{h}{\pi}$$

ILLUSTRATION 6.8

If the average life time of an excited state of hydrogen is of the order of 10^{-8} s, estimate how many orbits an electron makes when it is in the state $n = 2$ and before it suffers a transition to state $n = 1$ (Bohr radius $a_0 = 5.3 \times 10^{-11}$ m)?

Sol. The angular momentum of an electron in n th orbit is $nh/2\pi$.

By Bohr's hypothesis: $mvr = nh/2\pi$ where r is the radius of the orbit.

$$\text{or } v = \frac{nh}{2\pi mr} \quad \dots(i)$$

The time period of completing an orbit

$$T = \frac{2\pi r}{v} = \frac{2\pi r(2\pi mr)}{nh}$$

$$\text{or } T = \frac{4\pi^2 mr^2}{nh} \quad \dots(ii)$$

Since the radius of the orbit is proportional to n^2 , hence $r = a_0 n^2$

$$\therefore T = \frac{4\pi^2 m a_0^2 n^4}{nh} = \frac{4\pi^2 m a_0^2 n^3}{h}$$

Number of orbits completed in 10^{-8} s

$$\begin{aligned} &= \frac{10^{-8}}{T} = \frac{10^{-8} \times h}{4\pi^2 m a_0^2 n^3} \\ &= \frac{10^{-8} \times (6.6 \times 10^{-34})}{4(3.14)^2 (9.1 \times 10^{-31})(5.3 \times 10^{-11})^2 (2)^3} = 8 \times 10^6 \end{aligned}$$

Energy of Electron in n th Orbit

Kinetic energy: Since we have

$$\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{(Ze)(e)}{r^2} \quad \text{or} \quad \frac{1}{2}mv^2 = \frac{Ze^2}{8\pi\epsilon_0 r}$$

$$\text{or } K = \frac{Ze^2}{8\pi\epsilon_0 r}$$

$$\text{Potential energy: } U = -\frac{1}{4\pi\epsilon_0} \frac{(Ze)(e)}{r} \quad \text{or} \quad U = -\frac{Ze^2}{4\pi\epsilon_0 r}$$

$$\text{Total energy: } E = K + U = \frac{Ze^2}{8\pi\epsilon_0 r} - \frac{Ze^2}{4\pi\epsilon_0 r}$$

$$\text{or } E = -\frac{Ze^2}{8\pi\epsilon_0 r}$$

$$\text{So we conclude that total energy} = -K = \frac{U}{2}$$

$$\text{Using } r = \frac{n^2 h^2 \epsilon_0}{\pi m e^2 Z}, \text{ we get}$$

$$E = -\left(\frac{me^4}{8h^2\epsilon_0^2}\right) \frac{Z^2}{n^2} = -(13.6) \frac{Z^2}{n^2} \text{ eV}$$

$$\text{Also } E = -\left(\frac{me^4}{8\epsilon_0^2 ch^3}\right) ch \frac{Z^2}{n^2} = -(Rch) \frac{Z^2}{n^2}$$

$$\text{where } R = \text{Rydberg's constant} = \frac{me^4}{8\epsilon_0^2 ch^3} = 1.097 \times 10^7 \text{ m}^{-1}$$

Substituting values of m, e, ϵ_0 , and h with $n = 1$, we get the least energy of the atom in first orbit, which is -13.6 eV.

$Rch = \text{Rydberg's energy} \approx 2.17 \times 10^{-18} \text{ J} \approx 13.6 \text{ eV}$ is the electron energy in first orbit of H atom.

$$\text{Hence, } E_1 = -13.6 \text{ eV} \quad \dots(i)$$

$$\text{and } E_n = \frac{E_1}{n^2} = -\frac{13.6}{n^2} \text{ eV} \quad \dots(ii)$$

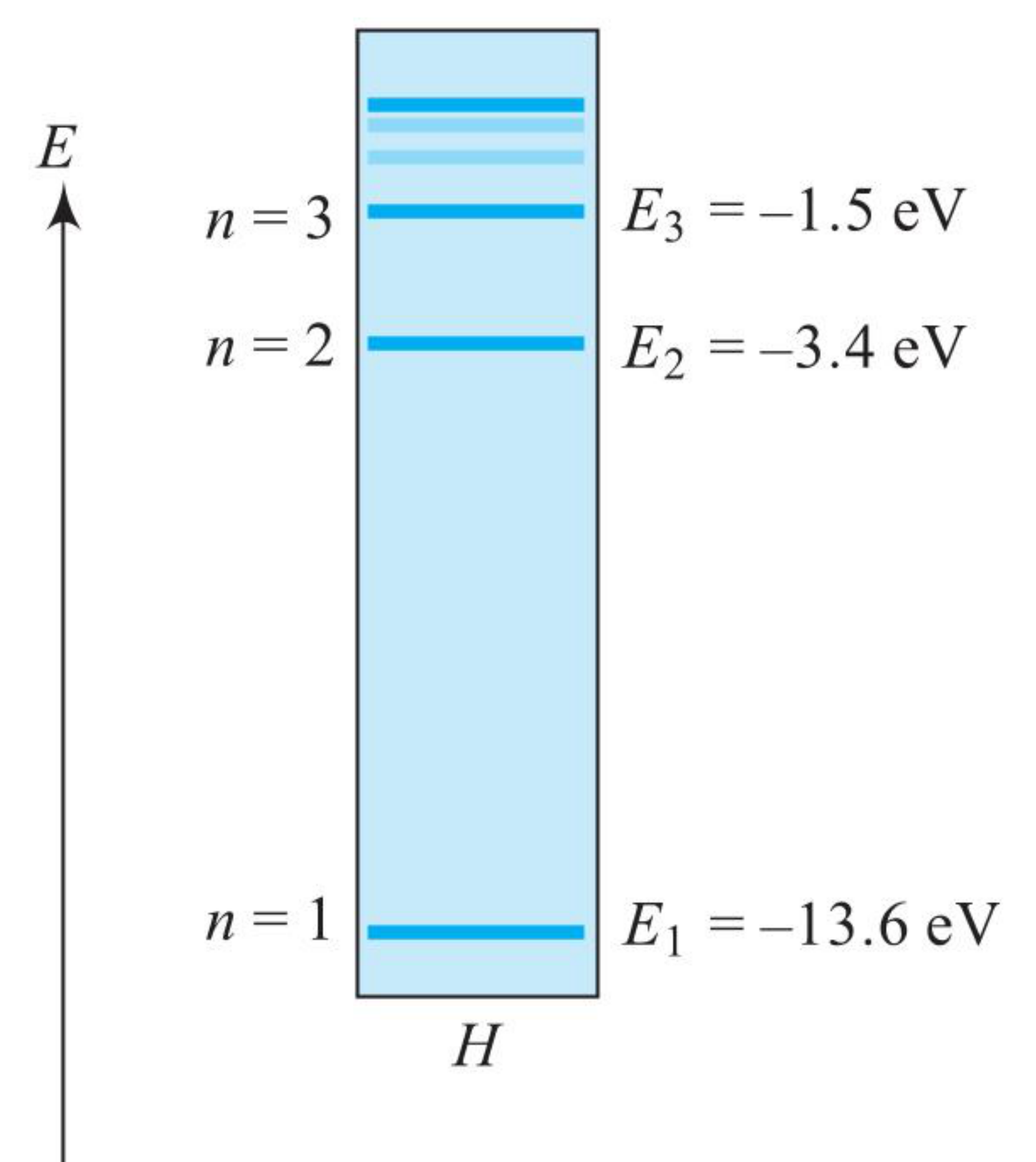
Substituting $n = 2, 3, 4, \dots$, etc., we get energies of atom in different orbits.

The significance of negative sign in Eq. (ii) is that the electron is bound to the nucleus by attractive forces and to separate the electron from the nucleus energy must be supplied to it. Giving different values to n , we can calculate the orbital energy or binding energy of the electron in different orbitals.

$$\begin{aligned} E_1 &= -13.6 \text{ eV} & \text{where } n &= 1 \text{ (K-shell)} \\ E_2 &= -3.4 \text{ eV} & n &= 2 \text{ (L-shell)} \\ E_3 &= -1.5 \text{ eV} & n &= 3 \text{ (M-shell)} \\ &\vdots & & \\ E_\infty &= 0 \text{ eV} & n &= \infty \text{ (Limiting case)} \end{aligned}$$

Now, we shall consider the energy level diagram. This diagram can be drawn in terms of various electron orbits to the scale of their radii but it is customary to draw horizontal lines to the energy scale as shown in figure.

The diagram is known as energy level diagram. The lowest energy level ($n = 1$) corresponds to normal unexcited state of hydrogen. This state is also called as ground state. In energy level diagram, the lower energies (more negative) are at the bottom while higher energies (less negative) are at the top. By such a consideration, the various electron jumps between allowed orbits will be vertical arrows between different energy levels. The energy of radiated photon is greater when the length of arrow is greater.



Frequency of Emitted Radiation

If electron makes a transition (jumps) from final state n_f to the initial state n_i , then frequency of emitted radiation f is given by

$$hf = E_f - E_i \quad \text{or} \quad f = \frac{E_f - E_i}{h} = -Z^2 R c \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

If λ is the wavelength of emitted radiation, then

$$f = \frac{c}{\lambda} = -Z^2 R c \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

So reciprocal of wavelength or wave number ($\bar{\nu}$) is given by

$$\bar{\nu} = \frac{1}{\lambda} = -Z^2 R \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

This relation holds for radiations emitted by hydrogen-like atoms, i.e.,

$$\text{H}(Z=1), \text{He}^+(Z=2), \text{Li}^{++}(Z=3), \text{and Be}^{+++}(Z=4)$$

If the electron makes a transition from $n = 1$ to the higher states, then absorption of radiation takes place.

ILLUSTRATION 6.9

Consider energy level diagram of a hydrogen atom. How will the kinetic energy and potential energy of electron vary if the electron moves from a lower level to a higher level?

Sol. Kinetic energy of an electron in a hydrogen atom is given by

$$K_n = \frac{e^2}{8\pi\epsilon_0 r_n} = \frac{me^4}{8\epsilon_0^2 n^2 h^2} = \frac{K_0}{n^2}, \quad \text{where} \quad K_0 = \frac{me^4}{8\epsilon_0^2 h^2}$$

Potential energy is given by

$$U_n = -\frac{e^2}{4\pi\epsilon_0 r_n} = -\frac{me^4}{4\epsilon_0^2 n^2 h^2} = -\frac{2K_0}{n^2}$$

The kinetic energy is always positive and it is inversely proportional to square of n . Hence, as the electron is excited to higher states, its kinetic energy decreases.

The potential energy is always negative for a particle in bound state. The magnitude of potential energy is twice that of kinetic energy. As the electron is raised to higher energy level, n increases. The potential energy becomes less negative that means the potential energy actually increases. Potential energy is maximum for $n \rightarrow \infty$, which corresponds to infinite separation between nucleus and electron.

BOHR MODEL TO DEFINE HYPOTHETICAL ATOMIC ENERGY LEVELS

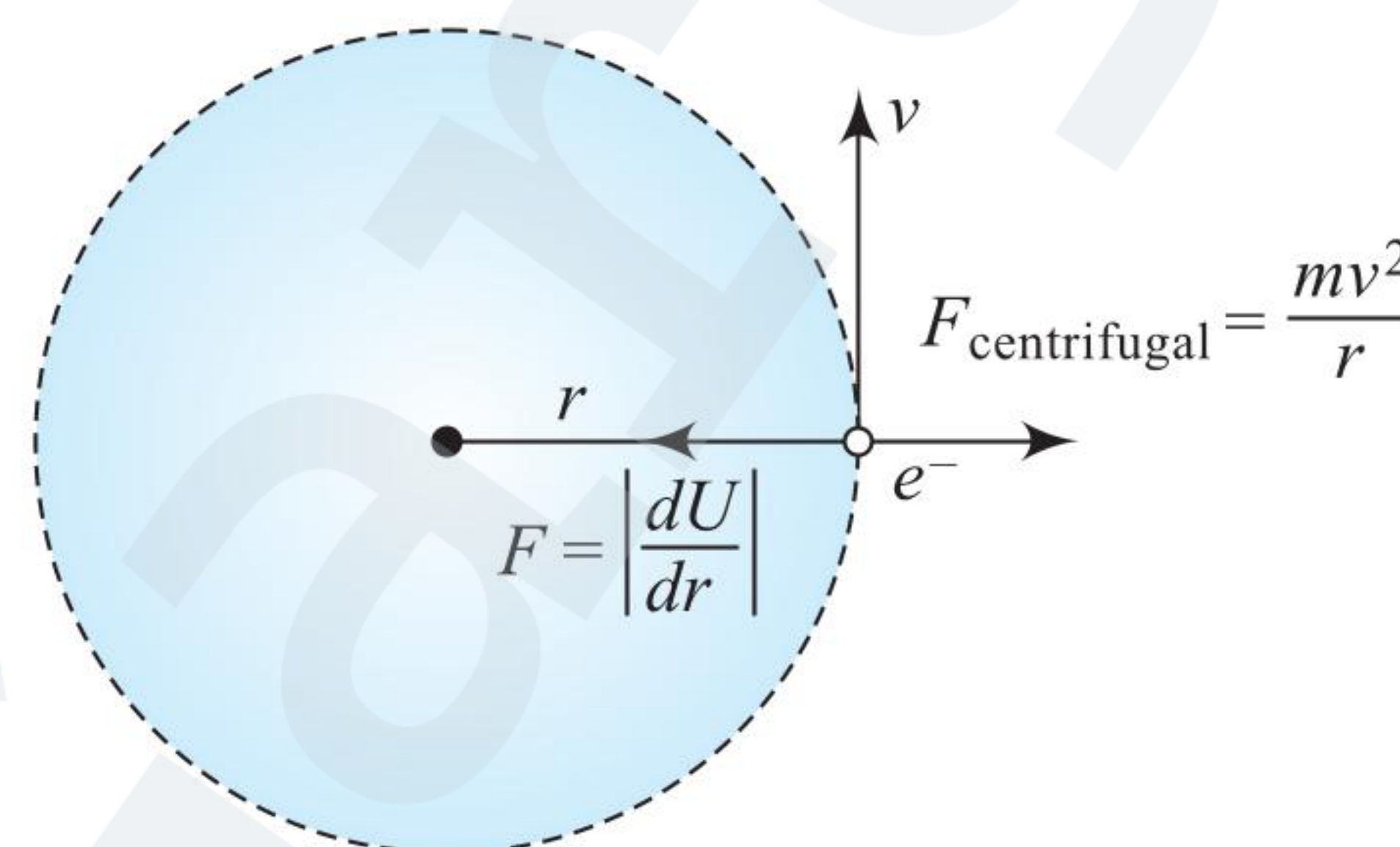
Originally, Bohr's model was to define only different energy levels for hydrogen atom.

According to Bohr's first postulate: $\frac{kZe^2}{r_n^2} = \frac{mv_n^2}{r_n}$... (i)

$$\left[\text{where } k = \frac{1}{4\pi\epsilon_0} \right]$$

and from Bohr's second postulate: $mv_n r_n = \frac{nh}{2\pi}$... (ii)

Equation (ii) gives explanation for balancing the inward Coulombian force on electron by the outward centrifugal force on it in the rotating frame of reference. There may be some situation in which it is given that electron of an atom is revolving under the influence of a hypothetical potential energy field function given as $U = f(r)$ [non-Coulombian field] and using Bohr model, we can develop the properties of the electron in this new atom. For this, first, we will develop the first postulate equation for this new atom. As the electron is orbiting in a new potential energy field given as $U = f(r)$, it will experience an inward force toward the center of orbit as shown in figure.



As we know, $F = -\frac{\partial U}{\partial r}$

The magnitude of force can be written as

$$F = \left| \frac{dU}{dr} \right| = \left| \frac{d}{dr} [f(r)] \right| \quad \dots (iii)$$

Thus, for a stable n^{th} orbit if electron is moving with speed v_n in the orbit of radius r_n , we have

$$\left| \frac{d}{dr} [f(r_n)] \right| = \frac{mv_n^2}{r_n} \quad \dots (iv)$$

Equation (iv) will be the new equation for the first postulate and from Bohr's model the second postulate is based on quantization of angular momentum of electron, its equation will remain same as

$$mv_n r_n = \frac{nh}{2\pi} \quad \dots (v)$$

Now, using Eqs. (iv) and (v), we can derive all the properties for electron motion such as radius of n^{th} orbit, velocity of electron in n^{th} orbit, angular velocity, frequency, time period, current, magnetic induction, magnetic moment, and the total energy of energy levels for this hypothetical atom in the same way we have derived these for properties for a general hydrogenic atom. Let us discuss a few examples to understand these concepts in a better way.

ILLUSTRATION 6.10

Suppose potential energy between electron and proton at separation r is given by $U = k \log r$, where k is a constant. For such a hypothetical hydrogen atom, calculate the radius of n^{th} Bohr's orbit and its energy levels.

Sol. For a conservative force field, $F = -\frac{dU}{dr} = -\frac{k}{r}$

This force $F = -k/r$ provides the centripetal force for the circular motion of electron.

$$\text{So, } \frac{mv^2}{r} = \frac{k}{r} \Rightarrow m^2 v^2 = mk \Rightarrow mv = \sqrt{mk} \quad \dots (i)$$

Applying Bohr's quantization rule, $mvr = \frac{nh}{2\pi}$... (ii)

From Eqs. (i) and (ii), we get $r = \frac{nh}{2\pi\sqrt{mk}}$

From Eq. (i), KE of electron $= \frac{1}{2}mv^2 = \frac{1}{2}k$

Total energy of electron

$$= \text{KE} + \text{PE} = \frac{1}{2}k + k \log r = \frac{k}{2} \left[1 + \log \frac{n^2 h^2}{4\pi^2 mk} \right]$$

HYDROGEN-LIKE ATOMS

We can extend the Bohr model to other one-electron atoms, such as singly ionized helium (He^+), doubly ionized lithium (Li^{2+}), and so on. Such atoms are called **hydrogen-like atoms**.

The Bohr model of hydrogen can be extended to hydrogen-like atoms, i.e., one-electron atoms, the nuclear charge is $+Ze$, where Z is the atomic number, equal to the number of protons in the nucleus.

The effect in the previous analysis is to replace e^2 everywhere by Ze^2 . Thus, the equations for r_n , v_n and E_n are altered as under:

$$r_n = \frac{\epsilon_0 n^2 h^2}{\pi m Ze^2} = a_0 \frac{n^2}{Z} \quad \text{or} \quad r_n \propto \frac{n^2}{Z} \quad \dots (i)$$

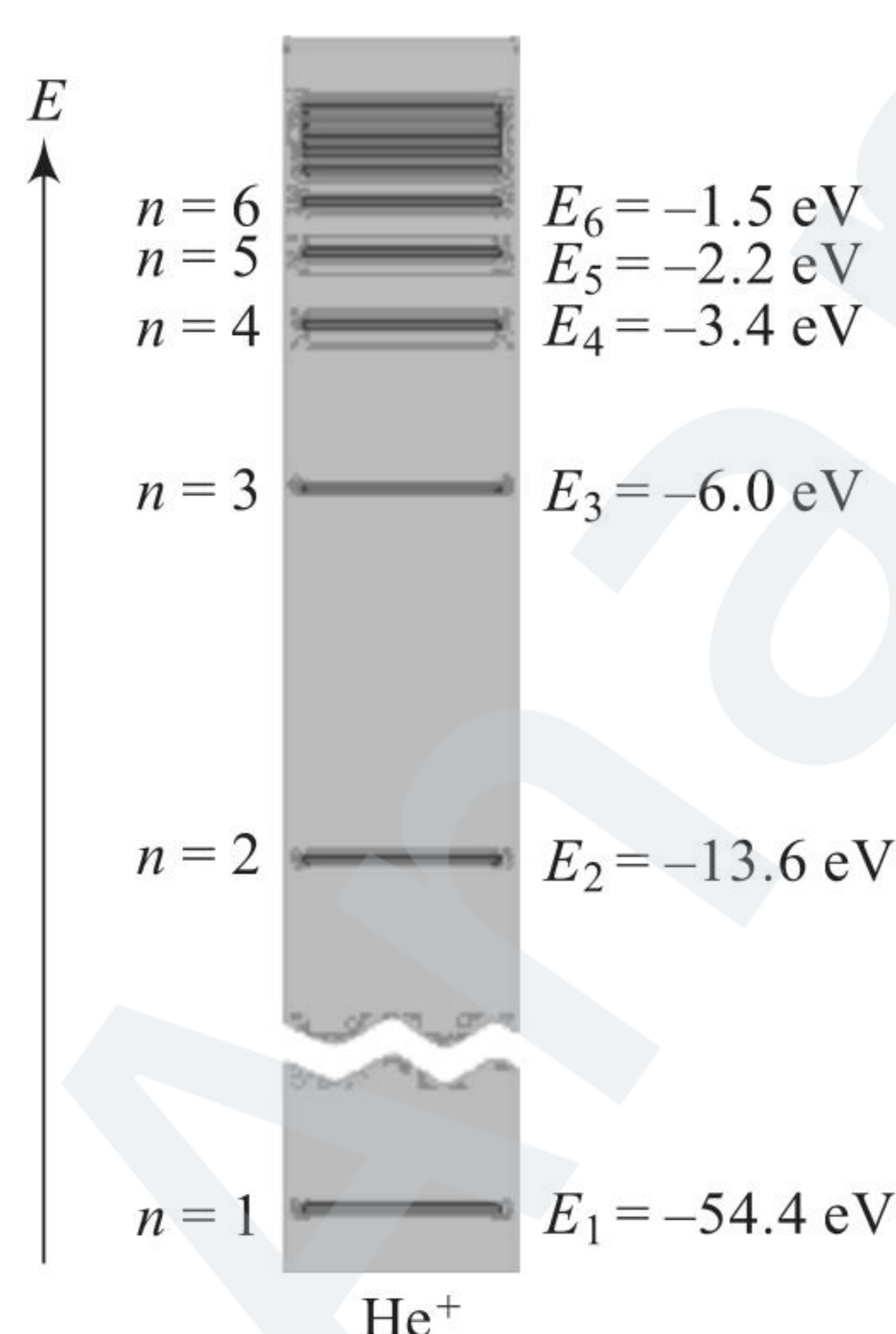
where $a_0 = 0.529 \text{ \AA}$ (radius of first orbit of H)

$$v_n = \frac{Ze^2}{2\epsilon_0 nh} = \frac{Z}{n} v_1 \quad \text{or} \quad v_n \propto \frac{Z}{n} \quad \dots (ii)$$

where $v_1 = 2.19 \times 10^6 \text{ m s}^{-1}$ (speed of electron in first orbit of H)

$$E_n = -\frac{mZ^2 e^4}{8\epsilon_0^2 n^2 h^2} = \frac{Z^2}{n^2} E_1 \quad \text{or} \quad E_n \propto \frac{Z^2}{n^2} \quad \dots (iii)$$

where $E_1 = -13.60 \text{ eV}$ (energy of atom in first orbit of H)



Energy level diagram of He

Important Point:

For single-electron system

Ground state: Lowest energy state of any atom or ion is called ground state of the atom.

Ground state energy of H atom $= -13.6 \text{ eV}$

Ground state energy of He^+ ion $= -54.4 \text{ eV}$

Ground state energy of Li^{++} ion $= -122.4 \text{ eV}$

ILLUSTRATION 6.11

Determine the minimum wavelength of a photon that can cause ionization of He^+ ion.

Sol. The ground state energy of He^+ is given by the equation

$$E_n = -(13.6) \left(\frac{Z^2}{n^2} \right)$$

With $n = 1$ and $Z = 2$, we get $E_1 = -(13.6)(4) = -54.4 \text{ eV}$

Thus, to ionize the He^+ ion, we should require 54.4 eV of energy. The minimum wavelength of photon that can cause ionization will have energy $E = hf = 54.4 \text{ eV}$ and wavelength

$$\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{(54.4)(1.6 \times 10^{-19})} = 22.8 \text{ nm}$$

If the atom absorbed a photon of greater energy (wavelength less than 22.8 nm), the atom could still be ionized and the freed electron would have some kinetic energy when it is away from the influence of He^+ ion.

ILLUSTRATION 6.12

Find the quantum number n corresponding to the excited state of He^+ ion if on transition to the ground state that ion emits two photons in succession with wavelengths 1026.7 and $304 \text{ \AA}$. ($R = 1.096 \times 10^7 \text{ m}^{-1}$)

Sol. Given $\frac{1}{\lambda_1} + \frac{1}{\lambda_2} = RZ^2 \left(1 - \frac{1}{n^2} \right)$

$$\begin{aligned} \frac{1}{n^2} &= 1 - \left[\frac{\lambda_1 + \lambda_2}{\lambda_1 \lambda_2} \times \frac{1}{RZ^2} \right] \\ &= 1 - \frac{1330.7 \times 10^{-10}}{1026.7 \times 304 \times 10^{-20} \times 4 \times 1.096 \times 10^7} \end{aligned}$$

$$\text{Thus, } \frac{1}{n^2} = 0.0275 \Rightarrow n = 6.03$$

Hence, the quantum number $= 6$

ILLUSTRATION 6.13

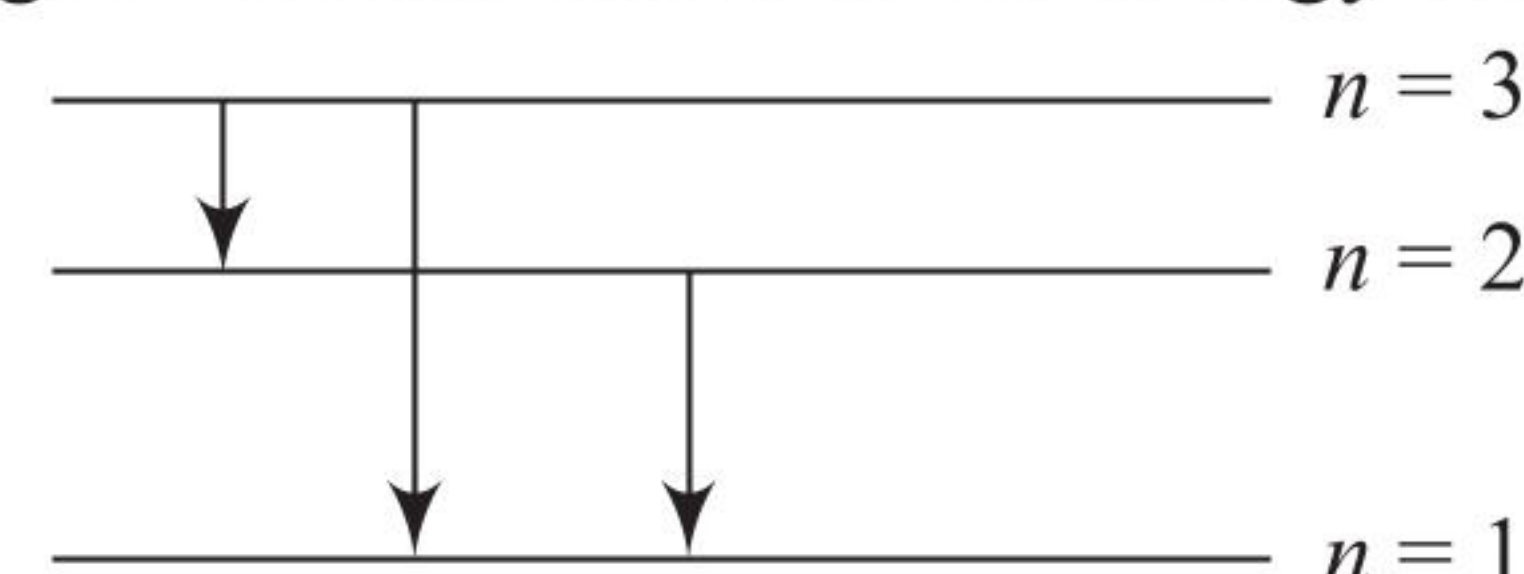
A gas of hydrogen-like atoms can absorb radiations of 68 eV . Consequently, the atoms emit radiations of only three different wavelengths. All the wavelengths are equal to or smaller than that of the absorbed photon.

- Determine the initial state of the gas atoms.
- Identify the gas atoms.
- Find the minimum wavelength of the emitted radiation.
- Find the ionization energy and the respective wavelength for the gas atoms.

Sol.

- Since three radiations are emitted, therefore, the final excited state of the gas is $n = 3$.

The initial state of the gas atoms is $N = 2$ as all the wavelengths are smaller and the energy will be higher.



$$(b) \quad 13.6Z^2 \left[\frac{1}{2^2} - \frac{1}{3^2} \right] = 68$$

$$\text{or } 13.6Z^2 \left[\frac{5}{36} \right] = 68 \Rightarrow Z = 6$$

- (c) The minimum wavelength corresponds to the transition $n = 3$ to $n = 1$

$$\frac{1}{\lambda_{\min}} = RZ^2 \left[\frac{1}{1^2} - \frac{1}{3^2} \right]$$

$$\text{or } \lambda_{\min} = \frac{9}{8RZ^2} = \frac{9}{8(1.097 \times 10^7)(6)^2} = 28.5 \text{ \AA}$$

- (d) The ionization energy of the gas atoms is

$$E = 13.6Z^2 = (13.6)(6)^2 = 489.6 \text{ eV}$$

$$\lambda = \frac{1}{RZ^2} = \frac{1}{(1.097 \times 10^7)(6)^2} = 25.32 \text{ \AA}$$

IONIZATION ENERGY (IE) AND IONIZATION POTENTIAL (IP)

Ionization energy (IE): Total energy zero of a hydrogen atom corresponds to infinite separation between electron and nucleus. Total positive energy implies that the atom is ionized and electron is in unbound (isolated) state moving with certain kinetic energy. Minimum energy required to move an electron from ground state to $n = \infty$ is called ionization energy of the atom or ion.

$$E_{\text{ionization}} = E_{\infty} - E_n = -E_n = \frac{13.6Z^2}{n^2} \text{ eV}$$

Ionization energy of H atom = 13.6 eV

Ionization energy of He^+ ion = 54.4 eV

Ionization energy of Li^{++} ion = 122.4 eV

Ionization potential (IP): Potential difference through which a free electron must be accelerated from rest such that its kinetic energy becomes equal to ionization energy of the atom is called ionization potential of the atom.

$$V_{\text{ionization}} = \frac{E_n}{e} = \frac{13.6Z^2}{n^2} \text{ V}$$

IP of H atom = 13.6 V

IP of He^+ ion = 54.4 V

IP of Li^{++} ion = 122.4 V

EXCITATION ENERGY AND EXCITATION POTENTIAL

Excitation: The process of absorption of energy by an electron so as to raise it from a lower energy level to some higher energy level is called excitation.

Excited state: The states of an atom other than the ground state are called its excited states.

- $n = 2$, first excited state
- $n = 3$, second excited state
- $n = 4$, third excited state
- $n = n_0 + 1$, n_0 th excited state

Excitation energy: Energy required to move an electron from ground state of the atom to any other excited state of the atom is called excitation energy of that state.

$$E_{\text{excitation}} = E_{\text{higher}} - E_{\text{lower}}$$

Energy in ground state of H atom = -13.6 eV

Energy in first excited state of H atom = -3.4 eV

The first excitation energy = $-3.4 - (-13.6) = 10.2 \text{ eV}$.

Excitation potential: Potential difference through which an electron must be accelerated from rest so that its kinetic energy becomes equal to excitation energy of any state is called excitation potential of that state.

$$V_{\text{excitation}} = \frac{E_{\text{excitation}}}{e}$$

First excitation energy = 10.2 eV, so First excitation potential = 10.2 V.

$n = 1$,	$E_1 = -13.6 \text{ eV}$	This is the ground state energy.
$n = 2$,	$E_2 = -3.4 \text{ eV}$	This is the first excited level.
$n = 3$,	$E_3 = -1.51 \text{ eV}$	This is the second excited level.
$\vdots$	$\vdots$	$\vdots$
$n = \infty$,	$E_{\infty} = 0$	The atom is said to be ionized.

BINDING ENERGY OR SEPARATION ENERGY

1. Energy liberated when constituents of a system are brought from infinity to assemble the system. The binding energy is negative of ionization energy.

$$E_{\text{binding}} = E_n$$

2. Energy required to move an electron from any state to $n = \infty$ is called binding energy of that state or energy released during formation of an H-like atom/ion from $n = \infty$ to some particular state n is called binding energy of that state. Binding energy of ground state of H atom is 13.6 eV.

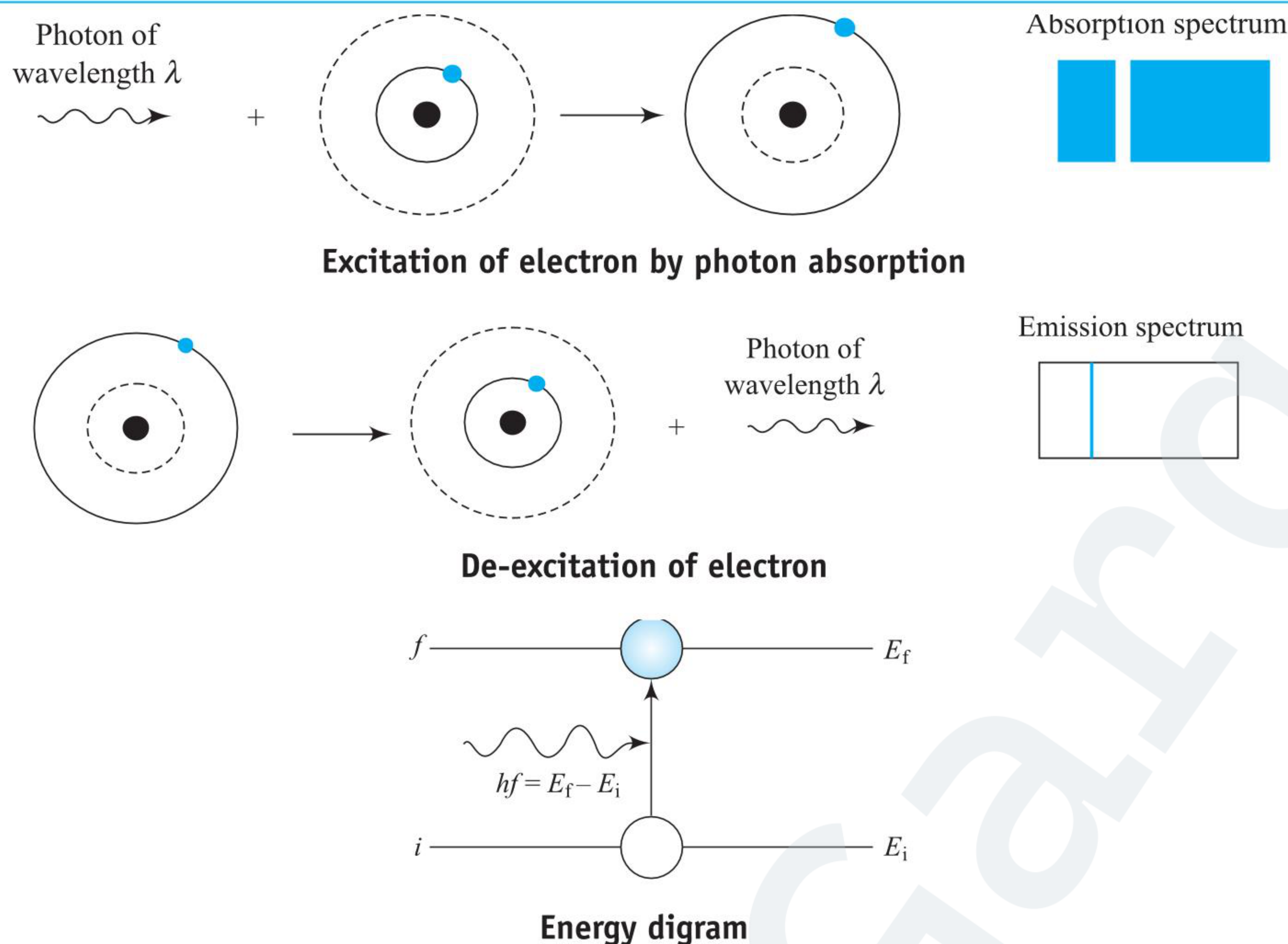
ATOMIC EXCITATION

An atom can be excited to an energy level above its ground state by the following two ways: (1) by photon absorption and (2) by collision.

When an atom absorbs photon, it becomes excited and returns to its ground state in an average of 10^{-6} s by emitting one or more photons (see figure). An atom will not absorb photon of any arbitrary energy. For a photon to be absorbed by an atom, it must have energy equal to difference of energy of ground state and any excited state. For example, when H atom absorbs a photon of wavelength 121.7 nm (10.2 eV), it will bring up H atom from $n = 1$ state to $n = 2$ state. This explains the origin of absorption spectra. On subsequent de-excitation, a photon of wavelength 121.7 nm is emitted when H atom drops from $n = 2$ to $n = 1$ state.

The second type of mechanism for excitation of atoms is by collision. For atomic excitation, collision has to be inelastic and kinetic energy lost in the process is utilized for atomic excitation. Numerical problems based on atomic excitation by collision are worked out using the following principles:

1. Conservation of linear momentum (COLM)
2. Conservation of energy (COE)

**ILLUSTRATION 6.14**

Find the ratio of ionization energy of Bohr's hydrogen atom and doubly ionized lithium ion (Li^{2+}).

Sol. The energy of ground state of Bohr's hydrogen-like atom is $E_n = -(13.6)Z^2$.

The ionization energy is equal in magnitude to energy of ground state; so $E_{\text{ionization}} = (13.6)Z^2$

$$\frac{(E_{\text{ionization}})_{\text{H}}}{(E_{\text{ionization}})_{\text{Li}^{2+}}} = \frac{(Z_{\text{H}})^2}{(Z_{\text{Li}^{2+}})^2} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

ILLUSTRATION 6.15

The Bohr model does not apply when more than one electron orbit the nucleus because it does not account for the electrostatic force that the electrons exert on one another. For instance, an electrically neutral lithium atom (Li) contains three electrons in orbit around a nucleus that includes three protons ($Z = 3$), and Bohr's analysis is not applicable. However, the Bohr model can be used for the doubly charged positive ion of lithium (Li^{2+}) that results when two electrons are removed from the neutral atom, leaving only one electron to orbit the nucleus. Obtain the ionization energy that is needed to remove the remaining electron from Li^{2+} .

Sol. The lithium ion Li^{2+} contains three times the positive nuclear charge as that of the hydrogen atom. Therefore, the orbiting electron is attracted more strongly to the nucleus in Li^{2+} than in the hydrogen atom. As a result, we expect that more energy is required to ionize Li^{2+} than the 13.6 eV required for atomic hydrogen.

The Bohr energy levels for Li^{2+} are given by $E_n = -(13.6 \text{ eV}) \frac{Z^2}{n^2}$
 With $Z = 3$ and $n = 1$, $E_1 = -(13.6 \text{ eV}) \frac{3^2}{1^2} = -122.4 \text{ eV}$

To remove the electron from Li^{2+} , 122.4 eV of energy must be supplied. So the ionization energy for Li^{2+} is 122.4 eV.

ILLUSTRATION 6.16

The energy levels of a hypothetical one electron atom are given by

$$E_n = -\frac{18.0}{n^2} \text{ eV}, \quad \text{where } n = 1, 2, 3, \dots$$

- Compute the four lowest energy levels and construct the energy level diagram.
- What is the excitation potential of the state $n = 2$?
- What wavelengths ($\text{\AA}$) can be emitted when these atoms in the ground state are bombarded by electrons that have been accelerated through a potential difference of 16.2 V?
- If these atoms are in the ground state, can they absorb radiation having a wavelength of 2000 $\text{\AA}$?
- What is the photoelectric threshold wavelength of this atom?

Sol.

$$\begin{aligned} \text{(a)} \quad E_1 &= -\frac{18.0}{1^2} = -18.0 \text{ eV}, \quad E_2 = -\frac{18.0}{2^2} = -4.5 \text{ eV}, \\ E_3 &= -\frac{18.0}{3^2} = -2.0 \text{ eV}, \quad E_4 = -\frac{18.0}{4^2} = -1.125 \text{ eV} \end{aligned}$$

The energy level diagram is shown in the figure.

$$\begin{array}{l} \text{————— } E_4 = -1.125 \text{ eV} \\ \text{————— } E_3 = -2.0 \text{ eV} \\ \text{————— } E_2 = -4.5 \text{ eV} \\ \text{————— } E_1 = -18.0 \text{ eV} \end{array}$$

- The excitation potential of state $n = 2$ is:
 $18.0 - 4.5 = 13.5 \text{ V}$
- Energy of the electron accelerated by a potential difference of 16.2 V is 16.2 eV. With this energy, the electron can excite the atom from $n = 1$ to $n = 3$ as

$$\begin{aligned} E_4 - E_1 &= 1.125 - (-18.0) = 16.875 \text{ eV} > 16.2 \text{ eV} \\ \text{and } E_3 - E_1 &= 2.0 - (-18.0) = 16.0 \text{ eV} < 16.2 \text{ eV} \end{aligned}$$

$$\lambda_{32} = \frac{12375}{E_3 - E_2} = \frac{12375}{-2.0 - (-4.5)} = 4950 \text{ \AA}$$

$$\lambda_{31} = \frac{12375}{E_3 - E_2} = \frac{12375}{16} = 773 \text{ \AA}$$

$$\text{and } \lambda_{21} = \frac{12375}{E_2 - E_1} = \frac{12375}{-4.5 - (-18.0)} = 917 \text{ \AA}$$

(d) No. The energy corresponding to $\lambda = 2000 \text{ \AA}$;

$$E = \frac{12375}{2000} = 6.1875 \text{ eV}$$

The minimum excitation energy is 13.5 eV ($n = 1$ to $n = 2$).

(e) Threshold wavelength for photoemission to take place

$$\text{from such an atom is, } \lambda_{\max} = \frac{12375}{18} = 687.5 \text{ \AA}$$

ILLUSTRATION 6.17

A doubly ionized lithium atom is hydrogen-like with atomic number 3. Find the wavelength of the radiation required to excite the electron in Li^{++} from the first to third Bohr orbit. The ionization energy of the hydrogen atom is 13.6 eV.

Sol. $E_n = -\frac{13.6Z^2}{n^2} \text{ eV}, \quad E_1 = -\frac{13.6 \times 3^2}{1^2} = -122.4 \text{ eV},$

$$E_3 = -\frac{13.6 \times 3^2}{3^2} = -13.6 \text{ eV}$$

The corresponding wavelength is $\lambda = \frac{12375}{E_3 - E_1} = 113.74 \text{ \AA}$

ILLUSTRATION 6.18

A single electron orbits a stationary nucleus of charge $+Ze$, where Z is a constant and e is the magnitude of electronic charge. It requires 47.2 eV to excite the electron from the second orbit to the third Bohr orbit. Find:

- the value of Z .
- the energy required to excite the electron from the third to the fourth Bohr orbit.
- the wavelength of electromagnetic radiation required to remove the electron from the first Bohr orbit to infinity.
- find the KE, PE, and angular momentum of electron in the first Bohr orbit.
- the radius of the first Bohr orbit.

[The ionization energy of hydrogen atom = 13.6 eV, Bohr radius = $5.3 \times 10^{-11} \text{ m}$, velocity of light = $3 \times 10^8 \text{ m s}^{-1}$, Planck's constant = $6.6 \times 10^{-34} \text{ J-s}$]

Sol. The energy required to excite the electron from n_1 to n_2 orbit revolving round the nucleus with charge $+Ze$ is given by

$$E_{n_2} - E_{n_1} = Z^2 \times 13.6 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right] \text{ electron volt.}$$

(a) Since 47.2 eV energy is required to excite the electron from $n_1 = 2$ to $n_2 = 3$ orbit, therefore,

$$47.2 = Z^2 \times 13.6 \left[\frac{1}{2^2} - \frac{1}{3^2} \right]$$

$$\Rightarrow Z^2 = \frac{47.2 \times 36}{13.6 \times 5} = 25 \quad \text{or} \quad Z = 5$$

(b) The energy required to excite the electron from $n_1 = 3$ to $n_2 = 4$ orbit is given by

$$E_4 - E_3 = 25 \times 13.6 \times \left[\frac{1}{3^2} - \frac{1}{4^2} \right] = \frac{25 \times 13.6 \times 7}{144} = 16.53 \text{ eV}$$

(c) The energy required to remove the electron from the first Bohr orbit to infinity (∞) is given by

$$E_{\infty} - E_1 = 13.6 \times Z^2 \left[\frac{1}{1^2} - \frac{1}{\infty^2} \right] = 13.6 \times 25 \text{ eV}$$

In order to calculate the wavelength of radiation, we use Bohr's frequency relation

$$hf = \frac{hc}{\lambda} = 13.6 \times 25 \times (1.6 \times 10^{-19}) \text{ J}$$

$$\text{or } \lambda = \frac{(6.6 \times 10^{-34}) \times (3 \times 10^8)}{13.6 \times 25 \times (1.6 \times 10^{-19})} = 36.5 \text{ \AA}$$

(d) $\text{KE} = -\text{Total energy} = -(-13.6 \times 5^2) \times 1.6 \times 10^{-19} \text{ J}$
 $= 5.44 \times 10^{-17} \text{ J}$

$$\text{PE} = -2 \times \text{KE} = -2 \times 5.44 \times 10^{-17} = 10.88 \times 10^{-17} \text{ J}$$

$$\text{Angular momentum} = mv_1 r_1 = h/2\pi = 1.05 \times 10^{-13} \text{ Js}$$

(e) The radius r_1 of the first Bohr's orbit is given by

$$r_1 = \frac{\epsilon_0 h^2}{\pi m e^2} \frac{1}{Z} = \frac{0.53 \times 10^{-10}}{5} = 0.106 \times 10^{-10} \text{ m}$$

$$= 0.106 \text{ \AA}$$

$$\left(\because \frac{\epsilon_0 h^2}{\pi m e^2} = 0.53 \times 10^{-10} \right)$$

ILLUSTRATION 6.19

In a transition from state n to a state of excitation energy 10.19 eV, a hydrogen atom emits a 4890 \AA photon. Determine the binding energy of the initial state.

Sol. The energy of the emitted photon is $hf = \frac{12400}{4890} = 2.54 \text{ eV}$

The excitation energy is the energy to excite the atom to a level above the ground state. Therefore, the energy of the level to which the transition occurs is:

$$E_x = -13.6 + 10.19 = -3.41 \text{ eV}$$

The photon arises from the transition between energy states (say n to x), then

$$E_n - E_x = hf, \quad \text{hence } E_n - (-3.41) = hf$$

$$\Rightarrow E_n - (-3.41) = 2.54 \quad \text{or} \quad E_n = -0.87 \text{ eV}$$

Therefore, the binding energy of an electron in the state is 0.87 eV.

Note that the transition is from

$$n = \sqrt{\frac{E_1}{E_n}} = \sqrt{\frac{13.6}{0.87}} = 4 \quad \text{to} \quad x = \sqrt{\frac{E_1}{E_x}} = \sqrt{\frac{13.6}{3.41}} = 2$$

ILLUSTRATION 6.20

The first excitation potential of a hypothetical hydrogen-like atom is 15 V. Find the third excitation potential of the atom.

Sol. Let energy of ground state = $-E_0$, then $E_n = -\frac{E_0}{n^2}$

Given: $E_2 - E_1 = 15 \text{ eV} \Rightarrow -\frac{E_0}{4} + E_0 = 15 \Rightarrow \frac{3E_0}{4} = 15$

$$\Rightarrow E_0 = 20 \text{ eV}$$

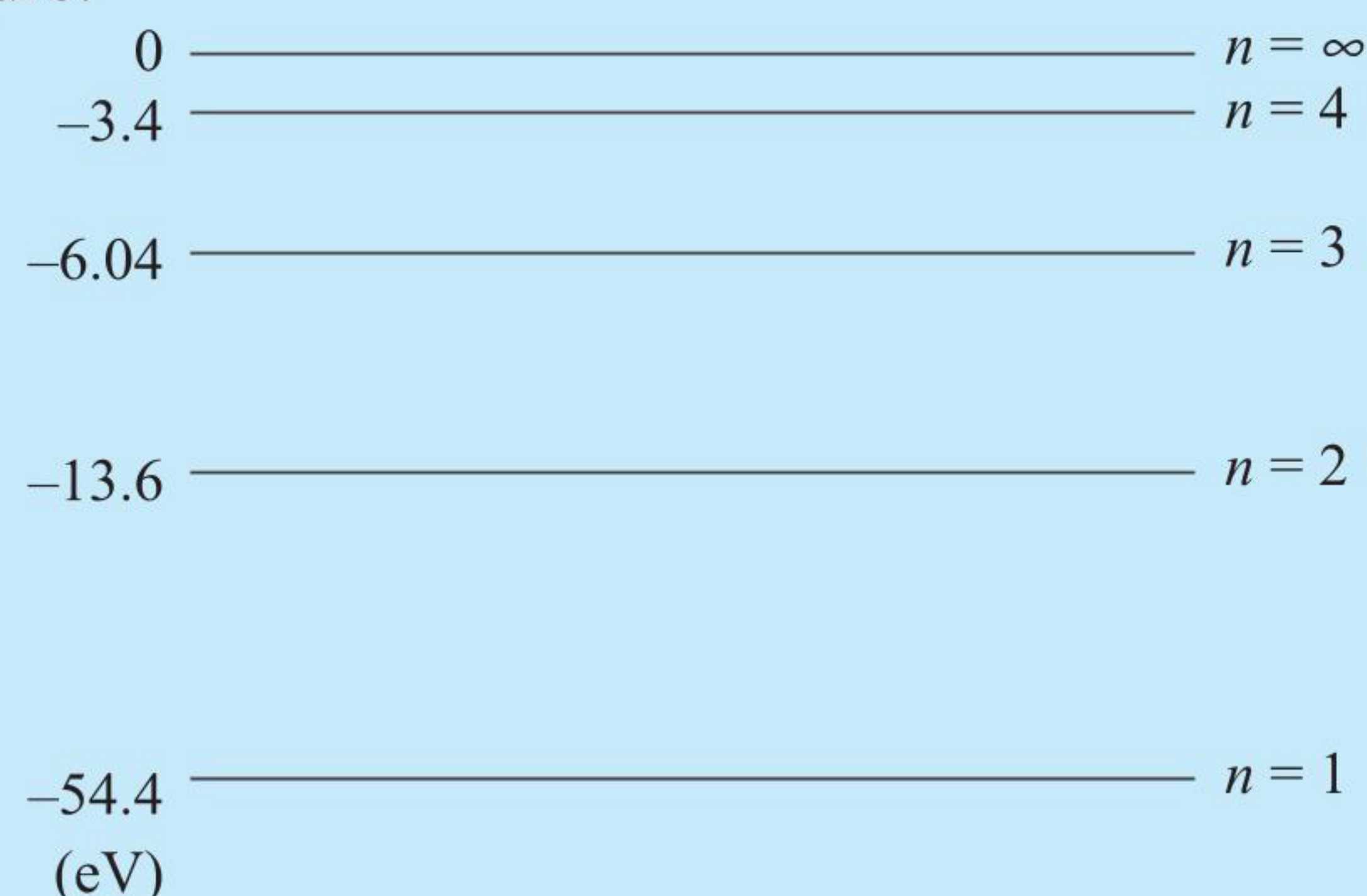
For the third excitation potential: $n = 4$, then

$$E_4 - E_1 = -\frac{E_0}{16} + E_0 = \frac{75}{4} \text{ eV}$$

Therefore, the third excitation potential is $(75/4) \text{ V}$.

ILLUSTRATION 6.21

The energy level diagram for a hydrogen-like atom is shown in figure.



- Find the value of Z .
- If initially the atom is in the ground state, then
 - determine its first excitation potential, and
 - determine its ionization potential.
- Can it absorb a photon of 42 eV?
- Can it absorb a photon of 56 eV?
- Calculate the radius of its first Bohr orbit.
- Calculate the kinetic energy and potential energy of an electron in the first orbit.

Sol.

- We know that $E_n = -13.6 \frac{Z^2}{n^2} \text{ eV}$

Here, for $n = 1$, $E_1 = -54.4 \text{ eV}$, therefore,

$$-54.4 = -13.6 \frac{Z^2}{1^2}$$

$$\therefore Z = \sqrt{\frac{54.4}{13.6}} = 2$$

- The first excitation energy is required to excite the electron from $n = 1$ to $n = 2$. Thus,

$$\Delta E_{12} = E_2 - E_1 = -13.6 - (-54.4) = 40.8 \text{ eV}$$

Therefore, the first excitation potential is 40.8 V.

- The ionization energy is required to eject the electron from $n = 1$ to $n = \infty$. Thus,

$$\Delta E_{1\infty} = E_\infty - E_1 = 0 - (-54.4) = 54.4 \text{ eV}$$

Therefore, the ionization potential is 54.4 V.

- No. By absorbing a photon of 42 eV, the energy of electron will become

$$E = -54.4 + 42 = -12.4 \text{ eV}$$

This energy level exists between $n = 2$ and $n = 3$ shells which is not possible. Hence, an electron in the first orbit cannot absorb it.

- Yes. By absorbing a photon of 56 eV, the electron comes out of the atom where the energy is not quantized. Hence, a photon of 56 eV can be absorbed.

- We know that $r_n = 0.53 \frac{n^2}{Z} (\text{\AA})$

$$\text{Here, } n = 1; Z = 2, \text{ therefore, } r = 0.53 \frac{(1)^2}{2} = 0.265 (\text{\AA})$$

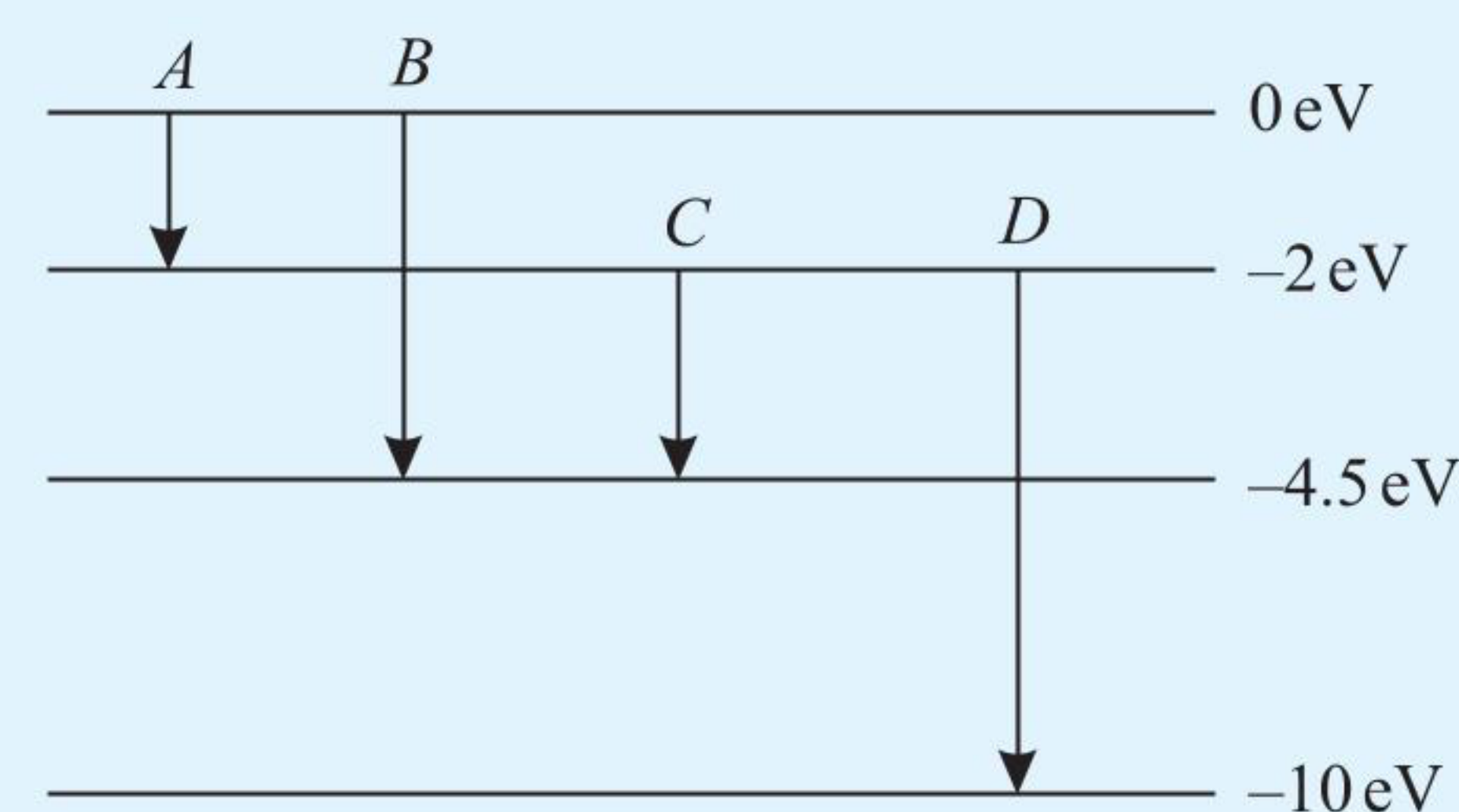
- Since $K = -E$ and $U = 2E$, therefore,

$$K = 54.4 \text{ eV} = 8.7 \times 10^{-18} \text{ J} \quad \text{and}$$

$$U = -108.8 \text{ eV} = 1.74 \times 10^{-17} \text{ J}$$

CONCEPT APPLICATION EXERCISE 6.2

- Which state of the triply ionized beryllium (Be^{+++}) has the same orbital radius as that of the ground state of hydrogen? Compare the energies of the two states.
- Which level of the doubly ionized lithium (Li^{++}) has the same energy as the ground state energy of the hydrogen atom? Compare the orbital radii of the two levels.
- Calculate the frequency of the photon, which can excite the electron to -3.4 eV from -13.6 eV .
- The ground state energy of hydrogen atom is -13.6 eV .
 - What are the potential energy and kinetic energy of an electron in the 3rd excited state?
 - If the electron jumps to the ground state from the third excited state, calculate the frequency of photon emitted.
- A photon of energy 12.09 eV is absorbed by an electron in ground state of a hydrogen atoms. What will be the energy level of electron? The energy of electron in the ground state of hydrogen atom is -13.6 eV .
- The first excitation potential of sodium atom is 2.1 V. Calculate the longest wavelength emitted by this atom.
- Find the frequency of revolution of the electron in the first stationary orbit of H-atom.
- The electron in a hydrogen atom, initially in a state of quantum number n_1 , makes a transition to a state whose excitation energy with respect to the ground state is 10.2 eV. If the wavelength associated with the photon emitted in this transition is 487.5 nm, find the (i) energy in eV and (ii) value of the quantum number n_1 of the electron in its initial state.
- The energy levels of an atom are as shown below. Which of them will result in the transition of a photon of wavelength 275 nm?



- Which transition corresponds to emission of radiation of (i) maximum wavelength and (ii) minimum wavelength?
- Using the known values for hydrogen atom, calculate:
 - radius of the third orbit for Li^{2+} .
 - speed of electron in the fourth orbit for He^+ .

11. Find the kinetic energy, potential energy, and total energy in the first and second orbits of hydrogen atom if potential energy in the first orbit is taken to be zero.
12. A small particle of mass m moves in such a way that the potential energy $U = ar^2$, where a is a constant and r is the distance of the particle from the origin. Assuming Bohr's model of quantization of angular momentum and circular orbits, find the radius of n^{th} allowed orbit.

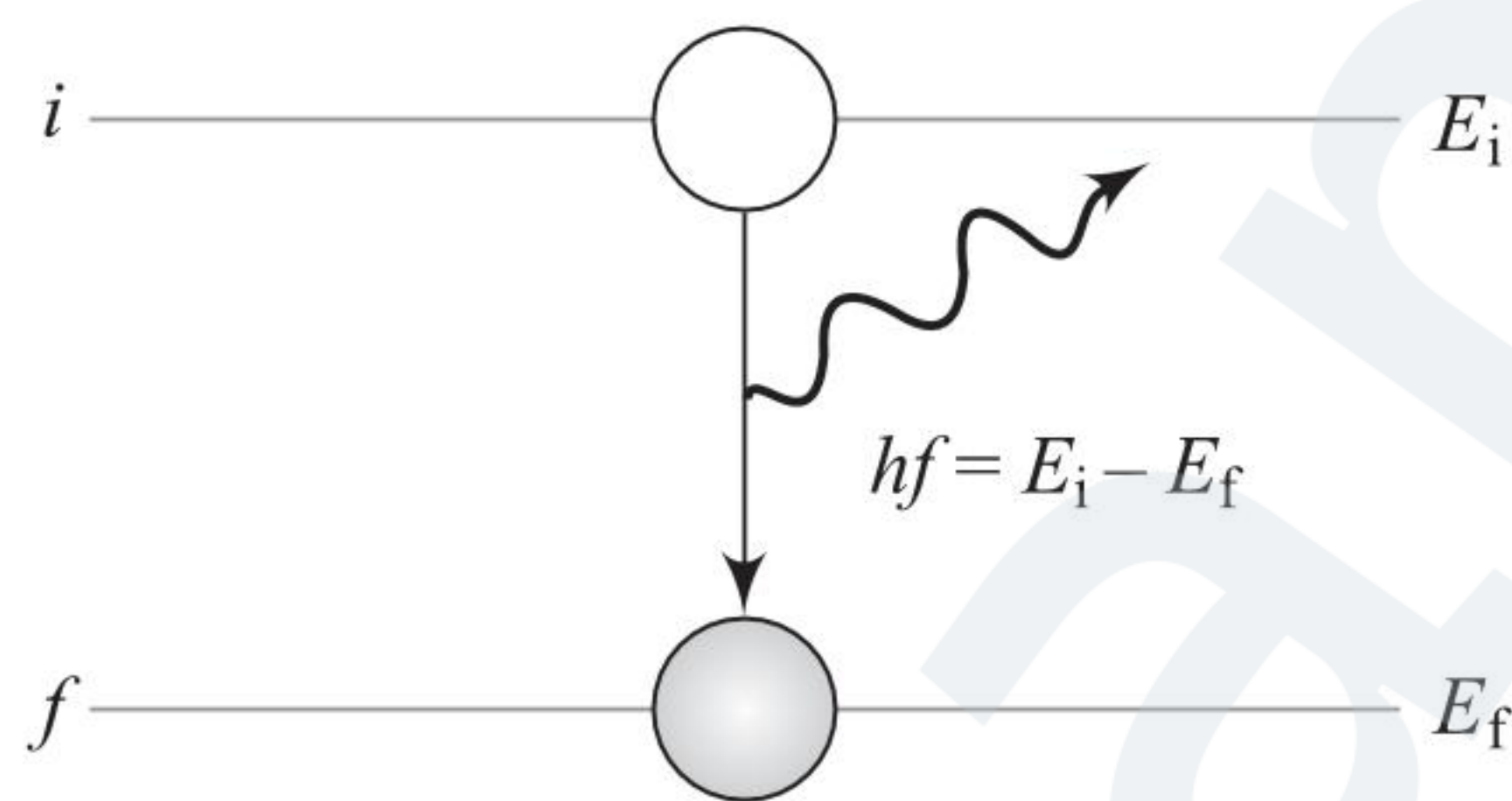
ANSWERS

1. $n = 2, \frac{E_{\text{Be}^{+++}}}{E_{\text{H}}} 4$ 2. $n = 3, \frac{r_{\text{Lithium}}}{r_{\text{Hydrogen}}} = 3$
3. $2.47 \times 10^{15} \text{ Hz}$ 4. (i) -1.7 eV , 0.85 eV (ii) $3 \times 10^{15} \text{ Hz}$
5. $n = 3$ 6. $5893 \text{ \AA}$ 7. $6.6 \times 10^{15} \text{ Hz}$
8. (i) -0.85 eV (ii) 4
9. (a) Transition B (b) (i) Transition A (ii) Transition D
10. (a) $1.587 \text{ \AA}$ (b) $1.095 \times 10^6 \text{ m s}^{-1}$ 12. $\left(\frac{n^2 h^2}{8um\pi^2} \right)^{1/4}$

WAVELENGTH OF PHOTON EMITTED IN DE-EXCITATION

According to Bohr, when an atom makes a transition from a higher energy level to a lower energy level, it emits a photon with energy equal to the energy difference between the initial and final levels. If E_i is the initial energy of the atom before such a transition, E_f is its final energy after the transition, and the photon's energy is $h\nu = hc/\lambda$, then conservation of energy gives,

$$h\nu = \frac{hc}{\lambda} = E_i - E_f \quad (\text{energy of emitted photon}) \quad \dots(i)$$



By 1913, the spectrum of hydrogen had been studied intensively. The visible line with longest wavelength, or lowest frequency is in red and is called H_α , the next line, in the blue-green is called H_β , and so on.

In 1885, Johann Balmer, a Swiss teacher found a formula that gives the wavelengths of these lines. This is now called the Balmer series. Balmer's formula is,

$$\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n^2} \right) \quad \dots(ii)$$

Here, $n = 3, 4, 5, \dots$, etc., $R = \text{Rydberg constant} = 1.097 \times 10^7 \text{ m}^{-1}$; and λ is the wavelength of light/photon emitted during transition. For $n = 3$, we obtain the wavelength of H_α line.

Similarly, for $n = 4$, we obtain the wavelength of H_β line. For $n = \infty$, the smallest wavelength ($= 3646 \text{ \AA}$) of this series is obtained. Using the relation $E = hc/\lambda$, we can find the photon energies corresponding to the wavelengths of the Balmer series.

$$E = \frac{hc}{\lambda} = hcR \left(\frac{1}{2^2} - \frac{1}{n^2} \right) = \frac{Rhc}{2^2} - \frac{Rhc}{n^2}$$

This formula suggests that, $E_n = \frac{Rhc}{n^2}$, $n = 1, 2, 3, \dots$... (iii)

The wavelengths corresponding to other spectral series (Lyman, Paschen, etc.) can be represented by formula similar to Balmer's formula.

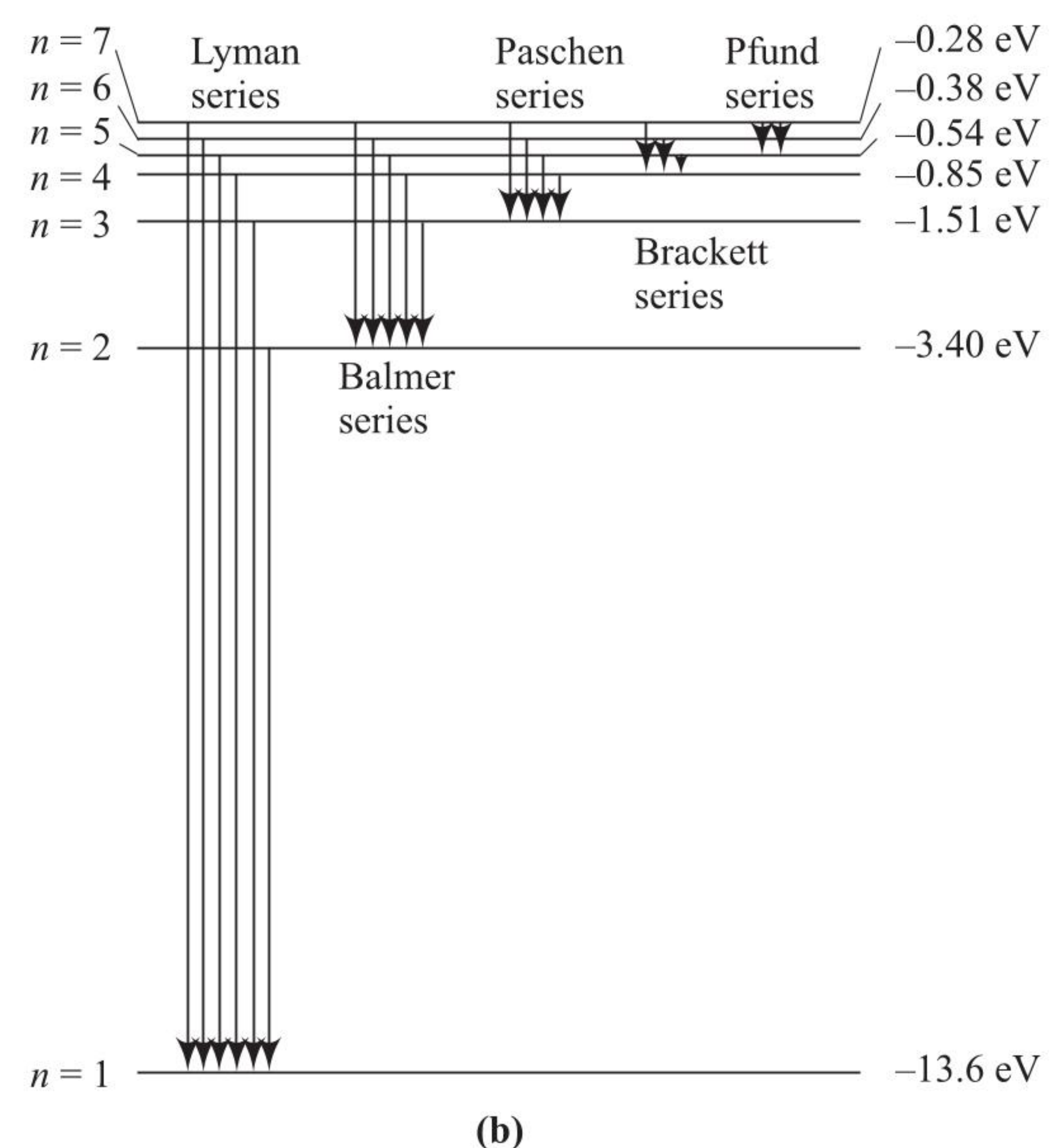
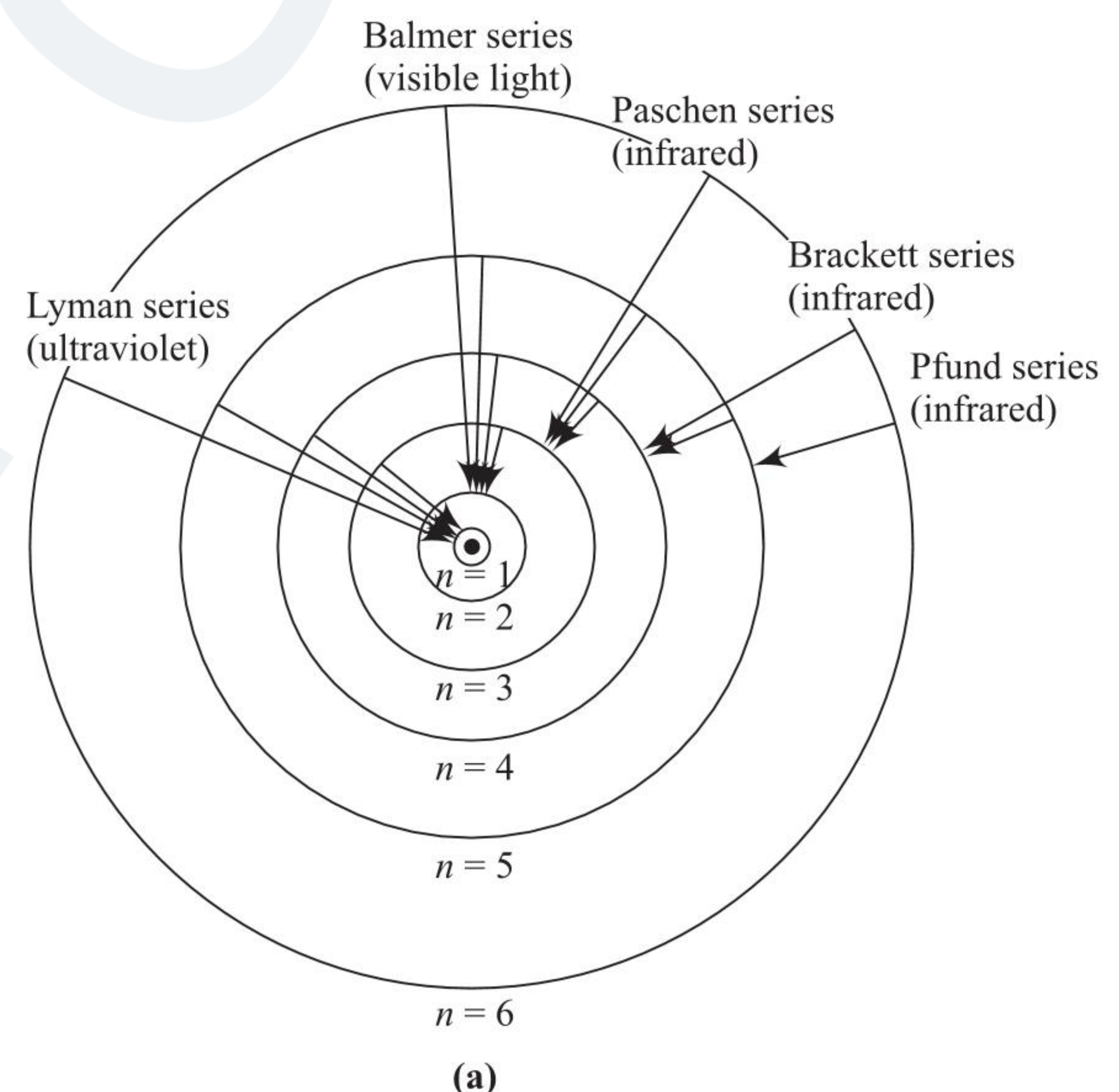
Lyman series: $\frac{1}{\lambda} = R \left(\frac{1}{1^2} - \frac{1}{n^2} \right)$, $n = 2, 3, 4, \dots$

Paschen series: $\frac{1}{\lambda} = R \left(\frac{1}{3^2} - \frac{1}{n^2} \right)$, $n = 4, 5, 6, \dots$

Brackett series: $\frac{1}{\lambda} = R \left(\frac{1}{4^2} - \frac{1}{n^2} \right)$, $n = 5, 6, 7, \dots$

Pfund series: $\frac{1}{\lambda} = R \left(\frac{1}{5^2} - \frac{1}{n^2} \right)$, $n = 6, 7, 8$

The Lyman series is in the ultraviolet, and the Paschen, Brackett, and Pfund series are in the infrared region.



Hydrogen spectrum

	Initial state	Final state	Wavelength formula	First member-second member	Series limit $n_i \rightarrow \infty$ to n_f	Maximum wavelength $(n_f + 1)$ to n_f	Lines found in
Lyman	$n_i = 2, 3, 4, 5, 6, \dots$	$n_f = 1$	$\frac{1}{\lambda} = R \left(\frac{1}{1^2} - \frac{1}{n_i^2} \right)$	$n_i = 2$ to $n_f = 1$ $n_i = 3$ to $n_f = 1$	From ∞ to 1 $\lambda = \frac{4}{R}$ $\lambda = 911 \text{ \AA}$	From 2 to 1 $\lambda = \frac{4}{3R}$ $\lambda = 121 \text{ \AA}$	UV region
Balmer	$n_i = 3, 4, 5, 6, 7, \dots$	$n_f = 2$	$\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n_i^2} \right)$	$n_i = 3$ to $n_f = 2$ $n_i = 4$ to $n_f = 2$	From ∞ to 2 $\lambda = \frac{4}{R}$ $\lambda = 3646 \text{ \AA}$	From 3 to 2 $\lambda = \frac{36}{5R}$ $\lambda = 6563 \text{ \AA}$	Visible region
Paschen	$n_i = 4, 5, 6, 7, 8, \dots$	$n_f = 3$	$\frac{1}{\lambda} = R \left(\frac{1}{3^2} - \frac{1}{n_i^2} \right)$	$n_i = 4$ to $n_f = 3$ $n_i = 5$ to $n_f = 3$	From ∞ to 3 $\lambda = \frac{9}{R}$ $\lambda = 8204 \text{ \AA}$	From 4 to 3 $\lambda = \frac{144}{7R}$ $\lambda = 18753 \text{ \AA}$	IR region
Brackett	$n_i = 5, 6, 7, 8, 9, \dots$	$n_f = 4$	$\frac{1}{\lambda} = R \left(\frac{1}{4^2} - \frac{1}{n_i^2} \right)$	$n_i = 5$ to $n_f = 4$ $n_i = 6$ to $n_f = 4$	From ∞ to 4 $\lambda = \frac{16}{R}$ $\lambda = 1485 \text{ \AA}$	From 5 to 4 $\lambda = \frac{400}{9R}$ $\lambda = 40515 \text{ \AA}$	IR region
Pfund	$n_i = 6, 7, 8, 9, 10, \dots$	$n_f = 5$	$\frac{1}{\lambda} = R \left(\frac{1}{5^2} - \frac{1}{n_i^2} \right)$	$n_i = 6$ to $n_f = 5$ $n_i = 7$ to $n_f = 5$	From ∞ to 5 $\lambda = \frac{25}{R}$ $\lambda = 22790 \text{ \AA}$	From 6 to 5 $\lambda = \frac{900}{11R}$ $\lambda = 74583 \text{ \AA}$	Far IR region

ILLUSTRATION 6.22

Brackett series of lines are produced when electrons excited to high energy levels make transitions to the $n = 4$ level.

- (a) Determine the longest wavelength in this series.
 (b) Determine the wavelength that corresponds to the transition from $n_f = 6$ to $n_i = 4$.

Sol.

- (a) The longest wavelength corresponds to transition from $n_f = 5$ to $n_i = 4$; the smallest energy change.

$$\text{So, } \frac{1}{\lambda_{\max}} = (1.097 \times 10^7) \left(\frac{1}{4^2} - \frac{1}{5^2} \right)$$

$$= 2.468 \times 10^5 \text{ m}^{-1}$$

$$\lambda_{\max} = \frac{1}{2.468 \times 10^5} = 4051 \text{ nm}$$

- (b) The wavelength for transition $n_f = 6$ to $n_i = 4$ is

$$\frac{1}{\lambda} = (1.0947 \times 10^7) \left(\frac{1}{4^2} - \frac{1}{6^2} \right) = 3.801 \times 10^5$$

$$\Rightarrow \lambda = \frac{1}{3.801 \times 10^5} = 2630 \text{ nm}$$

ILLUSTRATION 6.23

Electrons of energy 12.09 eV can excite hydrogen atoms. To which orbit is the electron in the hydrogen atom raised and what are the wavelengths of the radiations emitted as it drops back to the ground state?

Sol. The energies of the electron in different states are:

$$E_1 = -13.6 \text{ eV} \quad \text{for } n = 1,$$

$$E_2 = -3.4 \text{ eV} \quad \text{for } n = 2,$$

$$\text{and } E_3 = -1.51 \text{ eV} \quad \text{for } n = 3$$

Evidently, the energy needed by an electron to go to the E_3 level ($n = 3$ or M-level) is $13.6 - 1.51 = 12.09 \text{ eV}$. Thus, the electron is raised to the third orbit of principal quantum number $n = 3$.

Now, an electron in the $n = 3$ level can return to the ground state making the following possible jumps:

- (a) $n = 3$ to $n = 2$ and then from $n = 2$ to $n = 1$.
 (b) $n = 3$ to $n = 1$.

Thus, the corresponding wavelengths emitted are:

- (a) For $n = 3$ to $n = 2$:

$$\frac{1}{\lambda_1} = R \left[\frac{1}{2^2} - \frac{1}{3^2} \right] = \frac{5R}{36}$$

$$\text{or } \lambda_1 = \frac{36}{5R} = \frac{36}{5 \times 1.097 \times 10^7} = 6563 \text{ \AA}$$

This wavelength belongs to the Balmer series and lies in the visible region.

- (b) For $n = 2$ to $n = 1$:

$$\frac{1}{\lambda_2} = R \left[\frac{1}{1^2} - \frac{1}{2^2} \right] = \frac{3R}{4}$$

$$\text{or } \lambda_2 = \frac{4}{3R} = \frac{4}{3 \times 1.097 \times 10^7} = 1215 \text{ \AA}$$

λ_2 belongs to the Lyman series and lies in the ultraviolet region.

- (c) For the direct jump $n = 3$ to $n = 1$:

$$\frac{1}{\lambda_3} = R \left[\frac{1}{1^2} - \frac{1}{3^2} \right] = \frac{8R}{9}$$

$$\text{or } \lambda_3 = \frac{9}{8R} = \frac{9}{8 \times 1.097 \times 10^7} = 1026 \text{ \AA}$$

which also belongs to the Lyman series and lies in the ultraviolet region.

ILLUSTRATION 6.24

A hydrogen atom is in the third excited state. It makes a transition to a different state and a photon is either absorbed or emitted. Determine the quantum number n_f of the final state and the energy of the photon if it is

- emitted with the shortest possible wavelength,
- emitted with the longest possible wavelength, and
- absorbed with the longest possible wavelength.

Sol.

- (a) When a photon is emitted, it carries energy with it, so the final energy of the atom is less than its initial energy. So, the final quantum number is less than the initial when a photon is emitted.

An atom gains energy when a photon is absorbed. So, the final quantum number is greater than the initial when a photon is absorbed.

The largest possible energy arises when the electron jumps from the initial state ($n = 4$) to the ground state ($n_1 = 1$) as shown by transition *A* in figure. Therefore, the quantum number of final state is $n = 1$. The energy of the n^{th} state is given by

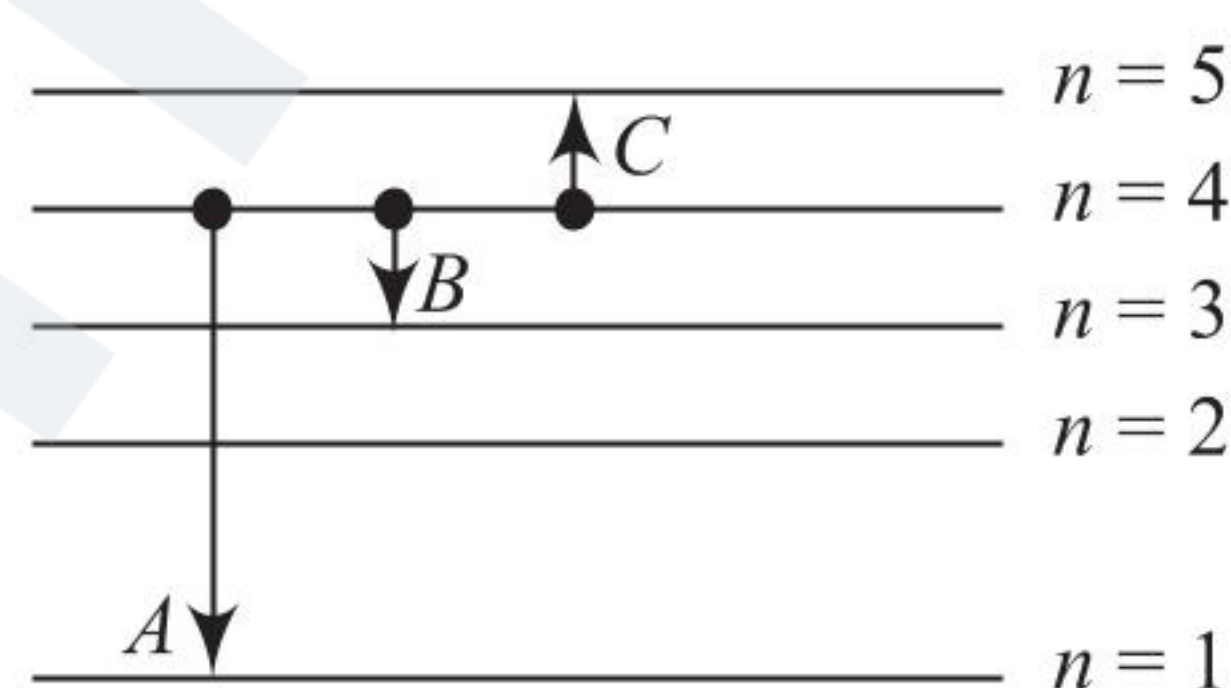
$$E_n = \left(\frac{Z^2}{n^2} \right)$$

The energy of the photon emitted corresponding to transition from excited state quantum number n_u to lower energy state n_l can be calculated from

$$\frac{hc}{\lambda} = E_u - E_l = (13.6)(1)^2 \left[\frac{1}{n_l^2} - \frac{1}{n_u^2} \right]$$

So, the energy of the photon with minimum wavelength is

$$\begin{aligned} \Delta E &= E_4 - E_1 \\ &= (13.6)(1)^2 \left(\frac{1}{1^2} - \frac{1}{4^2} \right) = 12.75 \text{ eV} \end{aligned}$$



- (b) The longest possible wavelength photon corresponds to minimum energy. This happens when the electron jumps from the initial state ($n_u = 4$) to the next lower state ($n_l = 3$) as shown by transition *B* in the figure. Therefore, the quantum number of the final state is $n = 3$. The energy of the photon is

$$\Delta E = E_4 - E_3 = (13.6)(1)^2 \left(\frac{1}{3^2} - \frac{1}{4^2} \right) = 0.661 \text{ eV}$$

- (c) The absorbed photon has the longest possible wavelength when its energy is the smallest, i.e., the electron jumps from the initial state ($n_l = 4$) to the next higher state ($n_u = 5$). This is shown by transition *C* in the figure. Therefore, the quantum number of the final state is $n = 5$. The energy of the photon is

$$\Delta E = E_5 - E_4 = (13.6)(1)^2 \left(\frac{1}{4^2} - \frac{1}{5^2} \right) = 0.306 \text{ eV}$$

ILLUSTRATION 6.25

The Balmer series for the hydrogen atom corresponds to electron transitions that terminate in the state of quantum number $n = 2$.

- Find the longest wavelength photon emitted and determine its energy.
- Find the shortest wavelength photon emitted in the Balmer series.

Sol.

- (a) The longest wavelength photon in the Balmer series results corresponding to transition from $n = 3$ to $n = 2$. Using the equation

$$\begin{aligned} \frac{1}{\lambda} &= R \left(\frac{1}{n_l^2} - \frac{1}{n_u^2} \right) \Rightarrow \frac{1}{\lambda_{\max}} = R \left(\frac{1}{2^2} - \frac{1}{3^2} \right) = \frac{5}{36} R \\ \Rightarrow \lambda_{\max} &= \frac{36}{5R} = \frac{36}{5(1.097 \times 10^7)} = 656.3 \text{ nm} \end{aligned}$$

This wavelength lies in the red region of the visible spectrum.

The energy of emitted photon is

$$\begin{aligned} E &= \frac{hc}{\lambda_{\max}} = \frac{(6.626 \times 10^{-34})(3 \times 10^8)}{(656.3 \times 10^{-9})} \\ &= 3.03 \times 10^{-19} \text{ J} = 1.89 \text{ eV} \end{aligned}$$

- (b) The shortest wavelength photon in the Balmer series is emitted when the electron makes a transition from $n = \infty$ to $n = 2$.

$$\begin{aligned} \frac{1}{\lambda_{\min}} &= R \left(\frac{1}{2^2} - \frac{1}{\infty} \right) = \frac{R}{4} \\ \Rightarrow \lambda_{\min} &= \frac{4}{R} = \frac{4}{1.097 \times 10^7} = 364.6 \text{ nm} \end{aligned}$$

This wavelength lies in the ultra-violet region.

ILLUSTRATION 6.26

How many different wavelengths may be observed in the spectrum from a hydrogen sample if the atoms are excited to a state with principal quantum number n ?

Sol. From the n th state, the atom may go to $(n-1)$ th state, ..., second state or first state. So, there are $(n-1)$ possible transitions starting from the n th state. The atoms reaching $(n-1)$ th state may make $(n-2)$ different transitions and so on. In general, the total number of possible transitions is

$$(n-1) + (n-2) + (n-3) + \dots + 2 + 1 = \frac{n(n-1)}{2}$$

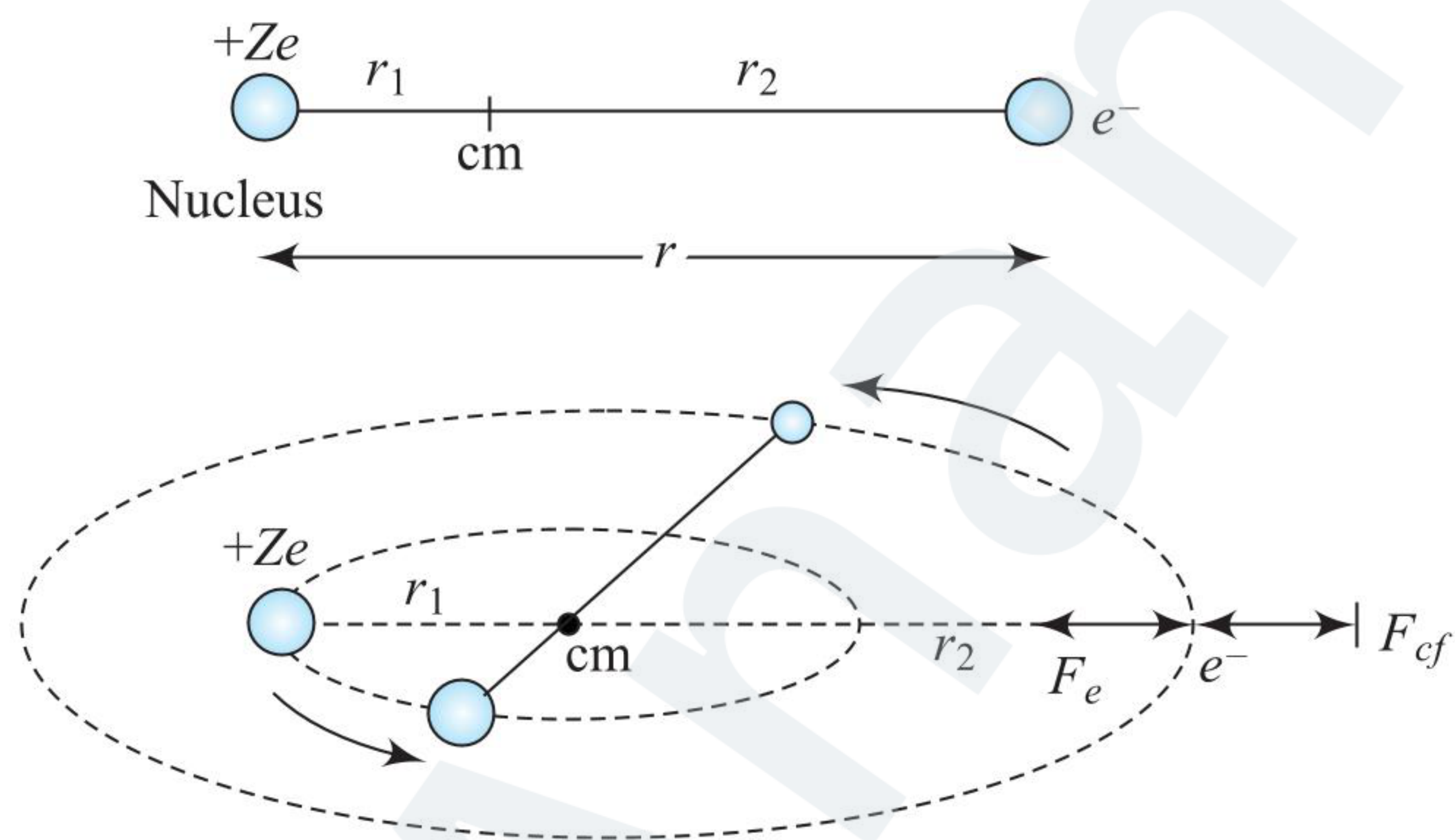
EFFECT OF NUCLEAR MOTION ON THE ENERGY OF ATOM

EFFECT OF MASS OF NUCLEUS ON BOHR MODEL

Up to this level, we have discussed about the structure of a hydrogenic atom by considering nucleus at rest and electron revolving around the nucleus. In fact, we know that as no external force is acting on a nucleus–electron system, hence the centre of mass of the nucleus–electron system must remain at rest. Theoretically, mass of electron is negligible or small compared to that of nucleus and due to this we assume that centre of mass of the atom is almost situated at nucleus. That is why in Bohr atomic model it was assumed that in an atom, nucleus remains at rest and electron revolves around it. But practically the situation is a bit different. Actually, centre of mass of nucleus–electron system is close to nucleus as it is heavy and to keep the centre of mass at rest, both electron and nucleus revolve around their centre of mass like a double star system as shown in figure. If r is the distance of electron from nucleus, the distances of nucleus and electron from the centre of mass, r_1 and r_2 , can be given as

$$r_1 = \frac{m_e r}{m_N + m_e}$$

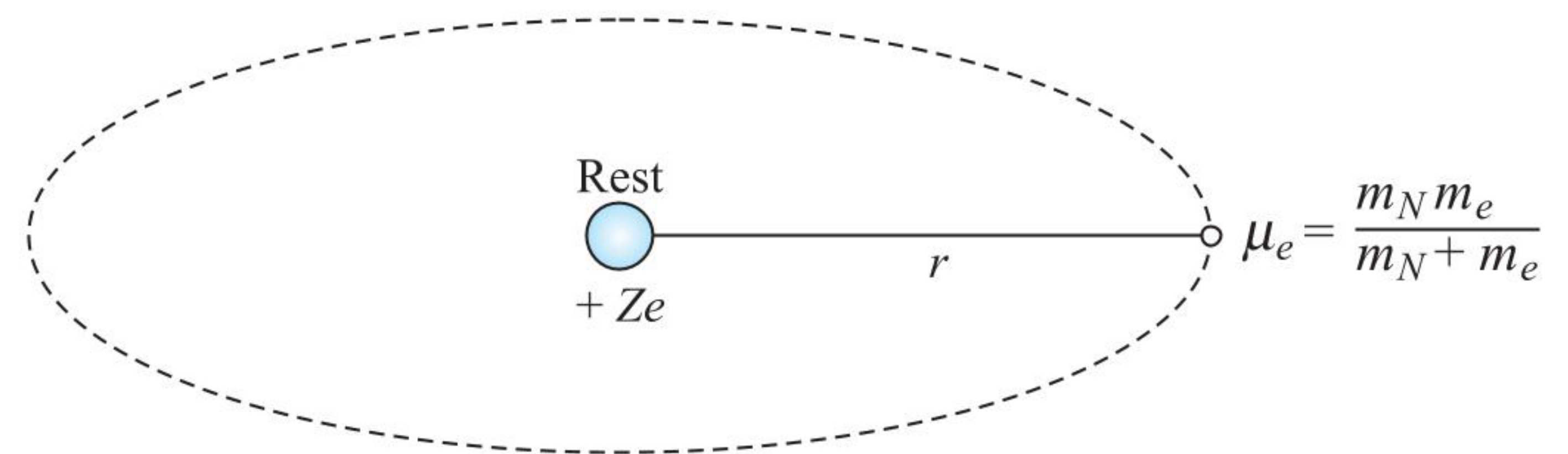
$$\text{and } r_2 = \frac{m_N r}{m_N + m_e}$$



We can see that in the atom, nucleus and electron revolve around their centre of mass in concentric circles of radii r_1 and r_2 to keep the centre of mass at rest. In above system, we can analyze the motion of electron with respect to nucleus by assuming nucleus to be at rest and the mass of electron replaced by its reduced mass μ_e , given as

$$\mu_e = \frac{m_N m_e}{m_N + m_e}$$

Now, the relative picture of atom will be same what we have considered earlier as shown in figure but electron mass is replaced by its reduced mass.



We can use all those relations which we have derived for the Bohr model just by replacing m_e by μ_e , such as the radius of electron in n th orbit of Bohr's model is given as

$$r_n = \frac{n^2 h^2}{4\pi^2 k Z e^2 m_e} \quad \dots(i)$$

But if we take the motion of nucleus into account, the radius of n th orbit will be given as

$$r'_n = \frac{n^2 h^2}{4\pi^2 k Z e^2 \mu_e} \Rightarrow r'_n = \frac{n^2 h^2 (m_N + m_e)}{4\pi^2 k Z e^2 m_e m_N} \quad \dots(ii)$$

$$\text{or } r'_n = r_n \times \frac{m_e}{\mu_e} \Rightarrow r = (0.529 \text{ \AA}) \frac{m n^2}{\mu Z} \quad \dots(iii)$$

Similarly, if we find the speed of electron in n th Bohr orbit, it can be given by

$$v_n = \frac{2\pi k Z e^2}{n h} \quad \dots(iv)$$

Here, we can see that in the above expression of speed the term m_e (mass of electron) is not present. Thus, speed of electron as n th orbit does not depend on electron mass. Here, no change will be there in the electron's speed of revolution if we take the motion of nucleus into account.

Similarly, we know the expression of energy of electron in n th orbit of the Bohr model is given as

$$E_n = -\frac{2\pi^2 k^2 Z^2 e^4 m_e}{n^2 h^2} \quad \dots(v)$$

$$\text{or } E'_n = -\frac{2\pi^2 k^2 Z^2 e^4 m_N m_e}{n^2 h^2 (m_N + m_e)} \quad \dots(vi)$$

$$\text{or } E'_n = E_n \times \frac{\mu_e}{m_e} \Rightarrow E_n = -(13.6 \text{ eV}) \frac{Z^2}{n^2} \left(\frac{\mu}{m} \right) \quad \dots(vii)$$

Thus, we can say that the energy of electron will be slightly less compared to what we have derived earlier. But for numerical calculations this small change can be neglected unless in a given problem it is asked to consider the effect of motion of nucleus.

ILLUSTRATION 6.27

A positronium “atom” is a system that consists of a positron and an electron that orbit each other. Compare the wavelengths of the spectral lines of positronium with those of ordinary hydrogen.

Sol. Here, the two particles have the same mass m , so the reduced mass is $\mu = \frac{m m}{m + m} = \frac{m^2}{2m} = \frac{m}{2}$

where m is the electron mass.

$$\text{We know that } E_n \propto m \Rightarrow \frac{E'_n}{E_n} = \frac{\mu}{m} = \frac{1}{2}$$

Hence, energy levels of positronium atom are half of corresponding energy level in a hydrogen atom.

As a result, the wavelengths in the positronium atom spectral lines are all twice those of the corresponding lines in the hydrogen spectrum.

ILLUSTRATION 6.28

A muon is an unstable elementary particle whose mass is $207m_e$ and whose charge is either $+e$ or $-e$. A negative muon (μ^-) can be captured by a hydrogen nucleus (or proton) to form a muonic atom.

- Find the radius of the first Bohr orbit of this atom.
- Find the ionization energy of the atom.

Sol.

- Here $m = 207m_e$ and $M = 1836m_e$; so the reduced mass is

$$\mu = \frac{mM}{m+M} = \frac{(207m_e)(1836m_e)}{207m_e + 1836m_e} = 186m_e$$

According to equation $\lambda = h/p$, the orbit radius corresponding to $n = 1$ is $r_1 = \frac{h^2 \epsilon_0}{\pi m_e e^2} = 5.29 \times 10^{-11} \text{ m}$

Hence, the radius r' that corresponds to the reduced mass μ is

$$r'_1 = \left(\frac{m}{\mu}\right)r_1 = \left(\frac{m_e}{1836m_e}\right)r_1 = 2.85 \times 10^{-13} \text{ m}$$

The muon is 186 times closer to the proton than an electron would be in the same orbit.

- We have $E_1 = -13.6 \text{ eV}$ for $n = 1$

$$E'_1 = \left(\frac{\mu}{m}\right)E_1 = 186 \times (-13.6) \text{ eV}$$

$$\Rightarrow E_1 = -2.53 \times 10^3 \text{ eV} = -2.53 \text{ keV}$$

The ionization energy is, therefore, 2.53 keV, 186 times that for an ordinary hydrogen atom.

ILLUSTRATION 6.29

A particle of charge equal to that of an electron, $-e$, and mass 208 times the mass of electron (called a μ -meson) moves in a circular orbit around a nucleus of charge $+3e$. (Take the mass of the nucleus to be infinite.) Assuming that Bohr's model of the atom is applicable to this system:

- Derive an expression for the radius of the n^{th} Bohr orbit.
- Find the value of n for which the radius of the orbit is approximately the same as that of the first Bohr orbit for the hydrogen atom.
- Find the wavelength of the radiation emitted when the μ -meson jumps from the third orbit to the first orbit. (Rydberg's constant = 10967800 m^{-1})

Sol.

- We have radius of n^{th} orbit of a hydrogen atom as

$$r_n = \frac{n^2 h^2}{4\pi^2 kZe^2 m}$$

If electron is replaced by a heavy particle of mass 208 times that of the electron, then the radius is given as

$$r_n = \frac{n^2 h^2}{4\pi^2 k(3)e^2 (208m)} = \frac{n^2 h^2}{2496\pi^2 ke^2 m} \quad \dots(i)$$

Here, we have not used reduced mass because it is given that mass of nucleus is assumed to be infinite and here we take $Z = 3$.

- The radius of first Bohr orbit is given as $r_1 = \frac{h^2}{4\pi^2 ke^2 m}$

$$\text{From Eq. (i), we have } \frac{n^2 h^2}{2496\pi^2 ke^2 m} = \frac{h^2}{4\pi^2 ke^2 m}$$

$$\text{or } n^2 = 624$$

$$\text{or } n \approx 25$$

- Rydberg constant for hydrogen-like atom is $R = \frac{2\pi^2 k^2 e^4 m}{ch^3}$

Now, when electron is replaced by μ -meson, Rydberg constant will change as

$$R' = \frac{2\pi^2 k^2 e^4 (208m)}{ch^3}$$

$$\text{or } = 208R = 208 \times 10967800 \text{ m}^{-1}$$

Now, the wavelength λ of the radiation emitted when μ -meson makes a transition from $n_2 = 3$ to $n = 1$ is

$$\frac{1}{\lambda} = R'Z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

$$\text{or } \frac{1}{\lambda} = R' \times 9 \left[\frac{1}{1^2} - \frac{1}{3^2} \right] \quad \text{or } \frac{1}{\lambda} = 8R'$$

$$\text{or } \lambda = \frac{1}{8R'}$$

$$\lambda = \frac{1}{8 \times 208 \times 10967800} \text{ m}$$

$$\text{or } \lambda = 0.548 \text{ \AA}$$

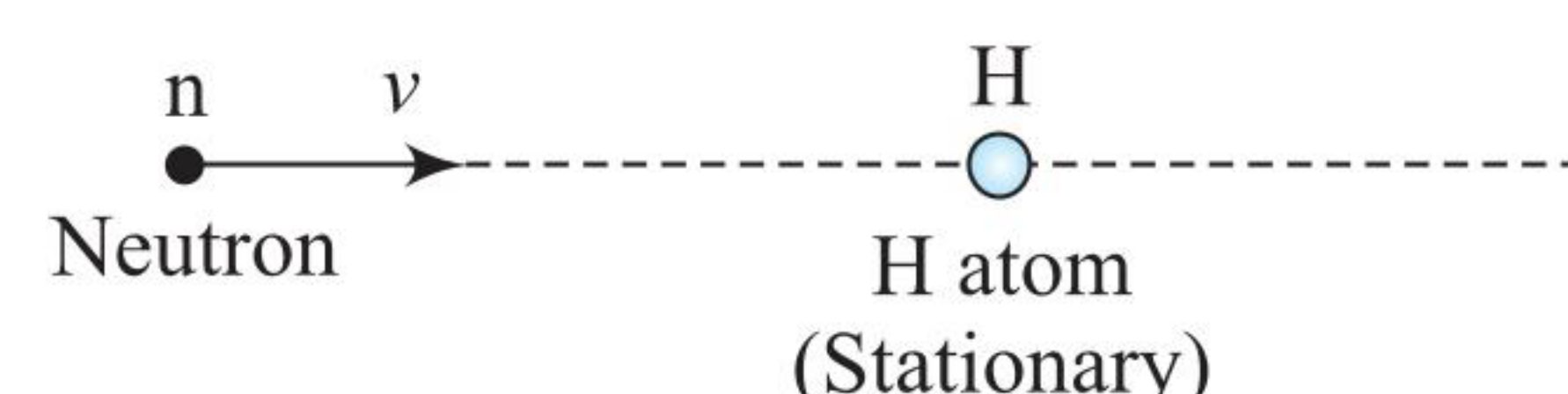
ATOMIC COLLISION

There are two ways to excite an electron in an atom:

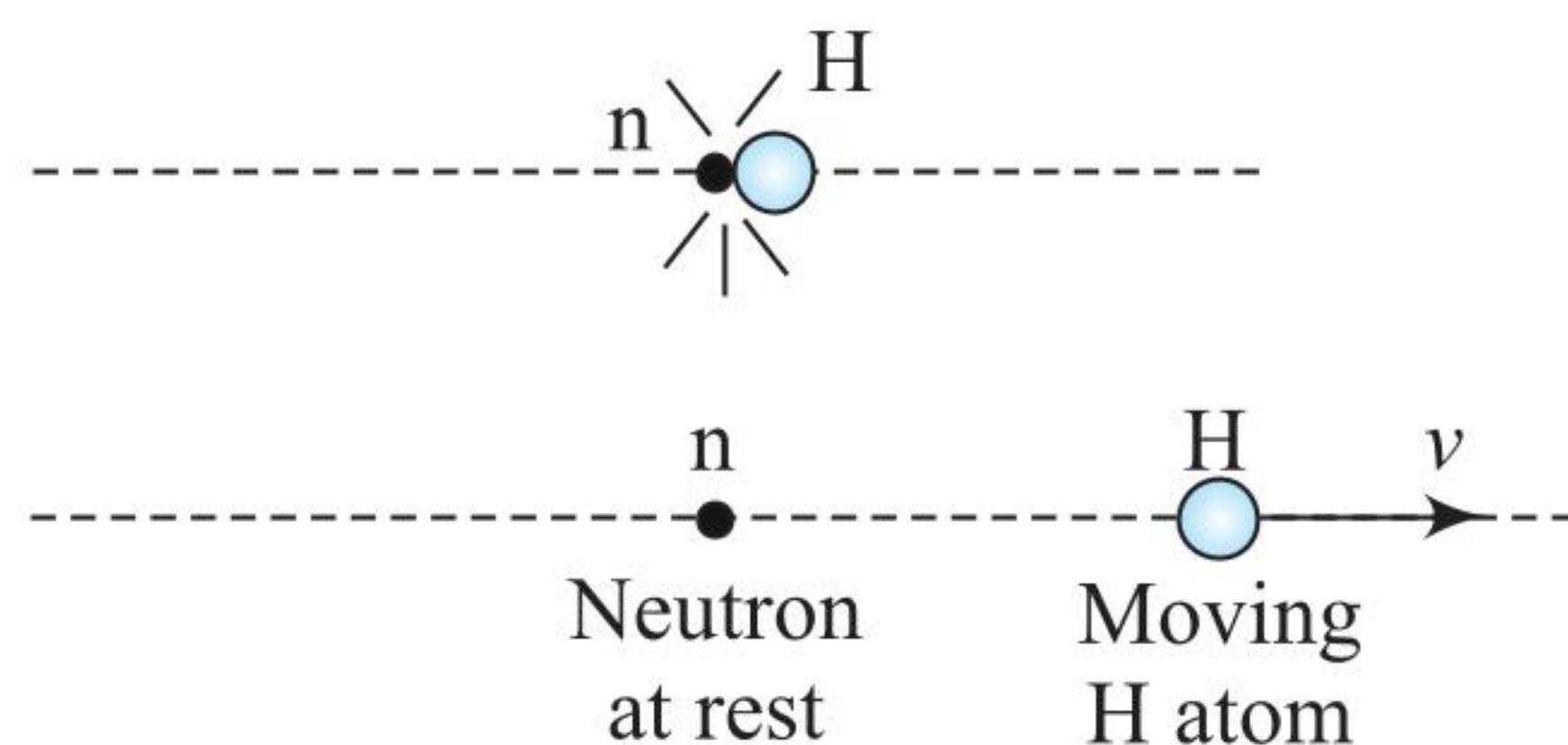
- By supplying energy to an electron through electromagnetic photons, which we have already discussed in photoelectric effect. An electron absorbs a photon only when the photon energy is equal to the difference in energies of the two energy levels of atom otherwise the photon will not be absorbed.
- The energy can be supplied to an electron by atomic collisions. In such collisions, assume that the loss in kinetic energy of the system is possible only if it becomes excited or gets ionized.

COLLISION OF A NEUTRON WITH AN ATOM

To understand the energy supplied by collisions, we consider an example of a head on collision of a moving neutron with a stationary hydrogen atom as shown in figure. Here, for mathematical analysis, let us assume the masses of neutron and H atom to be same.



Perfectly elastic collision: Here, when perfectly elastic collision takes place, then (for equal masses) the neutron will come to rest while the hydrogen atom will move with the same speed and kinetic energy with which the neutron was moving initially as shown in figure.



Perfectly inelastic collision: Now, if we consider the collision to be perfectly inelastic, then after collision both the neutron and H atom will move together with speed v_1 which can be given by law of conservation of momentum as

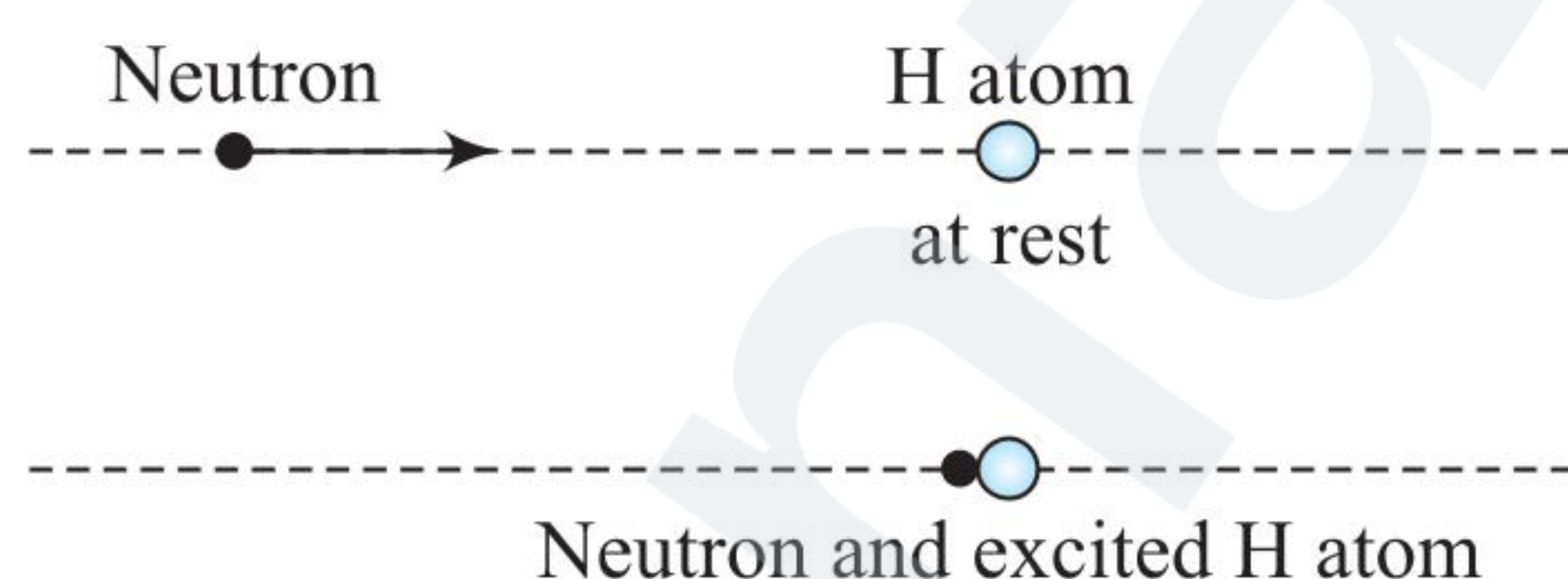
$$mv_0 = 2mv_1$$

or $v_1 = \frac{v}{2}$... (i)

In this case, the loss of energy can be given as the difference in initial and final kinetic energies of the neutron and H atom, given as

$$\begin{aligned} \Delta E &= E_i - E_f \\ &= \frac{1}{2}mv^2 - \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 \\ &= \frac{1}{2}mv^2 - \frac{1}{4}mv^2 \\ &= \frac{1}{4}mv^2 = \frac{1}{2}E_i \end{aligned} \quad \dots (ii)$$

Thus, half of the initial kinetic energy will be lost in the collision. One important point which we should understand carefully is that in collisions of elementary particles and atoms, no energy can be lost as heat because here we cannot consider the deformation of any lattice in the colliding body. The energy lost can only be absorbed by the atom involved in the collision and may get excited or ionized by this energy loss which takes place in case of inelastic collision.



In our case of collision of a neutron with an H atom, the energy loss is $(1/4)mv^2$ (half of initial energy of the neutron). This loss in energy can be absorbed by H atom only. We know that the minimum energy required to excite an H atom is 10.2 eV for $n = 1$ to $n = 2$. Thus, hydrogen atom can absorb only when this energy loss is equal to 10.2 eV. If this energy loss in perfectly inelastic, collision is more than 10.2 eV, then the H atom may absorb 10.2 eV energy for its excitation and rest of the energy will remain in the colliding particles (neutron and H atom) as their kinetic energy and the collision will not be perfectly inelastic in this case.

ILLUSTRATION 6.30

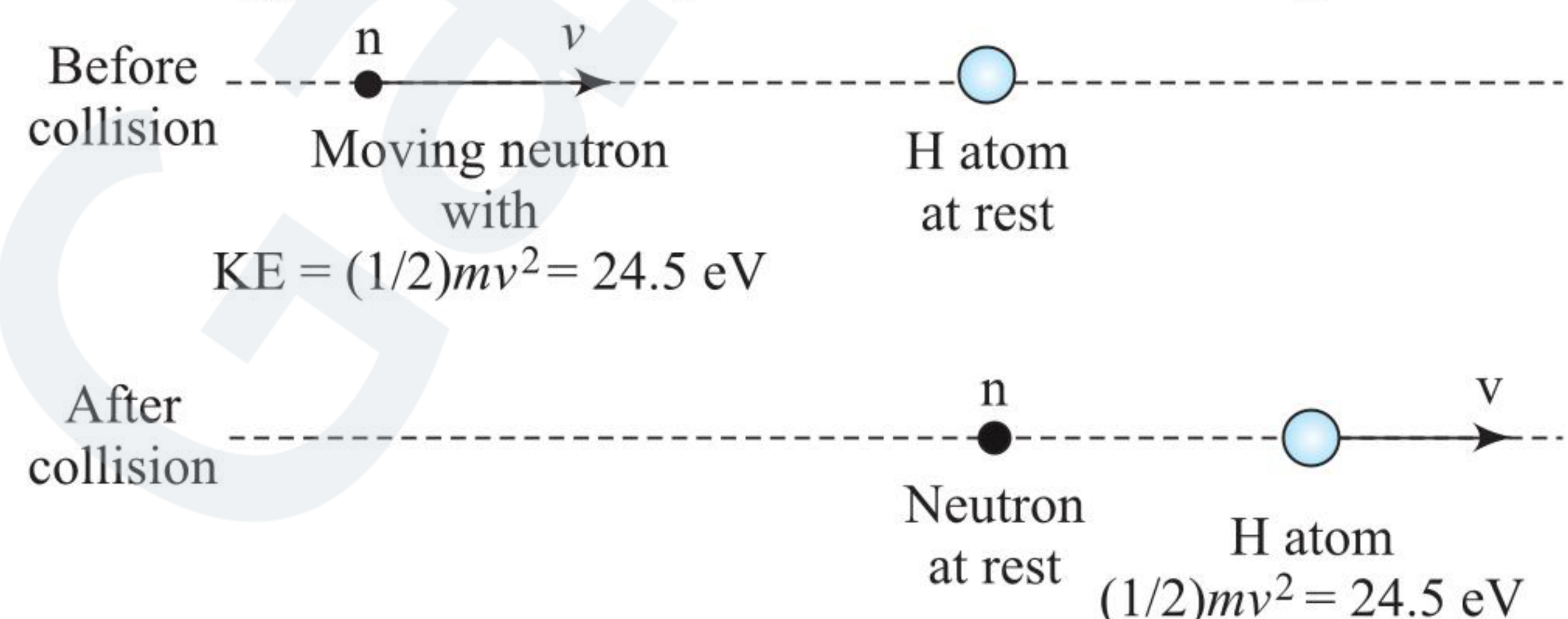
A neutron which is going to collide head on to a stationary H atom has initial kinetic energy of 24.5 eV. Discuss different possibilities of collision.

Sol. If perfectly inelastic collision takes place, then the maximum energy loss can be given as

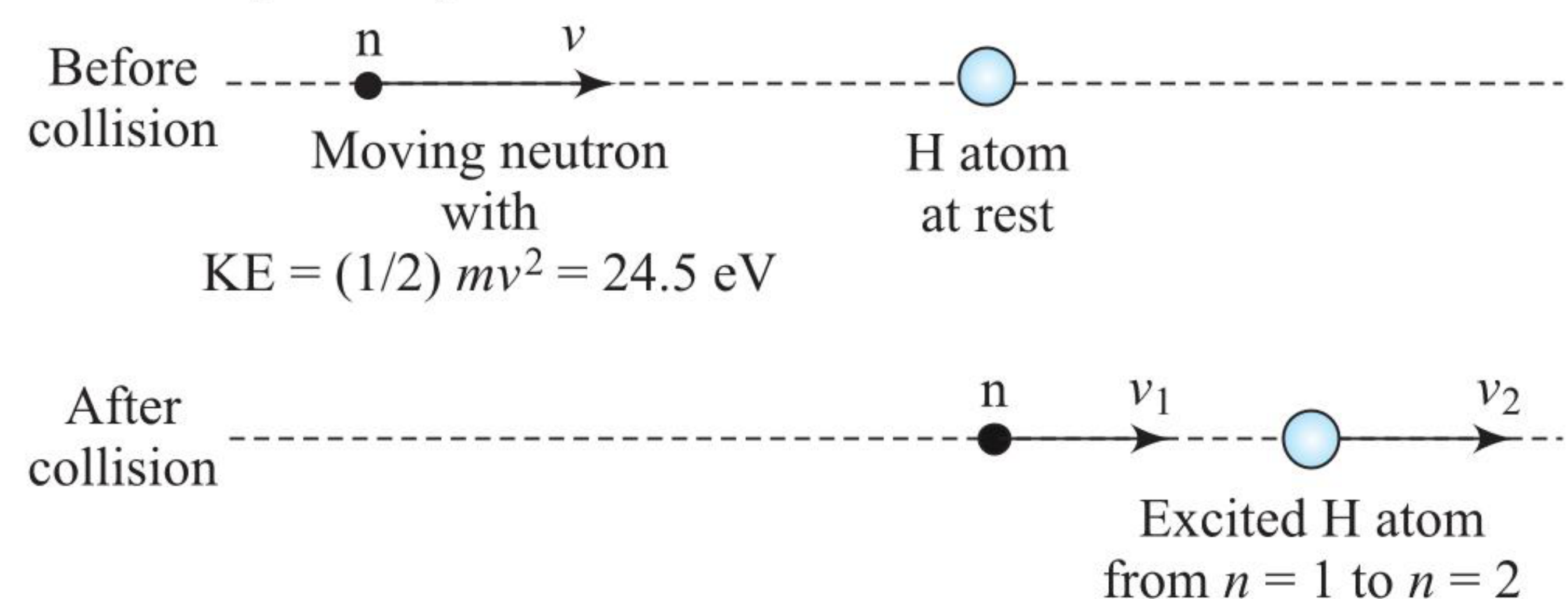
$$\Delta E_{\max} = \frac{1}{2}E_i = \frac{1}{2}(24.5) \text{ eV} = 12.25 \text{ eV}$$

From this energy, H atom can absorb either 10.2 eV or 12.09 eV for its excitation from $n = 1$ to $n = 2$ or from $n = 1$ to $n = 3$ energy level. Thus, in this case there may be three possibilities for collision. These are:

- (a) It may be possible that the collision is perfectly elastic and no energy is absorbed by H atom as shown in figure.



- (b) It may be possible that the H atom will absorb 10.2 eV energy during collision and both the neutron and H atom will be moving with kinetic energy $24.5 - 10.2 = 14.3$ eV after collision as shown in figure. In this case, the collision will be partially elastic.



$$\text{Final kinetic energy, } E_f = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = 14.3 \text{ eV}$$

In this case, the final speeds of neutron and H atom can be obtained by using equations of law of conservation of momentum and energy. Here, by momentum conservation law, we have

$$mv = mv_1 + mv_2$$

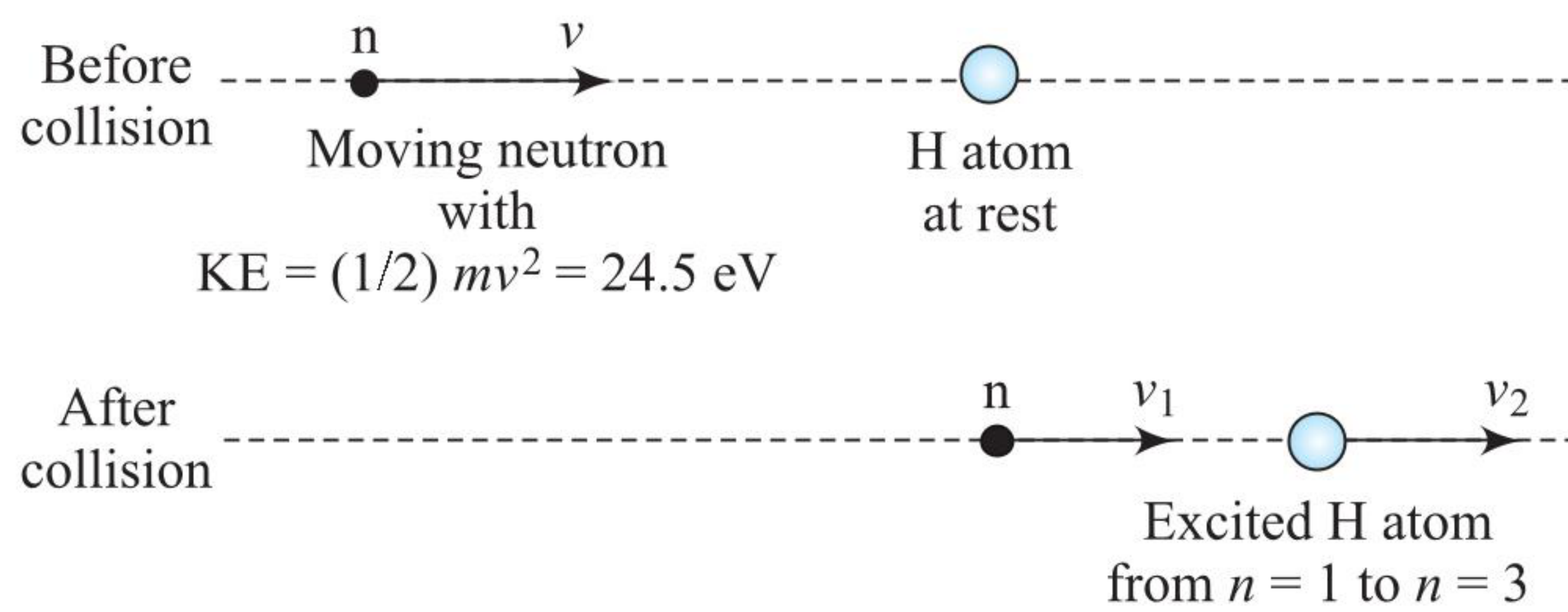
or $v = v_1 + v_2$... (i)

Using energy conservation, we have

$$\begin{aligned} \frac{1}{2}mv^2 - 10.2 \text{ eV} &= \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 \\ \text{or } 24.5 \text{ eV} - 10.2 \text{ eV} &= \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 \\ \text{or } \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 &= 14.3 \text{ eV} \end{aligned} \quad \dots (ii)$$

Now, solving Eqs. (i) and (ii), we will get the values of v_1 and v_2 which are the possible speeds of neutron and H atom after collision.

- (c) It may be possible that H atom will absorb 12.09 eV energy during collision and both the neutron and H atom will be moving with kinetic energy $24.5 - 12.09 = 12.41$ eV after collision as shown in figure. In this case, the collision will be partially elastic.



$$\text{Final kinetic energy, } E_f = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = 12.41 \text{ eV}$$

In this case, the final speeds of neutron and H atom can be obtained by using equations of law of conservation of momentum and energy. Here, by momentum conservation law, we have

$$mv = mv_1 + mv_2 \quad \text{or} \quad v = v_1 + v_2 \quad \dots(\text{iii})$$

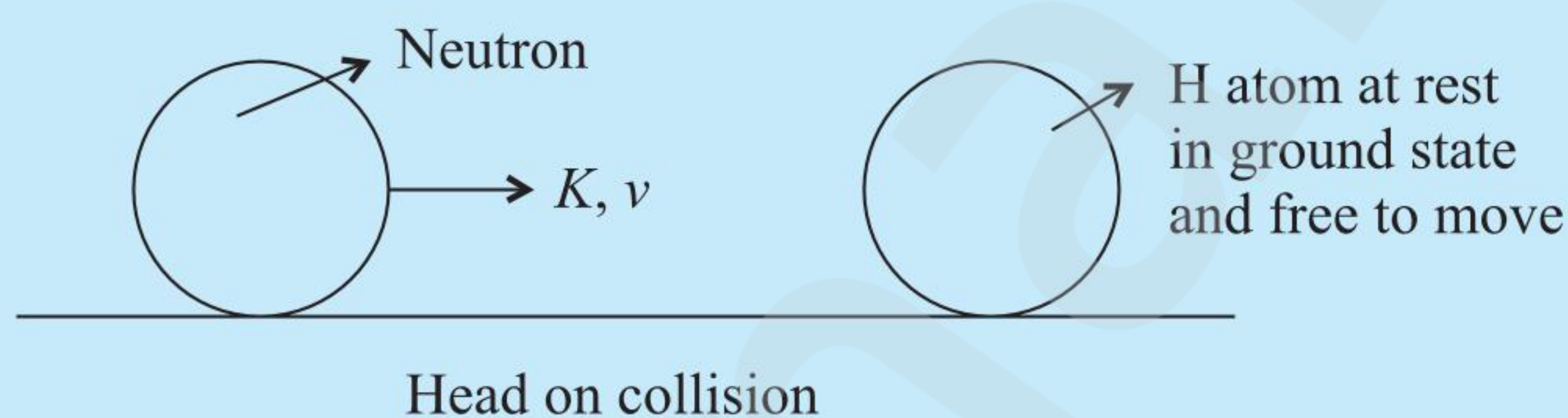
Using energy conservation, we have

$$\begin{aligned} \frac{1}{2}mv^2 - 12.09 \text{ eV} &= \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 \\ \text{or} \quad 24.5 \text{ eV} - 12.09 \text{ eV} &= \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 \\ \text{or} \quad \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 &= 12.41 \text{ eV} \quad \dots(\text{iv}) \end{aligned}$$

Now, solving Eqs. (iii) and (iv), we will get values of v_1 and v_2 which are the possible speeds of neutron and H atom after collision.

ILLUSTRATION 6.31

In the figure, what type of collision can be possible, if $K = 14$ eV, 20.4 eV, 22 eV, 24.18 eV (elastic/inelastic/perfectly inelastic).



Sol. Loss in energy (ΔE) during the collision will be used to excite the atom or electron from one level to another. According to Bohr's theory, for hydrogen atom,

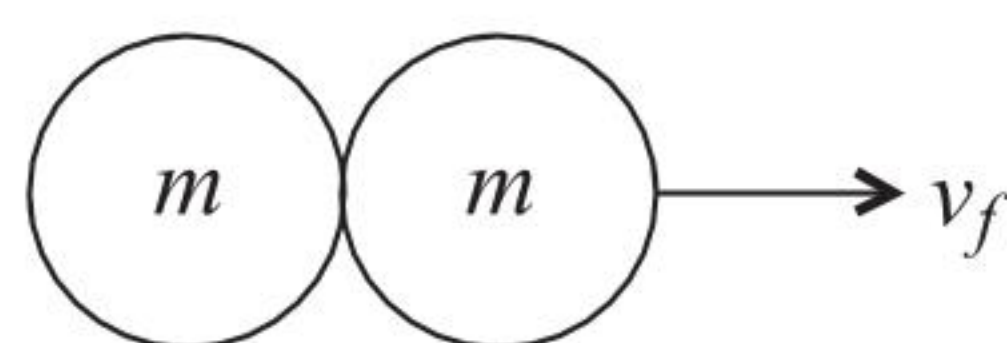
$$\Delta E = \{0, 10.2 \text{ eV}, 12.09 \text{ eV}, \dots, 13.6 \text{ eV}\}$$

According to Newtonian mechanics, minimum loss = 0 for elastic collision.

For maximum loss, collision will be perfectly inelastic.

If neutron collides perfectly inelastically, then applying momentum conservation:

$$mv_0 = 2mv_f \Rightarrow v_f = \frac{v_0}{2}$$



$$\text{Final KE} = \frac{1}{2} \times 2m \times \frac{v_0^2}{4} = \frac{\frac{1}{2}mv_0^2}{2} = \frac{K}{2}$$

$$\text{So, maximum loss} = \frac{K}{2}$$

According to classical mechanics, energy loss lies in the range:

$$\Delta E = \left[0, \frac{K}{2}\right]$$

- (a) $K = 14$ eV: According to quantum mechanics,

$$\Delta E = \{0, 10.2 \text{ eV}, 12.09 \text{ eV}\}$$

According to classical mechanics, $\Delta E = [0, 7 \text{ eV}]$

So, possible loss = 0, hence it is an elastic collision.

- (b) $K = 20.4$ eV: According to quantum mechanics,

$$\Delta E = \{0, 10.2 \text{ eV}, 12.09 \text{ eV}, \dots\}$$

According to classical mechanics, $\Delta E = [0, 10.2 \text{ eV}]$

If possible loss = 0 $\Rightarrow$ elastic collision,

If loss = 10.2 eV $\Rightarrow$ perfectly inelastic collision

- (c) $K = 22$ eV: According to quantum mechanics,

$$\Delta E = \{0, 10.2 \text{ eV}, 12.09 \text{ eV}, \dots\}$$

According to classical mechanics, $\Delta E = [0, 11 \text{ eV}]$

If possible loss = 0 $\Rightarrow$ elastic collision,

If loss = 10.2 eV $\Rightarrow$ inelastic collision (not perfectly)

- (d) $K = 24.18$ eV: According to quantum mechanics,

$$\Delta E = \{0, 10.2 \text{ eV}, 12.09 \text{ eV}, \dots\}$$

According to classical mechanics, $\Delta E = [0, 12.09 \text{ eV}]$

Possible loss = 0 $\Rightarrow$ elastic collision,

If loss = 10.2 eV $\Rightarrow$ inelastic collision (not perfectly),

and If loss = 12.09 eV $\Rightarrow$ perfectly inelastic collision

ILLUSTRATION 6.32

A He^+ ion is at rest and is in ground state. A neutron with initial kinetic energy K collides head on with the He^+ ion. Find minimum value of K so that there can be an inelastic collision between these two particles.



Sol. Here, the loss during the collision can only be used to excite the atoms or electrons.

So, according to quantum mechanics, possible losses can be

$$\{0, 40.8 \text{ eV}, 48.36 \text{ eV}, \dots, 54.4 \text{ eV}\} \quad \dots(\text{i})$$

$$\text{from } E_n = -13.6 \text{ eV} \frac{Z^2}{n^2}$$

Now, according to Newtonian mechanics: Minimum loss = 0

Maximum loss will be for perfectly inelastic collision.

Let v_0 be the initial speed of neutron and v_f be the final common speed.

$$\text{So, by momentum conservation: } mv_0 = mv_f + 4mv_f \Rightarrow v_f = \frac{v_0}{5}$$

where m = mass of neutron and mass of He^+ ion = $4m$

$$\text{Final kinetic energy of system: } KE = \frac{1}{2}mv_f^2 + \frac{1}{2}4mv_f^2$$

$$= \frac{1}{2}(5m)\frac{v_0^2}{25} = \frac{1}{5}\left(\frac{1}{2}mv_0^2\right) = \frac{K}{5}$$

$$\text{Maximum loss} = K - \frac{K}{5} = \frac{4K}{5}$$

$$\text{So, loss will be } \left[0, \frac{4K}{5}\right] \quad \dots(\text{ii})$$

For inelastic collision, there should be at least one common value other than zero in Eqs. (i) and (ii)

$$\therefore \frac{4K}{5} > 40.8 \text{ eV} \Rightarrow K > 51 \text{ eV}$$

Hence, minimum value of K for an inelastic collision:
 $K_{\min} = 51 \text{ eV}$.

ILLUSTRATION 6.33

In the previous question, find minimum value of K so that all types of collisions are possible.

Sol. For all three types of collisions, in set (i) and (ii),

$$\frac{4K}{5} = 48.36 \text{ eV} \Rightarrow K = 60.45 \text{ eV}$$

ILLUSTRATION 6.34

An H atom in ground state is moving with initial kinetic energy K . It collides head on with He^+ ion in ground state kept at rest but free to move. Find minimum value of K so that both the particles can excite to their first excited state.

Sol. Here, energy loss during the collision is used to excite the atom or ion. Now, according to quantum mechanics, loss in energy (ΔE) for H atom

$$\{0, 10.2 \text{ eV}, 12.09 \text{ eV}, \dots, 13.06 \text{ eV}\}$$

For He^+ ion: $\{0, 40.8 \text{ eV}, 48.36 \text{ eV}, \dots, 54.4 \text{ eV}\}$

To excite the hydrogen atom and He^+ ion in the first excited state, minimum energy = $40.8 + 10.2 = 51 \text{ eV}$

Now, according to Newtonian mechanics, minimum loss is 0 (for elastic collision). Maximum loss will be when there is perfectly inelastic collision.

Now, let mass of H atom be m , then mass of He^+ ion = $4m$.

Let v_0 be the initial speed of H atom and v_f the final common speed. According to momentum conservation:

$$mv_0 = 4mv_f + mv_f \Rightarrow v_f = \frac{v_0}{5}$$

$$\text{Kinetic energy} = \frac{1}{2}(5m)\frac{v_0^2}{25} = \frac{1}{5}\left(\frac{1}{2}mv_0^2\right) = \frac{K}{5}$$



Now, for minimum value of K so that the electron excite to the first excited state of H atom and He^+ ion.

$$\frac{4K}{5} = 51 \text{ eV}$$

$$\text{or } K = \frac{51 \times 5}{4} \text{ eV} \quad \text{or } K = 63.75 \text{ eV}$$

ILLUSTRATION 6.35

A moving hydrogen atom makes a head-on collision with a stationary hydrogen atom. Before collision, both atoms are in ground state and after collision they move together. What is the minimum value of the kinetic energy of the moving hydrogen atom, such that one of the atoms reaches one of the excitation state.

Sol. Let K be the kinetic energy of the moving hydrogen atom and K' the kinetic energy of combined mass after collision.

From conservation of linear momentum,

$$p = p' \Rightarrow \sqrt{2Km} = \sqrt{2K'(2m)}$$

$$\text{or } K = 2K' \quad \dots(\text{i})$$

From conservation of energy,

$$K = K' + \Delta E \quad \dots(\text{ii})$$

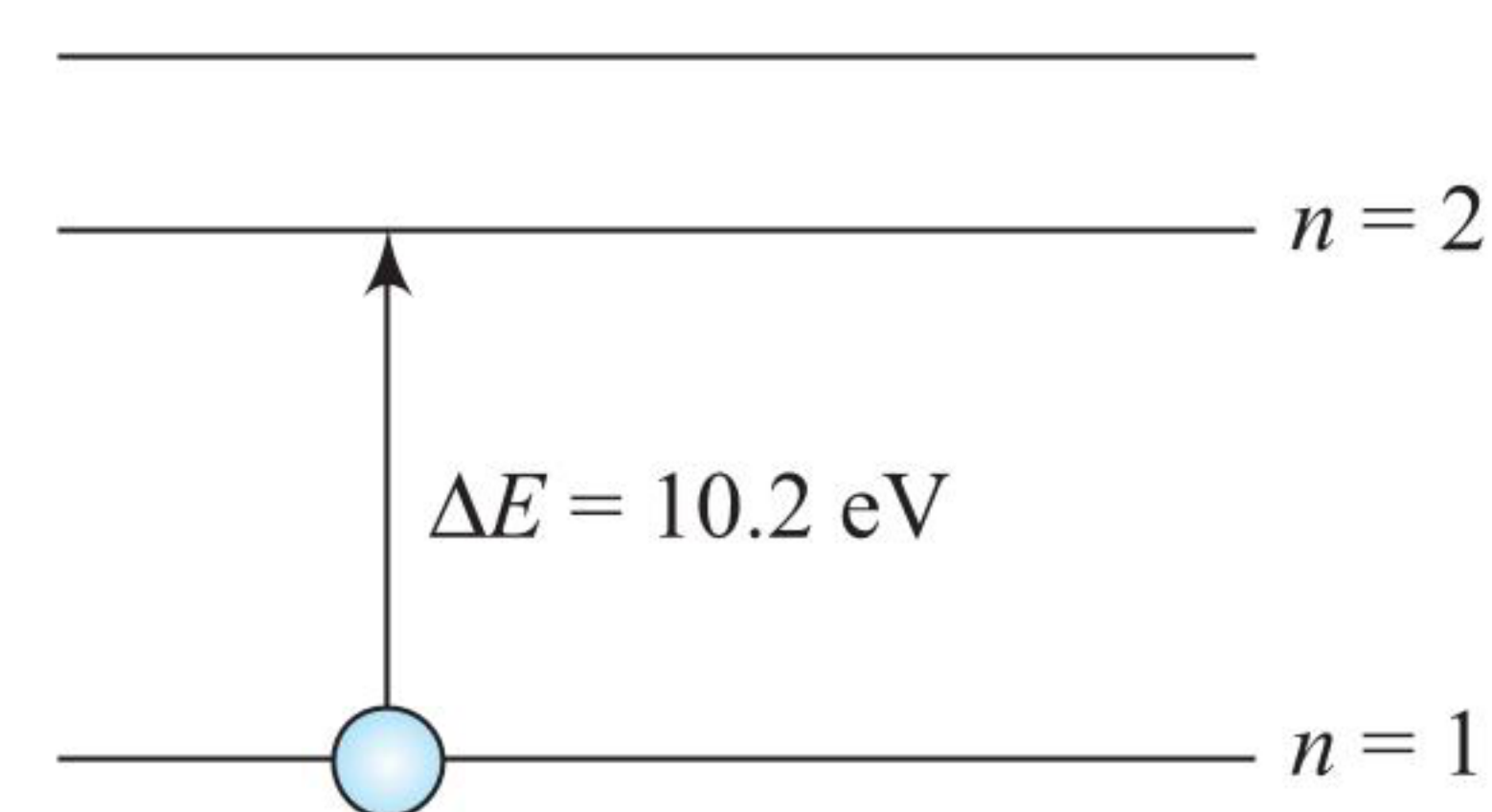
Solving Eqs. (i) and (ii), we get $\Delta E = \frac{K}{2}$

Now, minimum value of ΔE for hydrogen atom is 10.2 eV.

$$\text{or } \Delta E \geq 10.2 \text{ eV}$$

$$\therefore \frac{K}{2} \geq 10.2 \text{ eV}$$

$$\therefore K \geq 20.4 \text{ eV}$$

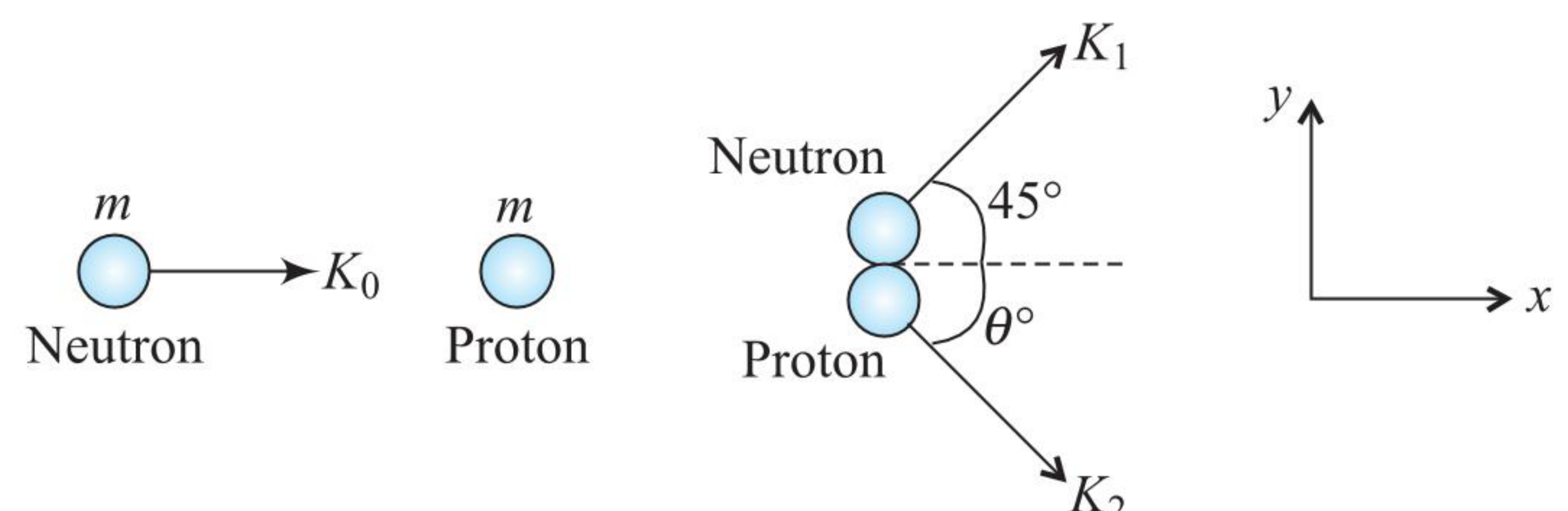


Therefore, the minimum kinetic energy of moving hydrogen is 20.4 eV.

ILLUSTRATION 6.36

A neutron with an energy of 4.6 MeV collides elastically with protons and is retarded. Assuming that upon each collision the neutron is deflected by 45° , find the number of collisions which will reduce its energy to 0.23 eV.

Sol. Mass of neutron = mass of proton = m



From conservation of momentum in y-direction,

$$\sqrt{2mK_1} \sin 45^\circ = \sqrt{2mK_2} \sin \theta \quad \dots(\text{i})$$

In x-direction,

$$\sqrt{2mK_0} = \sqrt{2mK_1} \cos 45^\circ + \sqrt{2mK_2} \cos \theta \quad \dots(\text{ii})$$

Squaring and adding Eqs. (i) and (ii), we have

$$K_2 = K_1 + K_0 - \sqrt{2K_0K_1} \quad \dots(\text{iii})$$

From conservation of energy,

$$K_2 = K_0 - K_1 \text{ [as collision is elastic]} \quad \dots(\text{iv})$$

Solving Eqs. (iii) and (iv), we get $K_1 = \frac{K_0}{2}$

i.e., after each collision energy remains half. Therefore, after n collisions,

$$K_n = K_0 \left(\frac{1}{2}\right)^n$$

$$\therefore 0.23 = (4.6 \times 10^6) \left(\frac{1}{2}\right)^n \Rightarrow 2^n = \frac{4.6 \times 10^6}{0.23}$$

Taking log and solving, we get $n \approx 24$

ILLUSTRATION 6.37

Explain why nearly all H atoms are in the ground state at room temperature and hence emit no light.

Sol. According to kinetic theory, the average kinetic energy of atoms or molecules in a gas is given by $K = (3/2)kT$ where $k = 1.38 \times 10^{-23} \text{ J K}^{-1}$ is Boltzmann's constant and T is Kelvin (absolute) temperature. Room temperature is about $T = 300 \text{ K}$. Hence,

$$\begin{aligned} K &= \frac{3}{2} (1.38 \times 10^{-23})(300) = 6.2 \times 10^{-21} \text{ J} \\ &= \frac{6.2 \times 10^{-21}}{1.6 \times 10^{-19}} \text{ eV} = 0.04 \text{ eV} \end{aligned}$$

The energy required to raise an electron from ground state to the next higher state is $13.6 - 3.4 = 10.2 \text{ eV}$. The average kinetic energy (0.04 eV) of atom is too small to excite an electron from ground state. Any atom in excited state emits photons and eventually falls to the ground state. Once in ground state, collision with other atoms can transfer energy of nearly 0.04 eV. Very high temperature is required to excite electron to upper status.

ILLUSTRATION 6.38

Consider a hydrogen-like atom whose energy in n th excited state is given by

$$E_n = \frac{13.6 Z^2}{n^2}$$

When this excited atom makes a transition from excited state to ground state, most energetic photons have energy $E_{\text{max}} = 52.224 \text{ eV}$ and least energetic photons have energy $E_{\text{min}} = 1.224 \text{ eV}$.

Find the atomic number of atom and the initial state of excitation.

Sol. Maximum energy is liberated for transition $E_n \rightarrow 1$ and minimum energy for $E_n \rightarrow E_{n-1}$. Hence,

$$\frac{E_1}{n^2} - E_1 = 52.224 \text{ eV}$$

$$\frac{E_1}{n^2} - \frac{E_1}{(n-1)^2} = 1.224 \text{ eV}$$

Solving the above equations simultaneously, we get

$$E_1 = -54.4 \text{ eV} \quad \text{and} \quad n = 5$$

$$\text{Now, } E_1 = -\frac{13.6 Z^2}{1^2} = -54.4 \text{ eV}$$

Hence, $Z = 2$, i.e., the gas is helium, originally excited to $n = 5$ energy state.

ILLUSTRATION 6.39

A hydrogen-like atom with atomic number Z is found to be in an excited state corresponding quantum number $2n$. It can emit a maximum energy photon of 204 eV. If it makes a transition to quantum state n , a photon of energy 40.8 eV is emitted. Find n , Z and the ground state energy (in eV) for this atom. Also, calculate the minimum energy (in eV) that can be emitted by this atom during de-excitation. Ground state energy of hydrogen atom is -13.6 eV .

Sol.

We define the energy levels of a hydrogen-like atom as

$$E_n = -\frac{Z^2 Rch}{n^2} = -(13.6 \text{ eV}) \frac{Z^2}{n^2} \quad \dots(\text{i})$$

The energy of emitted photon corresponding to transition $n = n_2 \rightarrow n = n_1$ with $n_2 > n_1$ is given by

$$E_{n_2 \rightarrow n_1} = -(13.6 \text{ eV}) Z^2 \left(\frac{1}{n_2^2} - \frac{1}{n_1^2} \right) \quad \dots(\text{ii})$$

If $n_2 = 2n$, the energy of the emitted photon will be maximum for transition $n_2 = 2n$ to $n_1 = 1$. Then maximum energy of the photon

$$\begin{aligned} E_{\text{max}} &= -(13.6 \text{ eV}) Z^2 \left(\frac{1}{(2n)^2} - \frac{1}{1^2} \right) \\ \Rightarrow E_{\text{max}} &= -(13.6 \text{ eV}) Z^2 \left(\frac{1}{4n^2} - 1 \right) \end{aligned}$$

We are given that $E_{\text{max}} = 204 \text{ eV}$, we use

$$204 \text{ eV} = -(13.6 \text{ eV}) Z^2 \left(\frac{1}{4n^2} - 1 \right) \quad \dots(\text{iii})$$

for the transition $n_2 = 2n$ to $n_1 = n$, we have

$$E_{2n \rightarrow n} = -(13.6 \text{ eV}) Z^2 \left(\frac{1}{4n^2} - \frac{1}{n^2} \right)$$

Given that $E_{2n \rightarrow n} = 40.8 \text{ eV}$, we use

$$40.8 \text{ eV} = -(13.6 \text{ eV}) Z^2 \left(\frac{1}{4n^2} - \frac{1}{n^2} \right) \quad \dots(\text{iv})$$

$$\text{Dividing (iii) by (iv), we get } \frac{204}{40.8} = \frac{\left(\frac{1}{4n^2} - 1 \right)}{\left(\frac{1}{4n^2} - \frac{1}{n^2} \right)}$$

$$\Rightarrow 5 = \frac{1 - 4n^2}{1 - 4}$$

which gives $4n^2 = 16$ or $n = 2$. Using this value of n in either (iii) or (iv) we get $Z^2 = 16$ or $Z = 4$.

The ground state energy of the atom corresponds to state $n = 1$. Putting $n = 1$ and $Z = 4$ in Eq. (i), the ground state energy of the atom is

$$E_1 = -(13.6 \text{ eV}) \times \frac{(4)^2}{(1)^2} = -217.6 \text{ eV}$$

Since $n = 2$, the given excited atom has $n_2 = 2n = 4$.

It will emit a photon of minimum energy for a transition $n_2 = 4$ to $n_1 = 3$. Using these values in (ii), we have

$$E_{\min} = (-13.6 \text{ eV})(4)^2 \left(\frac{1}{4^2} - \frac{1}{3^2} \right) = 10.58 \text{ eV}$$

ILLUSTRATION 6.40

A hydrogen-like atom (atomic number Z) is in a higher excited state of quantum number n . This excited atom can make a transition to the first excited state by successively emitting two photons of energies 10.20 eV and 17.00 eV, respectively.

Alternatively, the atom from the same excited state can make a transition to the second excited state by successively emitting two photons of energies 4.25 eV and 5.954 eV, respectively. Determine the values of n and Z (ionization energy of hydrogen atom = 13.6 eV).

Sol. For the transition from a higher state n to the first excited state $n_1 = 2$, the total energy released is

$$10.2 + 17.0 \text{ eV} \quad \text{or} \quad 27.2 \text{ eV}.$$

Thus, for $\Delta E = 27.2 \text{ eV}$, $n_1 = 2$, and $n_2 = n$, we have

$$27.2 = 13.6 Z^2 \left[\frac{1}{4} - \frac{1}{n^2} \right] \quad \dots(i)$$

For the eventual transition to the second excited state $n_1 = 3$, the total energy released is $(4.25 + 5.95) \text{ eV}$ or 10.2 eV. Thus,

$$10.2 = 13.6 Z^2 \left[\frac{1}{9} - \frac{1}{n^2} \right] \quad \dots(ii)$$

Dividing Eqs. (i) and (ii), we get $\frac{27.2}{10.2} = \frac{9n^2 - 36}{4n^2 - 36}$

Solving, we get $n^2 = 36$ or $n = 6$

Substituting $n = 6$ in any of the above equations, we obtain

$$Z^2 = 9 \quad \text{or} \quad Z = 3$$

Thus, $n = 6$ and $Z = 3$

ILLUSTRATION 6.41

Electrons in a hydrogen-like atom ($Z = 3$) make transitions from the fourth excited state to the third excited state and from the third excited state to the second excited state. The resulting radiations are incident on a metal plate and eject photoelectrons. The stopping potential for photoelectrons ejected by shorter wavelength is 3.95 eV.

Calculate the work function of the metal and stopping potential for the photoelectrons ejected by the longer wavelength.

Sol. Energy of a photon corresponding to transition from $n = 5$ (fourth excited state) to $n = 4$ (third excited state) is

$$h\nu = 13.6(3)^2 \left[\frac{1}{4^2} - \frac{1}{5^2} \right] = 2.75 \text{ eV} \quad \dots(i)$$

Similarly, energy of a photon corresponding to transition from $n = 4$ to $n = 3$ is

$$h\nu = 13.6(3)^2 \left[\frac{1}{3^2} - \frac{1}{4^2} \right] = 5.95 \text{ eV} \quad \dots(ii)$$

From Einstein's photoelectric effect equation,

$$h\nu = \phi + K_{\max}$$

$$K_{\max} = eV_s = h\nu - \phi$$

The shorter wavelength corresponds to greater energy difference between energy levels involved in the transition. So, shorter wavelength photons are emitted for transition $n = 4$ to $n = 3$. Thus, we have

$$3.95 = 5.95 - \phi \Rightarrow \phi = 2 \text{ eV}$$

and for longer wavelength photon, $eV_s = 2.75 - 2 = 0.75 \text{ eV}$

So, stopping potential = $\left(\frac{0.75 \text{ eV}}{e} \right) = 0.75 \text{ V}$

ILLUSTRATION 6.42

The radiation emitted when an electron jumps from $n = 3$ to $n = 2$ orbit in a hydrogen atom falls on a metal to produce photoelectrons. The electrons from the metal surface with maximum kinetic energy are made to move perpendicular to a magnetic field of $(1/320) \text{ T}$ in a radius of 10^{-3} m . Find (a) the kinetic energy of the electrons, (b) work function of the metal, and (c) wavelength of radiation.

Sol.

(a) Radius of the circle traced by a charged particle in a magnetic field is given by

$$R = \frac{mv}{Bq} = \frac{p}{Bq} = \frac{\sqrt{2mK}}{Bq}$$

$$\text{Hence, } K = \frac{(BqR)^2}{2m} = \frac{[10^{-3} \times 1.6 \times 10^{-19} \times (1/320)]^2}{2 \times 9.1 \times 10^{-31}}$$

$$= \frac{10^{-17}}{72.8} \text{ J} = \frac{10^{-17}}{72.8} \times \frac{1}{1.6 \times 10^{-19}} \text{ eV} = 0.86 \text{ eV}$$

(b) Energy of the photon emitted when the electron makes a transition from $n = 3$ to $n = 2$ energy level is

$$h\nu = E_3 - E_2 = 13.6 \left[\frac{1}{2^2} - \frac{1}{3^2} \right] \text{ eV} = 1.9 \text{ eV}$$

From Einstein's photoelectric effect equation,

$$h\nu = \phi + K_{\max}$$

So, $\phi = (1.9 - 0.86) \text{ eV}$

$$= 1.04 \text{ eV}$$

(c) Wavelength of radiation is

$$\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{1.9 \times 1.6 \times 10^{-19}} \text{ m} = 6542 \text{ \AA}$$

DRAWBACKS AND LIMITATIONS OF BOHR'S THEORY

The results of the Bohr's theory agree very well with the experimentally observed facts as:

- The value of Rydberg constant obtained earlier agrees with the one obtained on the basis of this theory.
- The spectral lines predicted by this theory were discovered experimentally by Lyman, Brackett and Pfund.
- Ionisation and resonance potentials for hydrogen predicted on the basis of this theory have been verified experimentally.
- A number of phenomena like X-ray production, fluorescence and phosphorescence can easily be explained on the basis of this theory.

But this is not all. This theory has its own **drawbacks and limitations** as listed below.

- It is not clear as to why we take only the circular orbits and not elliptical ones. In fact, an electron has also wave properties. It is, therefore, incorrect to talk of sharply defined orbits of an electron.
- This theory is successful only in explaining the spectra of hydrogenic atoms. It fails when applied to more complex atoms.
- This theory does not predict the relative intensities of various spectral lines.
- Even in case of hydrogen, some of the spectral lines have a fine structure, i.e., these lines are a combination of closely packed fine lines. The theory fails to explain this fine structure.
- Bohr's theory does not indicate whether an electron from a higher energy level (say 3) falls directly to the lower energy level 1 or successively through intermediate energy levels, i.e., from energy level 3 to 2 and then from 2 to 1.
- This theory cannot explain the appearance of a large number of spectral lines when the source is placed under the effect of an electric and magnetic fields (**Stark and Zeeman effects** respectively).
- The charge distribution of the electrons comes out to be different from that assumed in this theory.

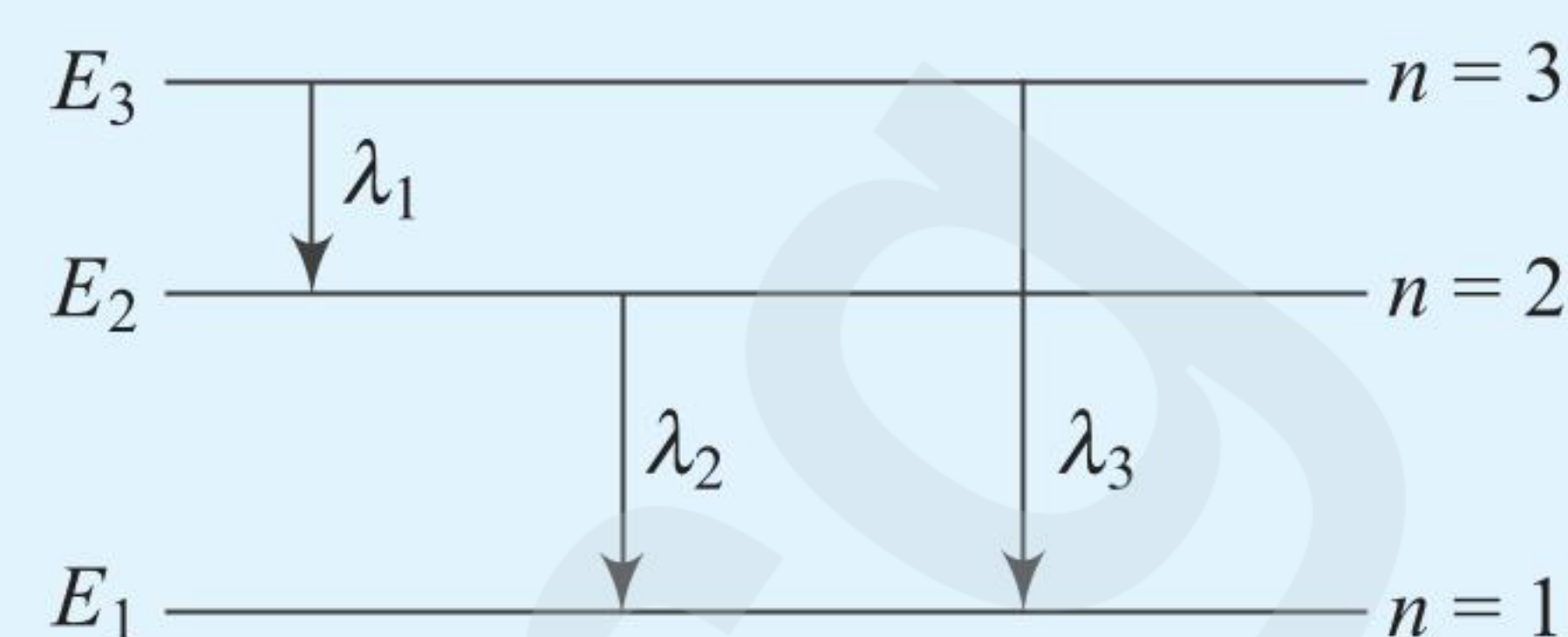
In short, the weakest aspect of Bohr's theory is its internal logical contradictions. It is neither a consistent classical theory nor a consistent quantum one. This theory is only a transition step towards the creation of a consistent theory of atomic phenomena.

However, some elements of Bohr's theory are a lasting legacy:

- angular momentum is quantised
- only certain discrete energies are allowed for atomic transitions
- electrons undergoing transitions between energy levels emit photons, which carry off the excess energy.

CONCEPT APPLICATION EXERCISE 6.3

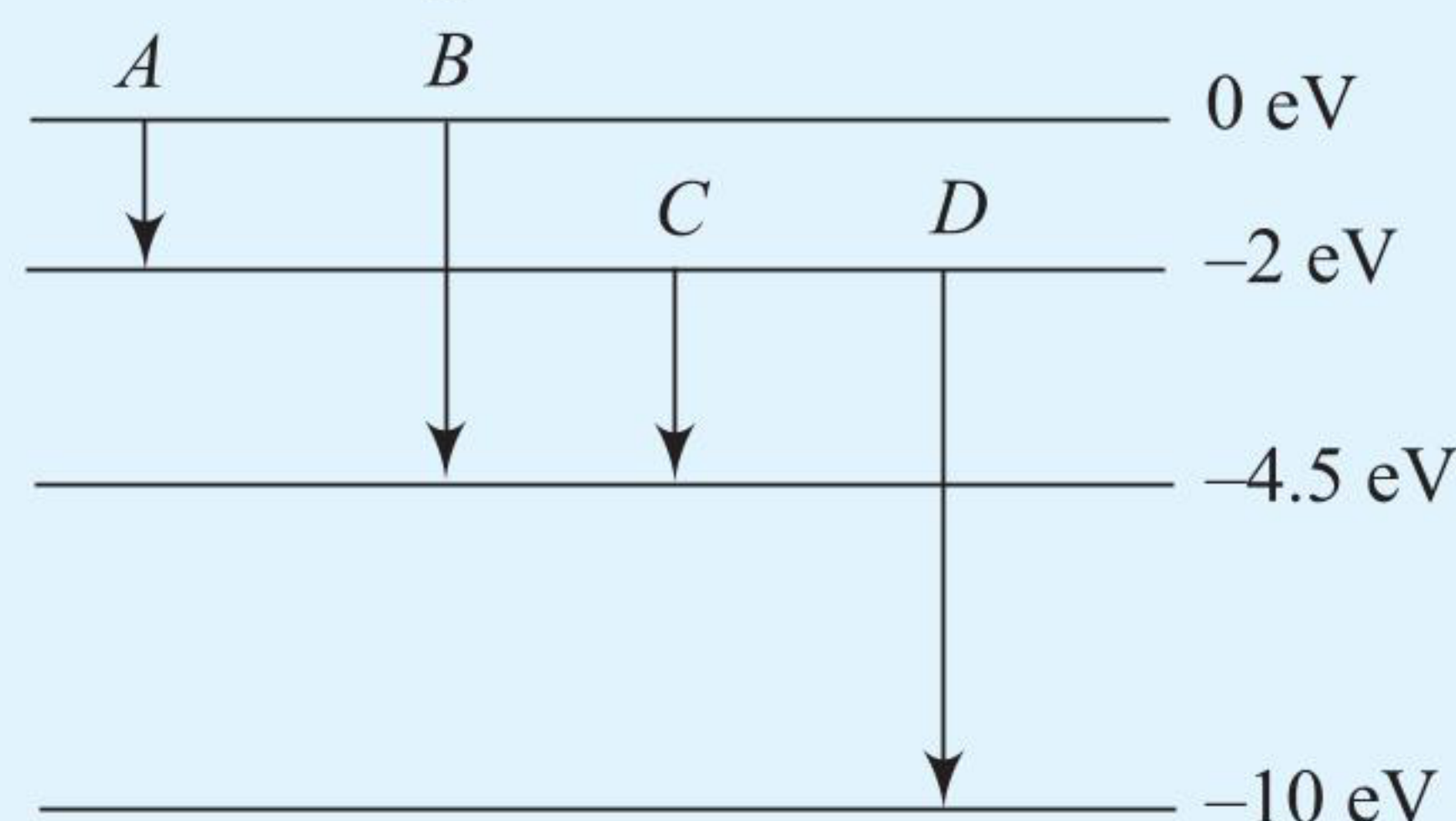
- Three energy levels of an atom are shown in figure. The wavelength corresponding to three possible transitions are λ_1 , λ_2 , and λ_3 . Find the value of λ_3 in terms of λ_1 and λ_2 .



- A hydrogen atom in a state of binding energy 0.85 eV makes a transition to a state of excitation energy of 10.2 eV.
 - What is the initial state of the hydrogen atom?
 - What is the final state of the hydrogen atom?
 - What is the wavelength of the photon emitted?
- Calculate (a) the wavelength and (b) the frequency of the H_β line of the Balmer series for hydrogen.
- Find the largest and shortest wavelengths in the Lyman series for hydrogen. In what region of the electromagnetic spectrum does each series lie?
- An imaginary particle has a charge equal to that of an electron and mass 100 times the mass of the electron. It moves in a circular orbit around a nucleus of charge $+4e$. Take the mass of the nucleus to be infinite. Assuming that Bohr's model is applicable to this system:
 - Derive an expression for the radius of n^{th} Bohr orbit.
 - Find the wavelength of the radiation emitted when the particle jumps from fourth orbit to second orbit.
- Hydrogen gas in the atomic state is excited to an energy level such that the electrostatic potential energy of H atom becomes -1.7 eV. Now, the photoelectric plate having $W = 2.3$ eV is exposed to the emission spectra of this gas. Assuming all the transitions to be possible, find the minimum de Broglie wavelength of ejected photoelectrons.
- Taking into account the motion of the nucleus of a hydrogen atom, find the expressions for the electron's binding energy in the ground state and for the Rydberg constant. How much (in percent) do the binding energy and the Rydberg constant, obtained without taking into account the motion of the nucleus, differ from the more accurate corresponding values of these quantities?
- An electron having energy 20 eV collides with a hydrogen atom in the ground state. As a result of the collision, the atom is excited to a higher energy state and the electron is scattered with reduced velocity. The atom subsequently returns to its ground state with emission of radiation of wavelength 1.216×10^{-7} m. Find the velocity of the scattered electron.
- According to classical physics, an electron in periodic motion will emit electromagnetic radiation with the same frequency as that of its revolution. Compute this value

for hydrogen atom in n th quantum state. Under what conditions does Bohr's quantum theory permit emission of such photons due to transitions between adjoining orbits? Discuss the result obtained.

10. The energy levels of an atom are as shown in figure. Which one of these transitions will result in the emission of a photon of wavelength 275 nm?



11. The ground state energy of hydrogen atom is -13.6 eV.
 (a) What is the kinetic energy of an electron in the second excited state?
 (b) If the electron jumps to the ground state from the second excited state, calculate the wavelength of the spectral line emitted.
12. Suppose that means were available for stripping 28 electrons from ${}_{29}\text{Cu}$ in vapour of this metal.
 (a) Compute the first three energy levels for the remaining electron.
 (b) Find the wavelengths of the spectral lines of the series for which $n_1 = 1, n_2 = 2, 3, 4$. What is the ionization potential for the last electron?

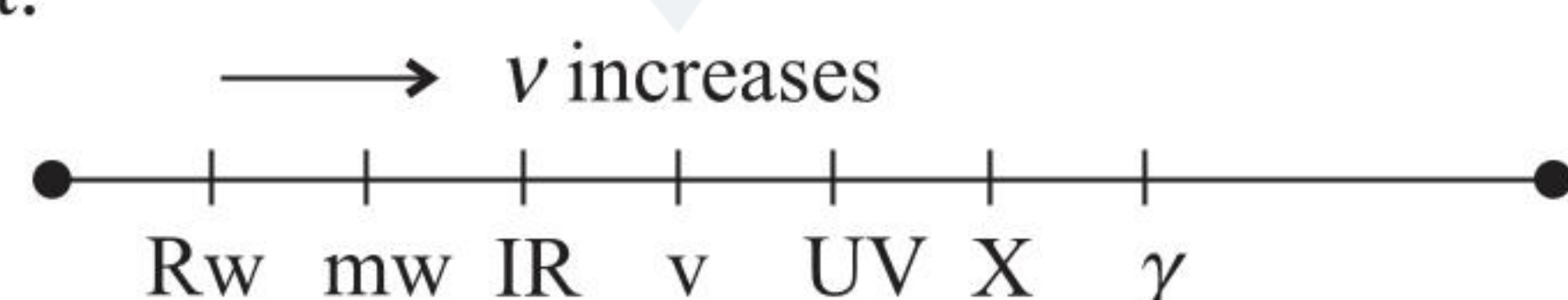
ANSWERS

1. $\frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$ 2. (a) 4 (b) 2 (c) $4860 \text{ \AA}$
 3. (a) $4.9 \times 10^{-7} \text{ m}$ (b) $6.12 \times 10^{14} \text{ Hz}$
 4. $215 \text{ \AA}, 911 \text{ \AA}$; UV region
 5. (a) $r_n = \frac{n^2 h^2 \epsilon_0}{400 \pi m e^2}$ (b) $3.03 \text{ \AA}$
 6. $3.8 \text{ \AA}$ 7. 0.055% 8. $1.86 \times 10^6 \text{ m s}^{-1}$
 10. Transition B 11. (a) 1.51 eV (b) $1.027 \text{ \AA}$
 12. $-11.44 \text{ keV}, -2.86 \text{ keV}, -1.27 \text{ keV}$; 11.44 kV

X-RAYS

X-rays are electromagnetic radiations of very short wavelength ($0.1 \text{ \AA}$ to $100 \text{ \AA}$) and high energy which are emitted when fast moving electrons or cathode rays strike a target of high atomic mass. These rays are invisible to eye. They are electromagnetic waves and have speed $c = 3 \times 10^8 \text{ m s}^{-1}$ in vacuum.

Its photons have energy around 1000 times more than the visible light.



When fast-moving electrons having energy of the order of several keV strike the metallic target, then X-rays are produced.

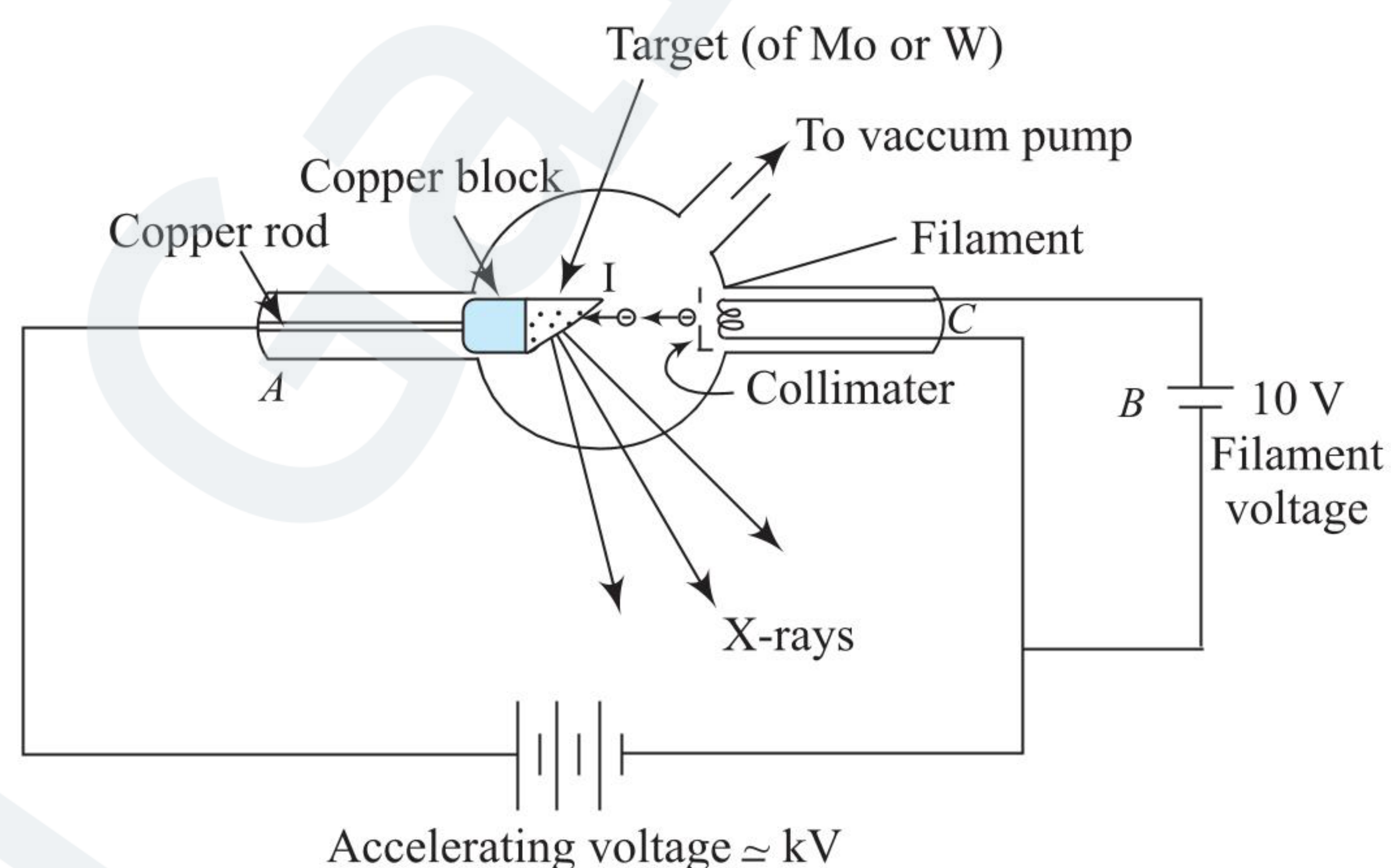
DISCOVERY OF X-RAYS

X-rays were discovered by Roentgen (1895) who found that a discharge tube, operating at low pressure and high voltage, emitted

a radiation that caused a fluorescent screen in the neighborhood to glow brightly. Crystals of barium platinocyanide also showed fluorescence. Results were same if the discharge tube was wrapped in black paper to prevent visible light. This indicated that some unknown radiation (X-rays) were responsible for fluorescence. Roentgen then confirmed that X-rays are emitted when cathode rays (electrons) strike the wall of discharge tube.

PRODUCTION OF X-RAYS (COOLIDGE'S TUBE)

X-rays are produced when energetic (fast moving) electrons strike a target such as a metal piece. When electrons collide with the atoms of solid, they lose their kinetic energy which is converted into radiant energy in the form of X-rays. Figure shows the essential features of a modern X-ray tube developed by Coolidge.



Coolidge's X-ray tube consists of a glass bulb exhausted to nearly perfect vacuum. The cathode C is the source of electrons by using a heated filament getting supply from battery B . The anode is made of solid copper bar A . A high melting metal like platinum or tungsten is embedded at the end of copper rod and serves as target T . A high dc voltage V (50 kV) is maintained between cathode and anode.

The melting point, specific heat capacity, and atomic number of the target should be high. When voltage is applied across the filament, then the filament on being heated emits electrons from it. For giving the beam shape to electrons, collimator is used. Now when the electrons strike the target, then X-rays are produced.

When electrons strike with the target, some part of energy is lost and converted into heat. Since the target should not melt or it should absorb the heat, so the melting point and specific heat of the target should be high.

Here a copper rod is attached so that heat produced can go behind and it can absorb heat and the target does not get heated very high.

For more energetic electrons, accelerating voltage is increased.

For more number of photons, voltage across the filament is increased.

The energetic electrons strike the target and the X-ray are produced. Only about 1–10% of the energy of the electrons is converted to X-rays and the rest is converted into heat. The target T as a result becomes very hot and therefore should have high melting point. The heat generated is dissipated through the copper rod and the anode is cooled by water flowing through the anode.

The nature of emitted X-rays depends on:

1. the current in the filament F , and
2. the voltage between the filament and the anode.
 - (a) An increase in the filament current increases the number of electrons it emits. Larger number of electrons means an intense beam of X-rays is produced. This way we can control the quantity of X-rays, i.e., intensity of X-rays.
 - (b) An increase in the voltage of the tube increases the kinetic energy of electrons [$eV = (1/2)mv^2$]. When such highly energetic beam of electrons is suddenly stopped by the target, an energetic beam of X-rays is produced. This way we can control the quality of X-rays, i.e., penetration power of X-rays.
 - (c) Based on penetrating power, X-rays are classified into two types: Hard X-rays and Soft X-rays. The ones having high energy and hence high penetration power are Hard X-rays and the ones with low energy and hence low penetration power are Soft X-rays.

PROPERTIES OF X-RAYS

1. These are highly penetrating rays and can pass through several materials which are opaque to ordinary light.
2. They ionize the gas through which they pass. While passing through a gas, they knock out electrons from several of the neutral atoms, leaving these atoms with +ve charge.
3. They cause fluorescence in several materials. A plate coated with barium platinocyanide, ZnS (zinc sulphide), etc. becomes luminous when exposed to X-rays.
4. They affect photographic plates especially designed for the purpose.
5. They are not deflected by electric and magnetic fields, showing that they are not charged particles.
6. They show all the properties of the waves except refraction. They show diffraction patterns when passed through a crystal which behaves like a grating.

APPLICATIONS OF X-RAYS

X-rays have important and useful applications in surgery, medicine, engineering, and studies of crystal structures.

Scientific applications: The diffraction of X-rays at crystals opened new dimension to X-rays crystallography. Various diffraction patterns are used in determining internal structure of crystals. The spacing and positions of atoms of a crystal can be precisely determined using Bragg's law.

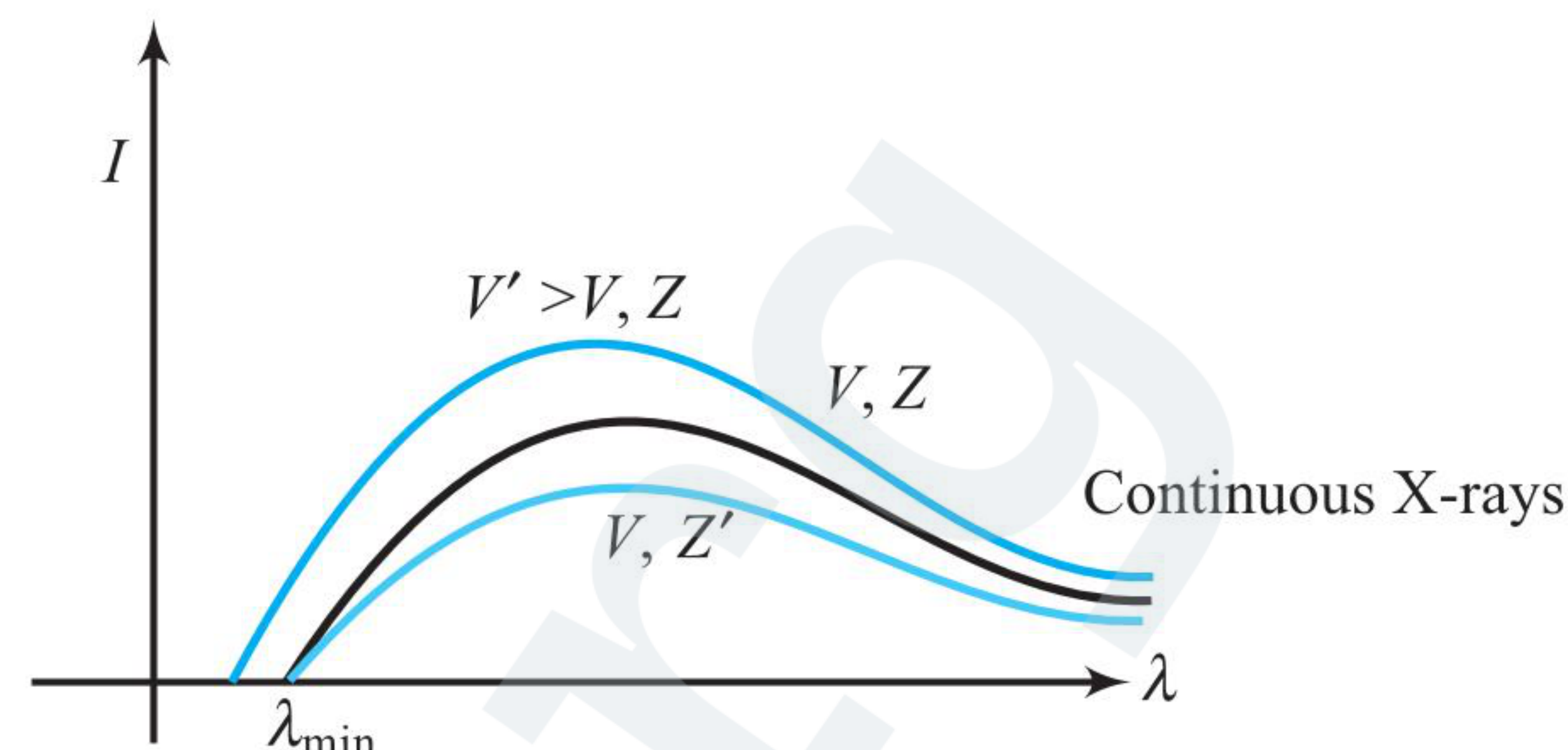
Industrial applications: Since X-rays can penetrate through various materials, they are used in industry to detect defects in metallic structures of big machines, railway tracks and bridges. X-rays are used to analyze the composition of alloys and pearls.

In radio therapy: X-rays can cause damage to the tissues of body (cells are ionized and molecules are broken). So, X-rays damage the malignant growths such as cancer and tumors which are dangerous to life, when it is used in proper and controlled intensities.

In medicine and surgery: X-rays are absorbed more in heavy elements than the lighter ones. Since bones (containing calcium and phosphorus) absorb more X-rays than the surrounding tissues (containing light elements such as H, C, O), their shadow

is casted on the photographic plate. So, the cracks or fracture in bones can be easily located. Similarly, intestine and digestive system abnormalities are also detected by X-rays.

Variation of intensity of X-rays with λ is plotted as shown in figure.



The minimum wavelength corresponds to the maximum energy of the X-rays which in turn is equal to the maximum kinetic energy (eV) of the striking electrons. Thus,

$$eV = h\nu_{\max} = \frac{hc}{\lambda_{\min}}$$

$$\lambda_{\min} = \frac{hc}{eV} = \frac{12400}{V_{\text{(in volts)}}} \text{ \AA}$$

We see that cut-off wavelength $\lambda_{\min}$ depends only on accelerating voltage applied between the target and filament. It does not depend upon material of the target, it is same for two different metals (Z and Z').

ILLUSTRATION 6.43

An X-ray tube operates at 20 kV. A particular electron loses 5% of its kinetic energy to emit an X-ray photon at the first collision. Find the wavelength corresponding to this photon.

Sol. Kinetic energy acquired by the electron is $K = 20 \times 10^3 \text{ eV}$. The energy of the photon is $0.05 \times 20 \times 10^3 \text{ eV} = 10^3 \text{ eV}$

$$\text{Thus, } \frac{hc}{\lambda} = 10^3 \text{ eV}$$

$$\Rightarrow \lambda = \frac{(4.14 \times 10^{-15} \text{ eV-s}) \times (3 \times 10^8 \text{ m s}^{-1})}{10^3 \text{ eV}}$$

$$= \frac{1242 \text{ eV-nm}}{10^3 \text{ eV}} = 1.24 \text{ nm}$$

ILLUSTRATION 6.44

An X-ray tube operates at 20 kV. Find the maximum speed of the electrons striking the anode, given the charge of electron is $1.6 \times 10^{-19} \text{ coulomb}$ and mass of electron is $9 \times 10^{-31} \text{ kg}$.

Sol. When an electron of charge e is accelerated through a potential difference V , it acquires energy eV . If m be the mass of the electron and $v_{\max}$ the maximum speed of electron, then

$$\frac{1}{2}mv_{\max}^2 = eV \text{ or } v_{\max} = \sqrt{\left(\frac{2eV}{m}\right)}$$

Substituting the given values, we get

$$v_{\max} = \sqrt{\left(\frac{2 \times (1.6 \times 10^{-19}) \times 20,000}{9 \times 10^{-31}}\right)} = 8.4 \times 10^7 \text{ m s}^{-1}$$

ILLUSTRATION 6.45

The voltage applied to an X-ray tube being increased $\eta = 1.5$ times, the short wave limit of an X-ray continuous spectrum shifts by $\Delta\lambda = 26$ pm. Find the initial voltage applied to the tube.

Sol. $\lambda_{\min} = \frac{ch}{eV}$

Here, $\lambda_1 = \frac{ch}{eV_1}$ and $\lambda_2 = \frac{ch}{eV_2}$

or $\frac{ch}{e} \left[\frac{1}{V_2} - \frac{1}{V_1} \right] = \lambda_2 - \lambda_1 = \Delta\lambda = 26 \times 10^{-12}$

or $\frac{ch}{e} \left[\frac{2}{3V_1} - \frac{1}{V_1} \right] = 26 \times 10^{-12} \quad (\because V_2 = 1.5 V_1)$

Solving for V_1 , we get $V_1 = 16000$ V or 16 kV

X-RAY ABSORPTION

The intensity of X-rays at any point may be defined as the energy falling per second per unit area held perpendicular to the direction of energy flow. The intensity of an X-ray beam decreases during its passage through the sheet of any material. The decrease in the intensity of X-rays is due to the absorption of X-rays by the material.

Let I_0 be the intensity of incident beam and I be the intensity of beam after penetrating a thickness x of a material, then $I = I_0 e^{-mx}$; where m is the coefficient of absorption or absorption coefficient of the material. The absorption coefficient depends upon wavelength of X-rays, density of material, and atomic number of material. The elements of high atomic mass and high density absorb X-rays to a higher degree.

X-RAY SPECTRA AND ORIGIN OF X-RAYS

Experimental observation and studies of spectra of X-rays reveal that X-rays are of two types and so are their respective spectra: characteristic X-rays and continuous X-rays.

Characteristic X-Rays

These X-rays are called characteristic X-rays because they are characteristic of the element used as target anode. Characteristic X-rays have a line spectral distribution unlike the continuous X-rays. The wavelength spectrum of the X-frequencies corresponding to these lines are the characteristic of the material or the target, i.e., anode material.

The spectrum of this group consists of several radiations with specific sharp wavelengths and frequency similar to the spectrum (line) of atoms like hydrogen. The wavelengths of this group show characteristic discrete radiations emitted by the atoms of the target material. The characteristic X-rays spectra help us to identify the element of target material.

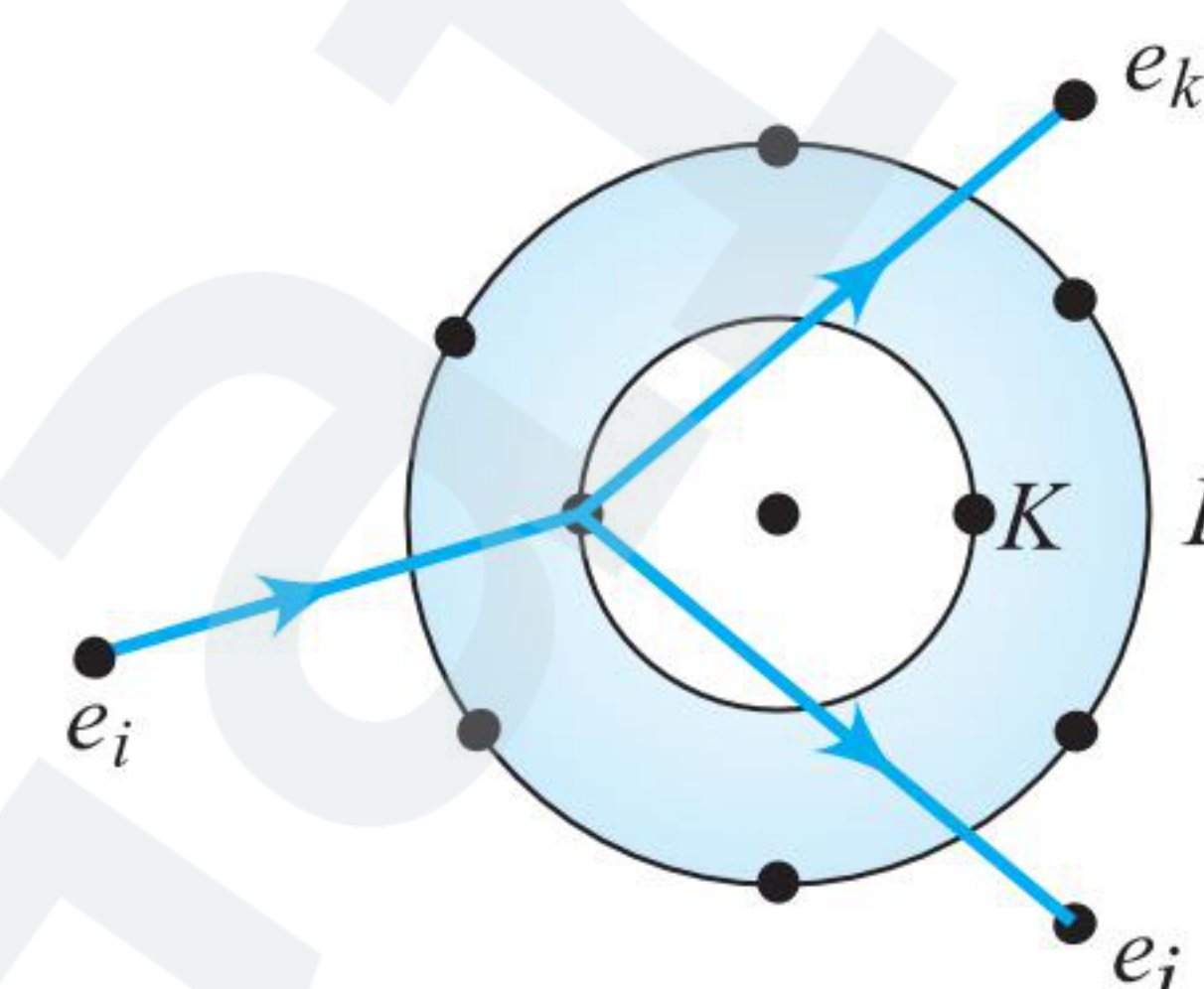
Origin of Characteristic X-Rays

1. When the atoms of the target material are bombarded with high energy electrons (or hard X-rays), which possess enough energy to penetrate into the atom, they knockout the electron of inner shell (say K shell, $n = 1$). When an electron is missing in the K shell, an electron from next

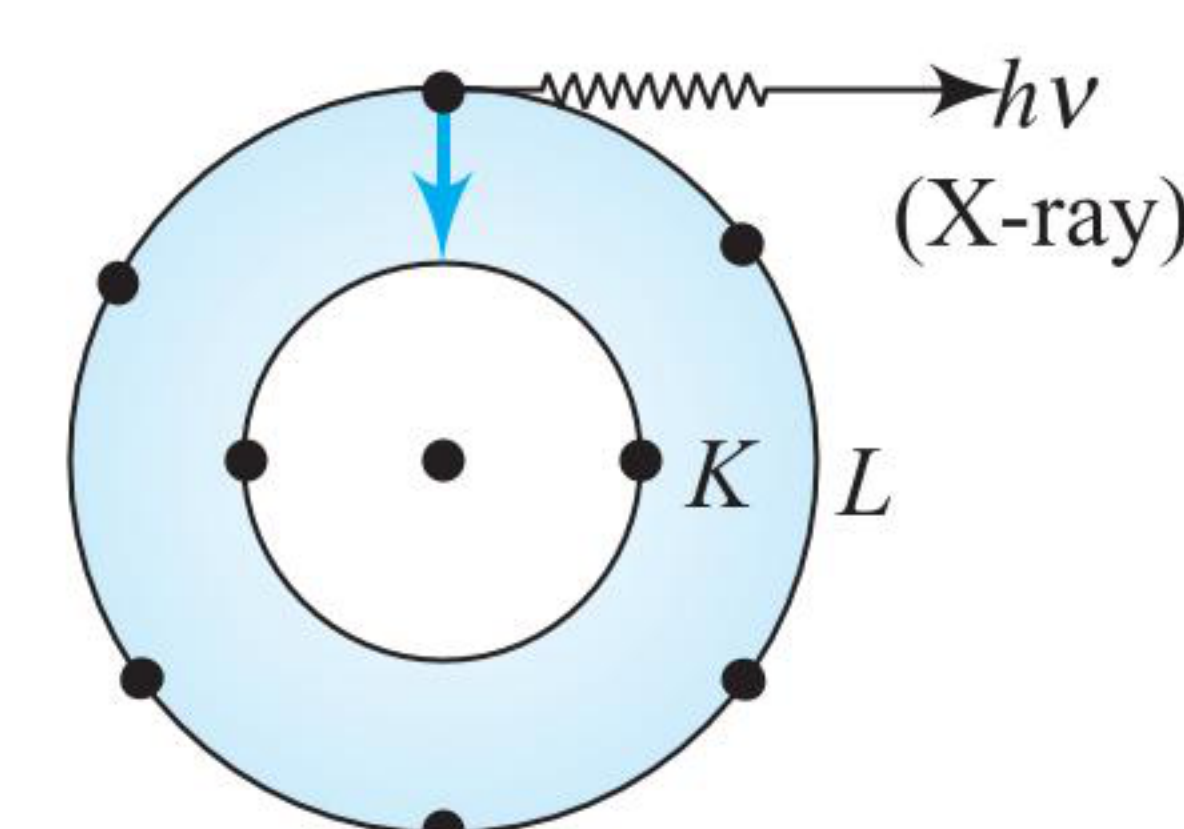
upper shell makes a quantum jump to fill the vacancy in the K shell. In the transition process, the electron radiates energy whose frequency lies in the X-ray region. The frequency of emitted radiation (i.e., of photon) is given by

$$\nu = RZ_e^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

where R is constant and Z_e is effective atomic number. Generally, Z_e is taken to be equal to $Z - s$, where Z is proton number or atomic number of the element and s is called the screening constant. Due to the presence of other electrons, the charge of the nucleus as seen by the electron will be different in different shells.

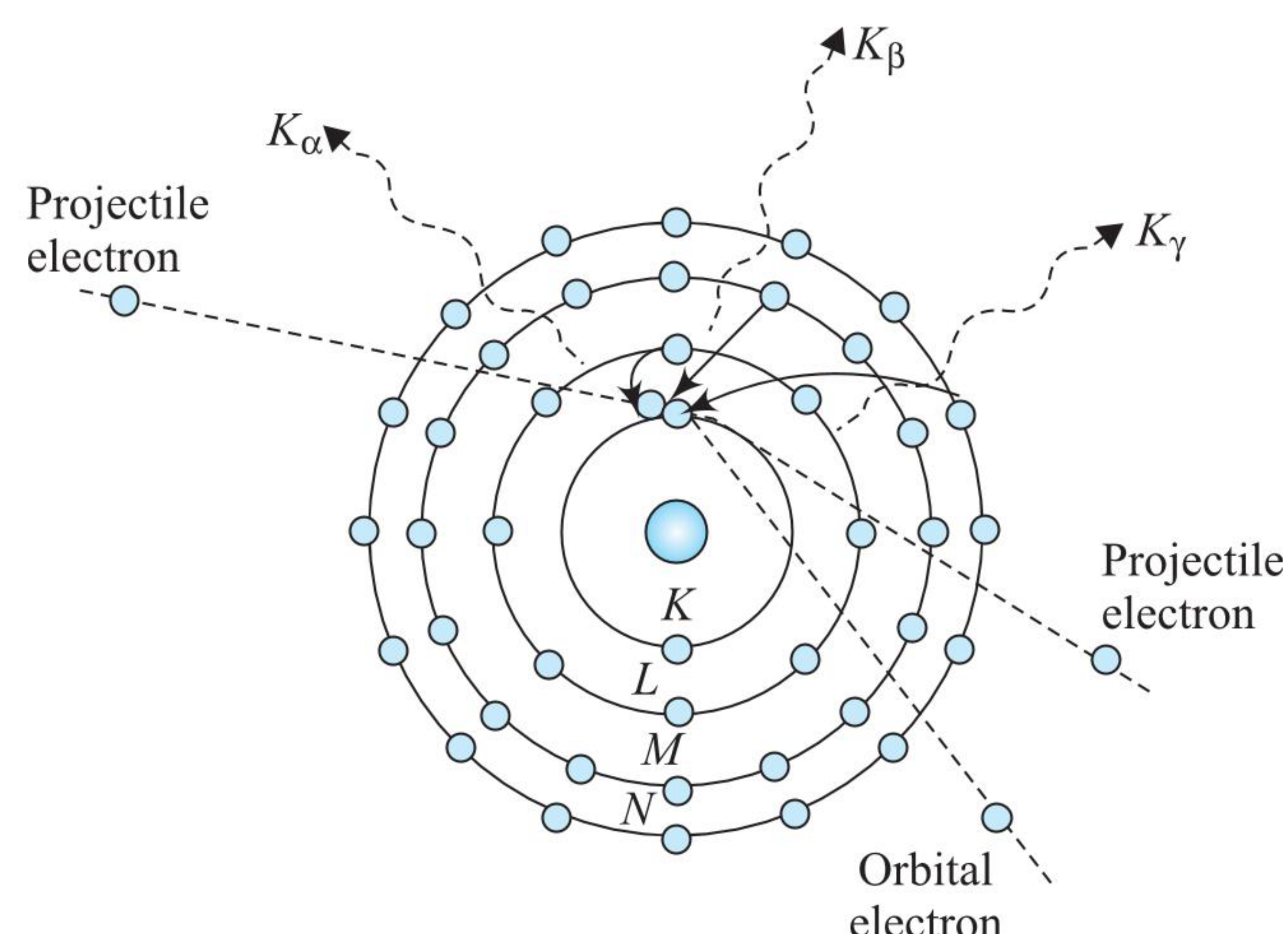


(a) Knocking out e' of K shell by incident electron e_i

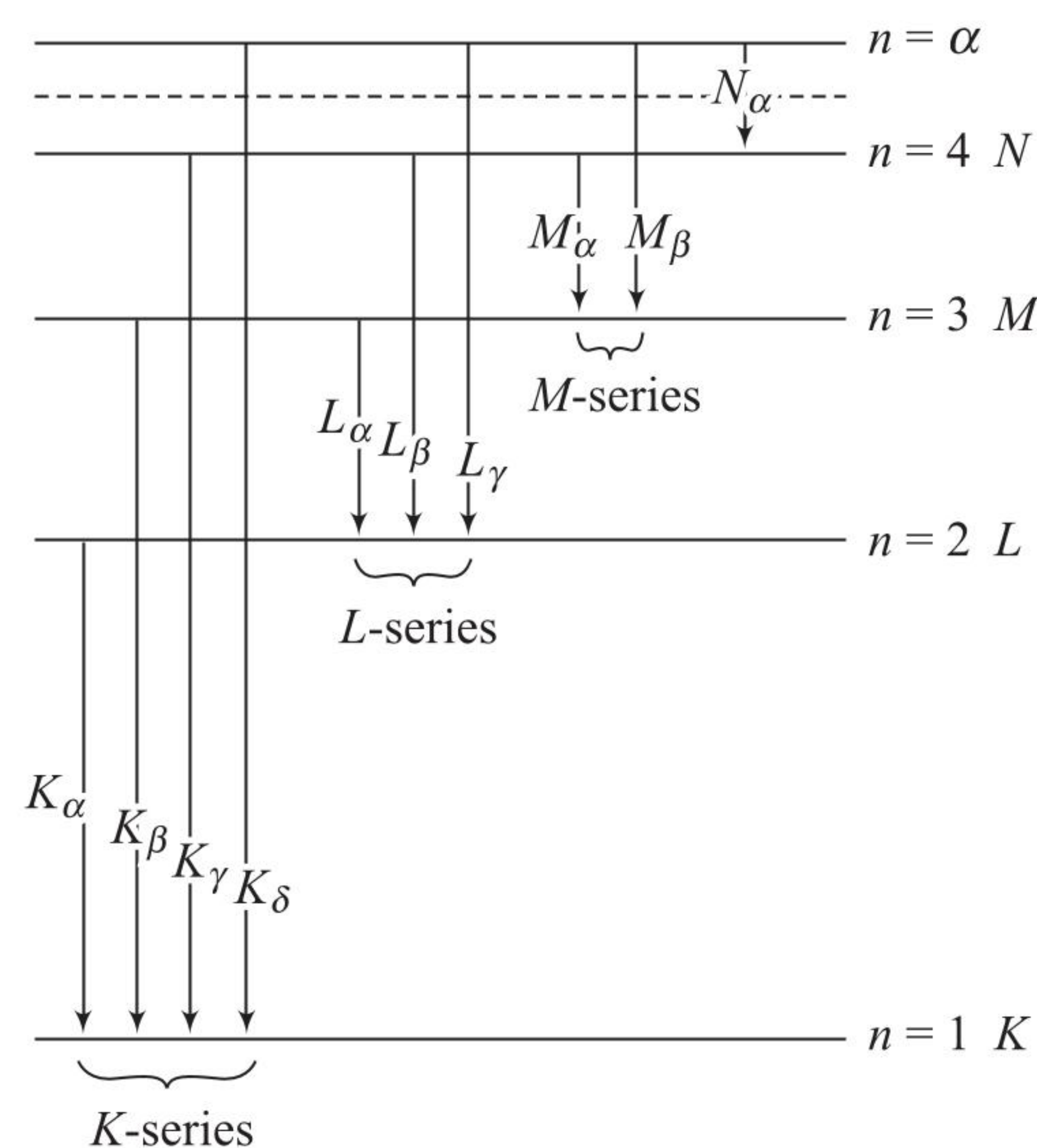


(b) Emission of X-ray photon (K_α series)

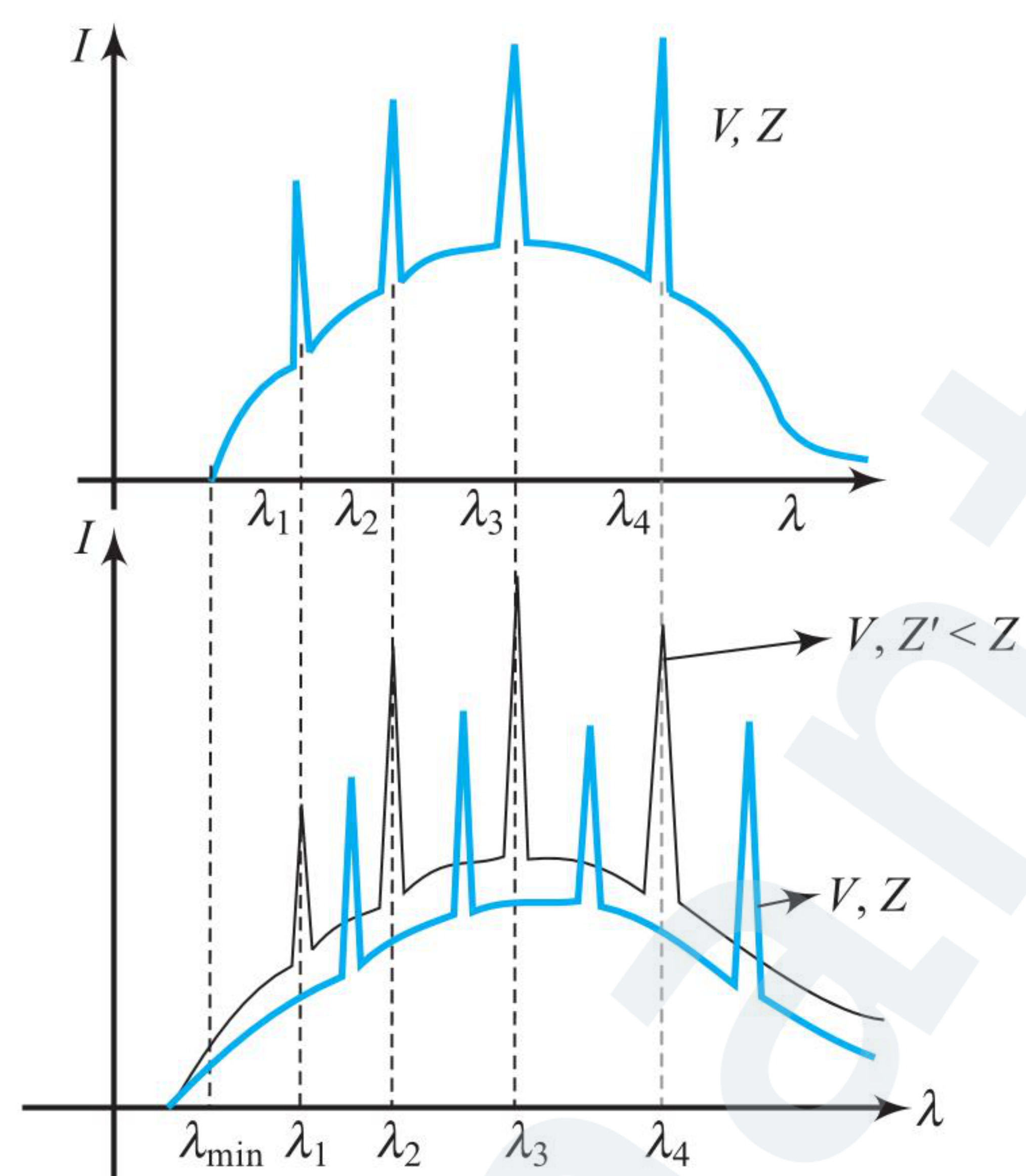
2. Another vacancy is now created in the L shell which is again filled up by another electron jump from one of the upper shell (M) which results in the emission of another photon, but of different X-ray frequency. This transition continues till outer shells are reached, thus, resulting in the emission of series of spectral lines.
3. The transitions of electrons from various outer shells to the inner most K shell produces a group of X-ray lines called as K -series. These radiations are most energetic and most penetrating. K -series is further divided into K_α , K_β , K_γ ,... depending upon the outer shell from which the transition is made (see figure).



4. If cavity is created in L shell, then according to the transition of electrons from high orbits there will be $L_\alpha, L_\beta, \dots$ lines, and this is called L -series. There may also be M -series as shown in Fig. (a). Figure (b) shows the wavelength spectrum for the characteristic X-rays when cavity is created in K -shell.



(a)



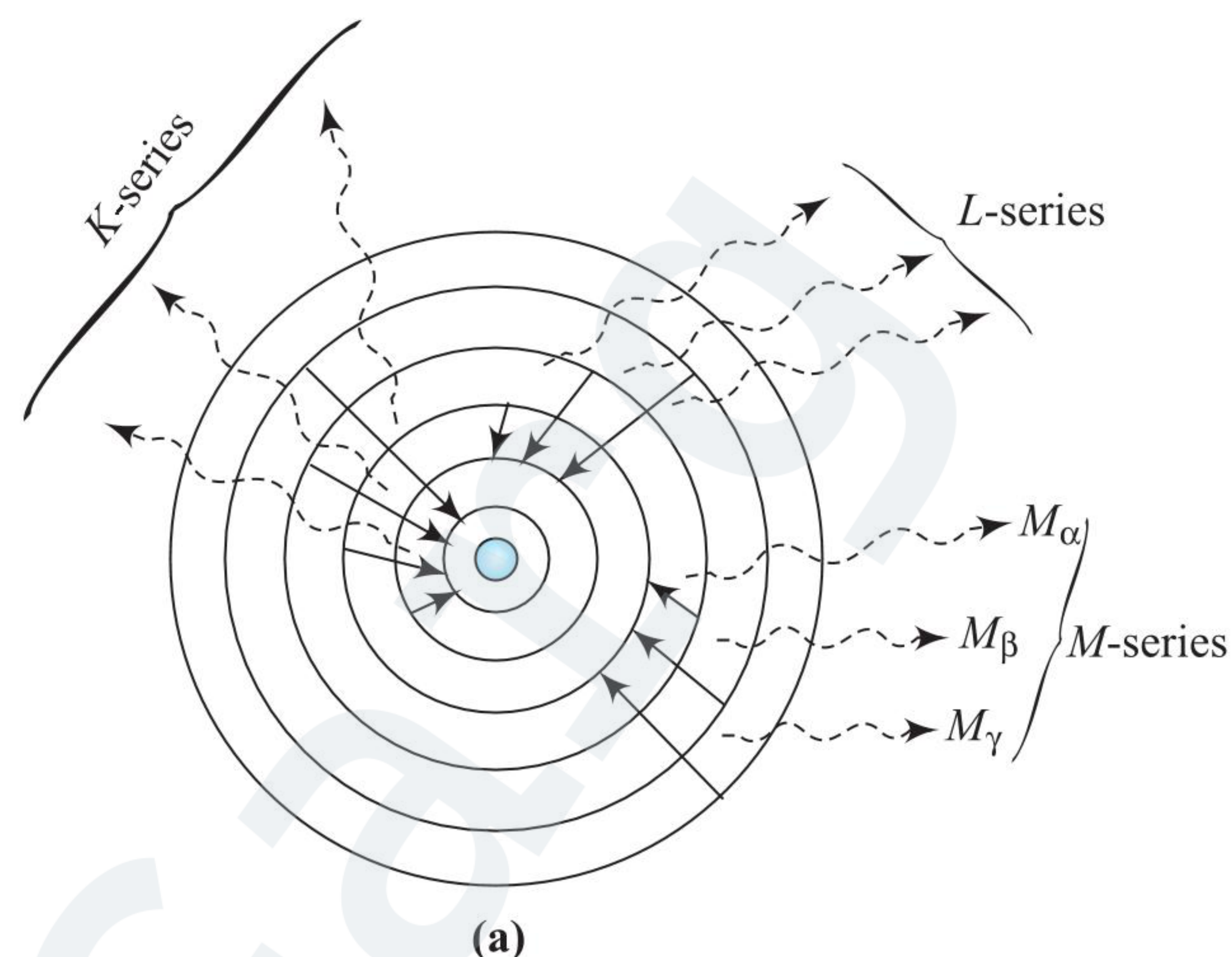
(b)

The sharp peaks obtained in graph are known as characteristic X-rays because they are characteristic of the target material.

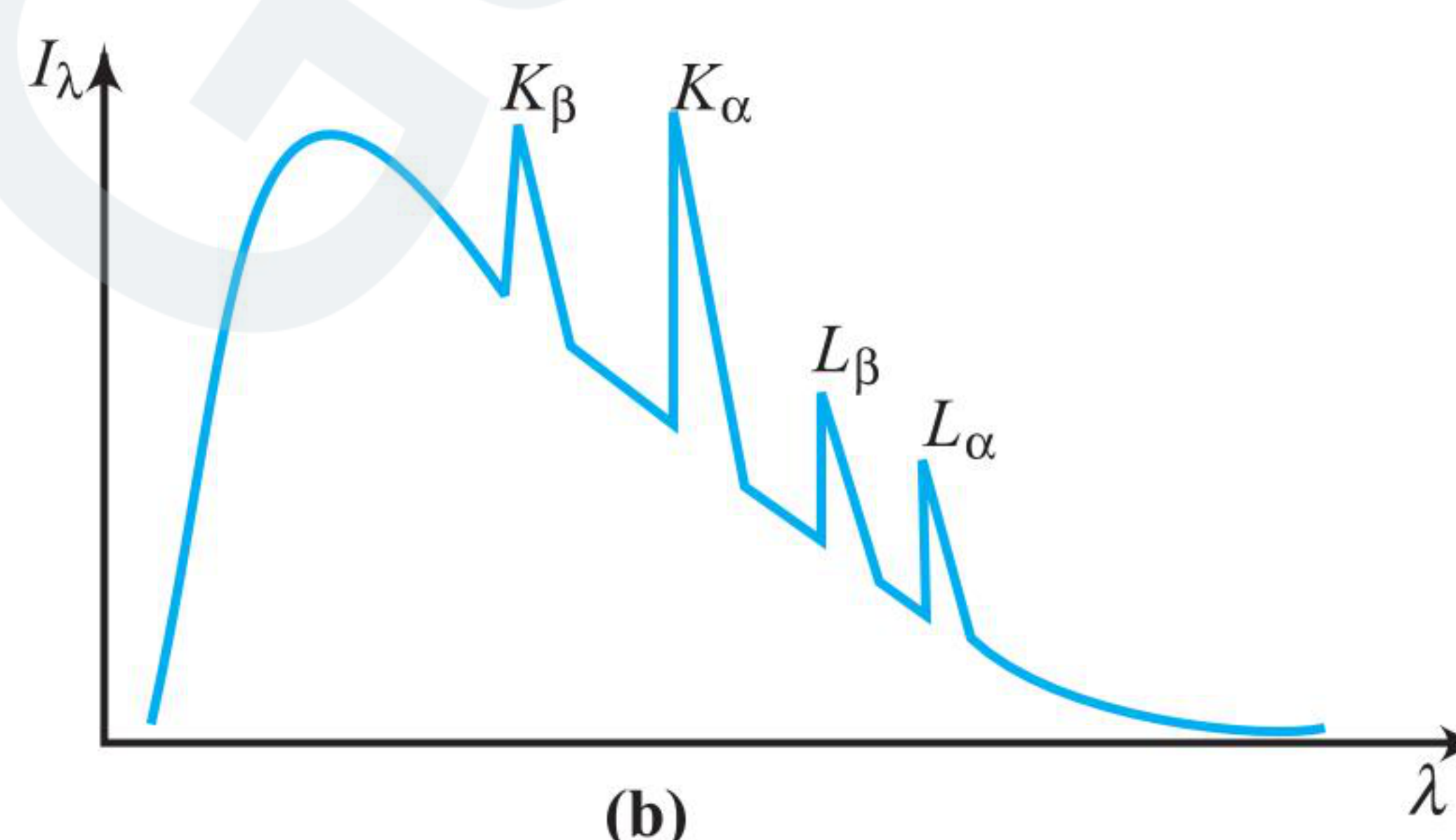
The characteristic wavelengths of the material having atomic number Z are called **characteristic X-rays** and the spectrum obtained is called **characteristic spectrum**. If a target material of atomic number Z' is used, then peaks are shifted.

Thus, the characteristic lines of the characteristic X-rays depend only on the target material and not on the accelerating voltage. One more thing we can see here that the characteristic X-rays are emitted only when the projectile electron makes a collision with another bound electron of the atom of the target material. When cathode rays pass through the target, then the probability of a projectile electron to collide with the bound electron is very less. Majority of the projectile electrons will pass

through the anode without making the collision and they produce continuous X-rays. So, always both types of X-rays, continuous and characteristic, will be emitted. X-rays of only a single type cannot be emitted.



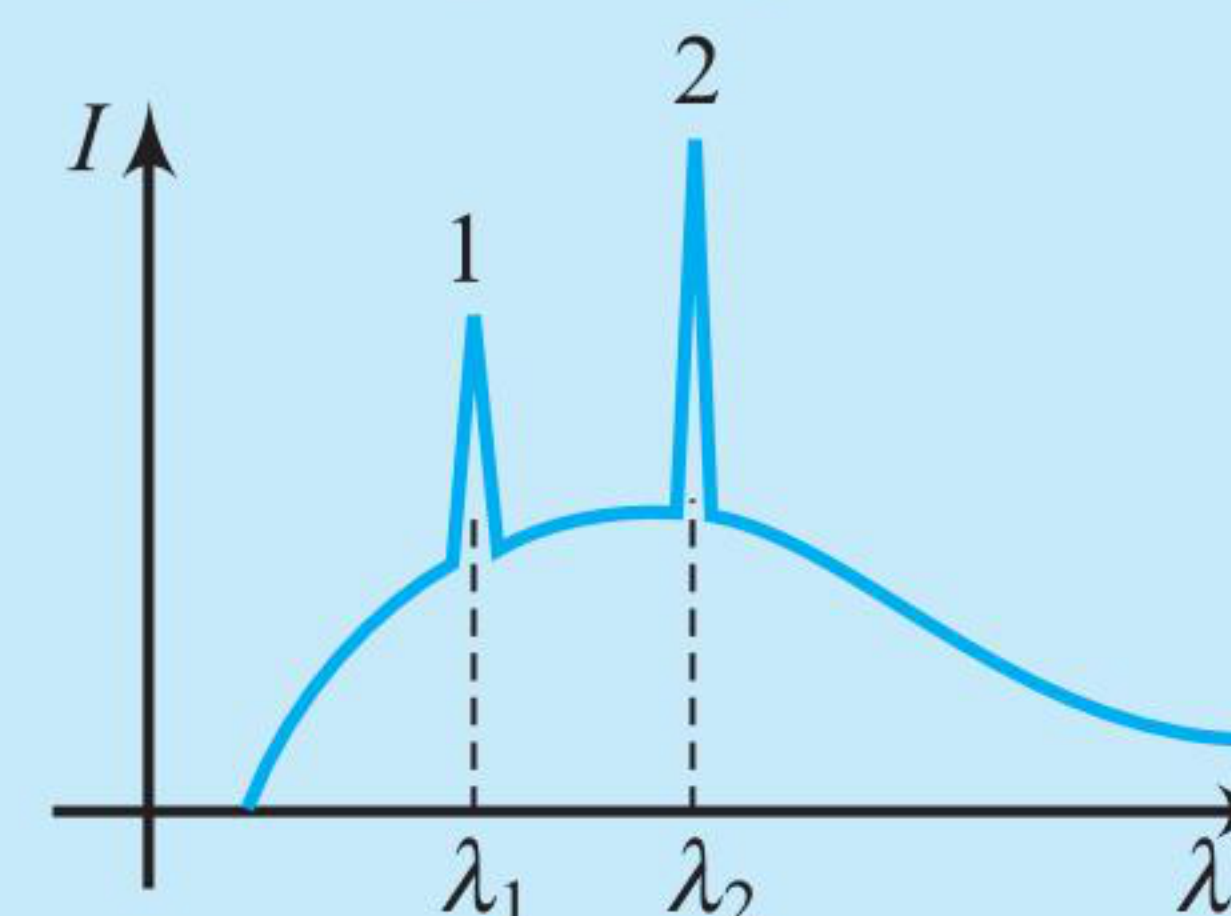
(a)



(b)

ILLUSTRATION 6.46

In figure, find which is K_α and K_β .



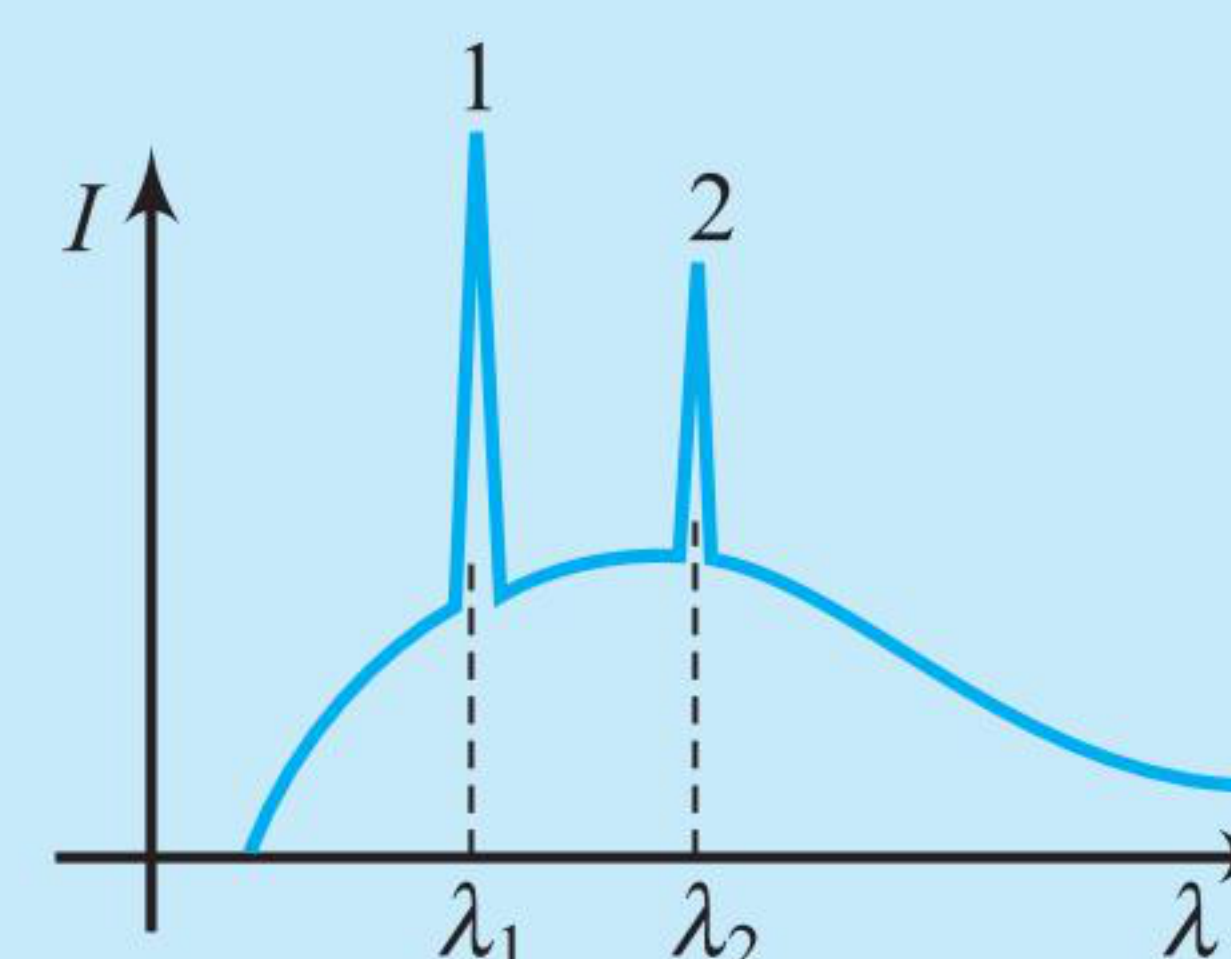
Sol. $\Delta E = \frac{hc}{\lambda}, \lambda = \frac{hc}{\Delta E}$

Since energy difference of K_α is less than K_β ,
i.e., $\Delta E_{K_\alpha} < \Delta E_{K_\beta}$, therefore, $\lambda_{K_\alpha} > \lambda_{K_\beta}$

So, peak 1 is K_β and peak 2 is K_α

ILLUSTRATION 6.47

In figure, find which is K_α and L_α .



Sol. $\Delta E_{K_\alpha} > \Delta E_{L_\alpha} \Rightarrow \lambda_{K_\alpha} < \lambda_{L_\alpha}$, so peak 1 is K_α and peak 2 is L_α

ILLUSTRATION 6.48

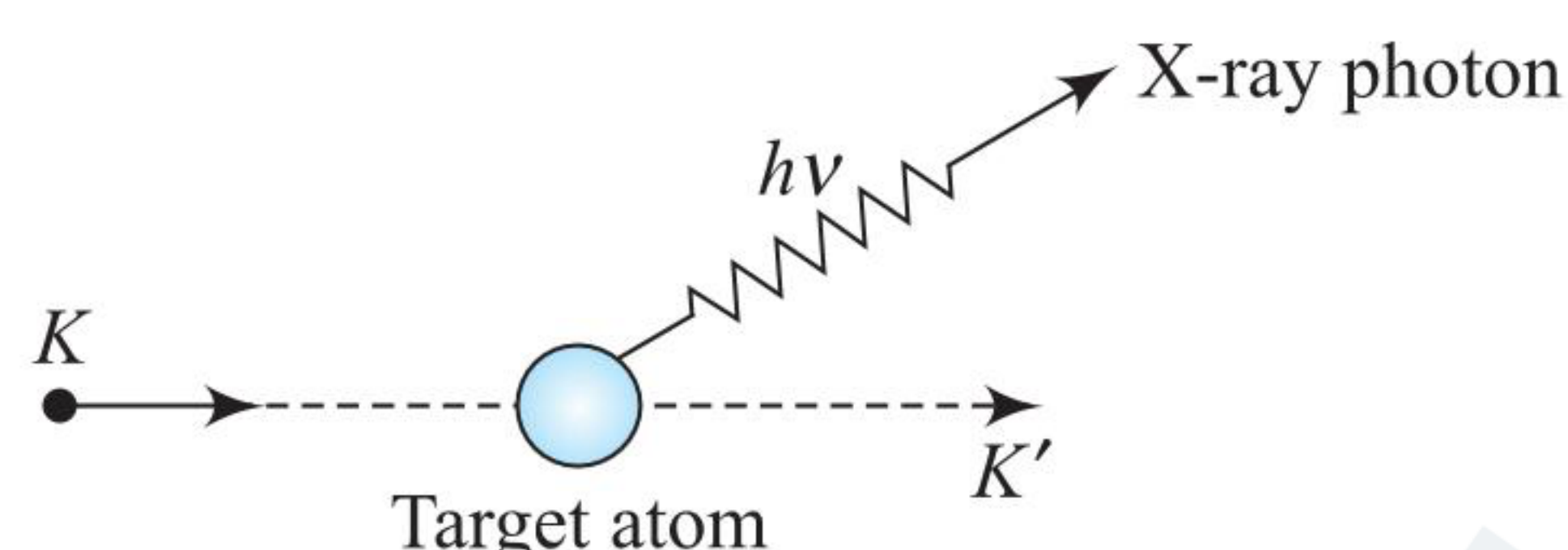
The K_α X-ray emission line of tungsten occurs at $\lambda = 0.021$ nm. What is the energy difference between K and L levels in this atom?

Sol. The origin of K_α X-ray can be understood on the basis of the Bohr model of atom. If there is only one electron in the K shell, and fill up this vacancy. An electron from higher shell will get de-excited. In this process of de-excitation of electron, a photon is emitted during this transition which is the X-ray.

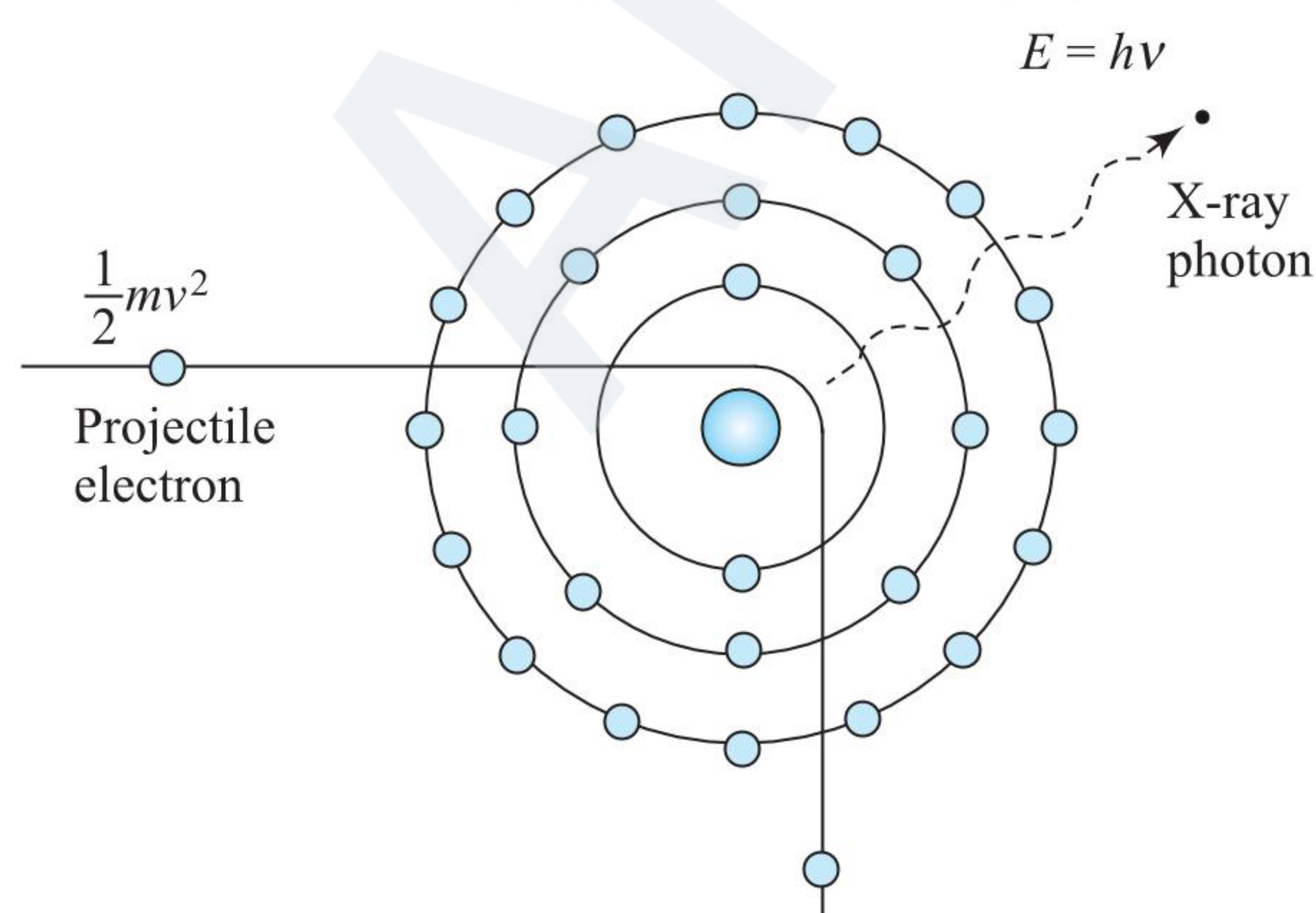
$$\begin{aligned}\Delta E &= \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{0.021 \times 10^{-9}} \\ &= 9.46 \times 10^{-15} \text{ J} \\ &= 5.9 \times 10^4 \text{ eV} = 59 \text{ keV}\end{aligned}$$

Continuous X-Rays

In addition to characteristic X-rays, tubes emit a continuous spectrum also. The characteristic line spectra is superimposed on a continuous X-rays spectra of varying intensities. The wavelengths of the continuous X-rays spectra are independent of the material. One important feature of continuous X-rays is that they end abruptly at a certain lower wavelength for a given voltage.

**Origin of Continuous X-Rays**

The mechanism for the production of continuous X-rays can be explained on the basis of figure. It shows an atom of the anode material of high atomic weight with its electron configuration. In the Coolidge tube, an electron is projected toward the anode with an accelerating voltage V . So, the kinetic energy of the projectile electron will be eV . As shown in the figure, when the projectile electron enters into the extremely high electric field of the nucleus of the atom of the anode material, it experiences strong electric force toward the nucleus of the atom and due to this strong attraction the velocity of this electron, when it emerge from the atom, will be highly reduced and negligible compared



with the initial velocity of the projectile electron. This electron, in the influence of the highly positive nucleus, experiences a very high acceleration and according to the classical theory every accelerated charge particle emits the electromagnetic radiations, so this electron will also emit electromagnetic radiations. These electromagnetic radiations are called X-rays. According to the law of conservation of energy, the energy of these electromagnetic radiations will be equal to the decrease in the kinetic energy of the projectile electron. This amount can be calculated.

If the initial velocity of the projectile electron is v , then as the velocity is gained due to the accelerating voltage V , we have

$$eV = \frac{1}{2}mv^2 \quad \dots(i)$$

$$\text{or } v = \sqrt{\frac{2eV}{m}} \quad \dots(ii)$$

When this electron comes out from the atom of anode material, the speed of this electron will be very less as compared to its initial speed. Thus, the difference of kinetic energy of this electron is emitted in the form of an X-ray photon from the anode atom.

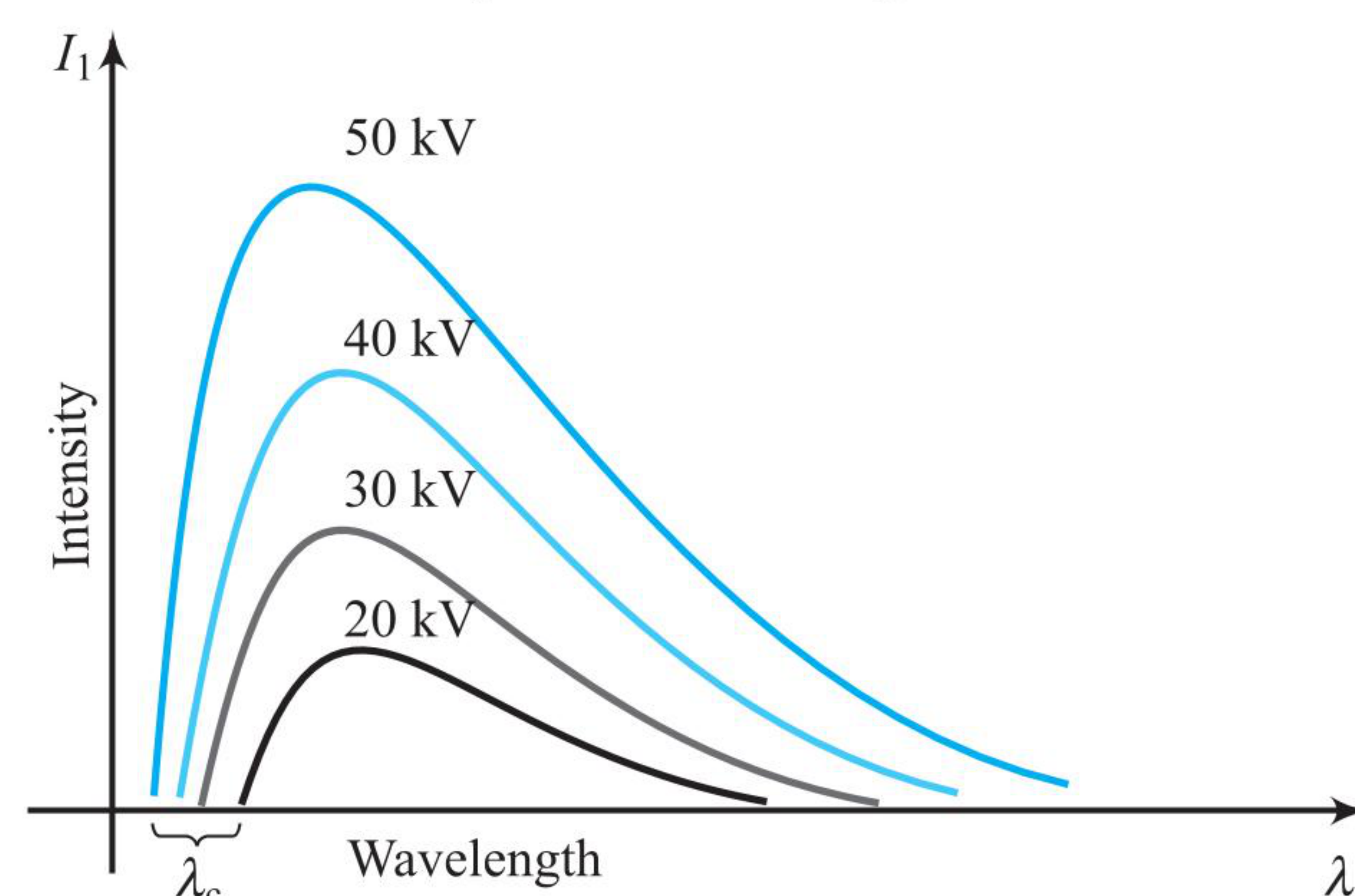
The electrons which pass through the atom very close to the nucleus, will be more accelerated and the energy corresponding to the electrons will be more as compared to those electrons which pass through the atom at relatively large distance from the nucleus. The maximum energy of X-ray photon will be corresponding to that electron which loses almost all of its energy during passing through the atom. The photon corresponding to this electron will have the shortest wavelength among all the photons radiated by other electrons. If this shortest wavelength is λ_c , then we have

$$\Delta E = \frac{1}{2}mv^2 = eV = \frac{hc}{\lambda_c}$$

$$\text{or } \lambda_c = \frac{hc}{eV} = \frac{12431}{V} \text{ \AA} \quad \dots(iii)$$

This is the minimum wavelength of X-rays emitted from an X-ray tube. Thus, from Eq. (iii), we can see that the maximum energy or minimum wavelength of X-rays emitted depends only on the potential difference applied across the discharge tube.

Thus, we can obtain X-rays in any range λ_c to ∞ by applying an appropriate voltage across the discharge tube, which will fix λ_c and other photons emitted from the tube will have wavelengths more than λ_c and ranging up to ∞ . That is why these X-rays are called continuous X-rays as shown in figure.



As we can see in the graph, the intensity of emitted X-rays will be maximum (maximum number of photons) for a particular value of wavelength at a particular accelerating voltage across the discharge tube. At a particular voltage, the intensity of X-rays can be varied by changing the current in the circuit because the intensity of X-rays (number of photons) is proportional to the number of electrons attacking the anode. The broad continuous spectrum beyond the peak intensity is referred as “Bremsstrahlung.”

ILLUSTRATION 6.49

When $0.50 \text{ \AA}$ X-rays strike a material, the photoelectrons from the K shell are observed to move in a circle of radius 23 mm in a magnetic field of $2 \times 10^{-2} \text{ tesla}$ acting perpendicular to direction of emission of photoelectrons. What is the binding energy of K -shell electrons?

Sol. The velocity of the photoelectrons is found by the relation:

$$evB = m \frac{v^2}{R} \quad \text{or} \quad v = \frac{e}{m} BR$$

The kinetic energy of the photoelectrons is

$$\begin{aligned} K &= \frac{1}{2} mv^2 = \frac{1}{2} \frac{e^2 B^2 R^2}{m} \\ &= \frac{1}{2} \frac{(1.6 \times 10^{-19})^2 (2 \times 10^{-2})^2 (23 \times 10^{-3})^2}{(9.1 \times 10^{-31})} \\ &= 2.97 \times 10^{-15} \text{ J} \\ &= (2.97 \times 10^{-15}) \frac{1}{1.6 \times 10^{-19}} = 18.36 \text{ KeV} \end{aligned}$$

The energy of the incident photon is $E_v = \frac{hc}{\lambda} = \frac{12.4}{0.50} = 24.8 \text{ KeV}$

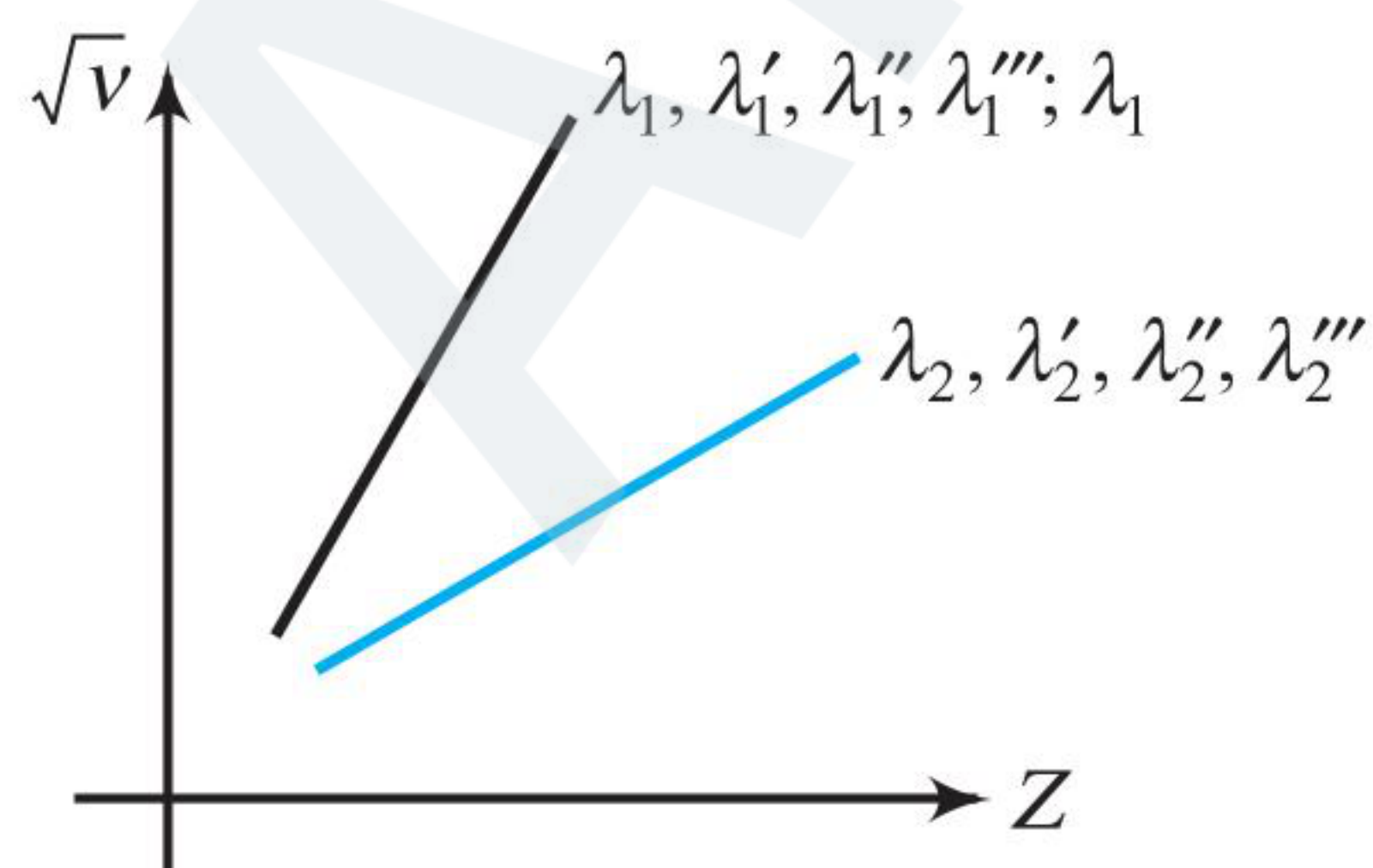
The binding energy is the difference between these two values:

$$\text{BE} = E_v - K = 24.8 - 18.6 = 6.2 \text{ keV}$$

MOSELEY'S LAW

Moseley measured the frequencies of characteristic X-rays for a large number of elements and plotted the square root of frequency against position number in periodic table. He discovered that the plot is very close to a straight line not passing through origin.

The relation of straight line is expressed as $\sqrt{\nu} = a(Z - b)$, where a and b are constants. This relation is called **Moseley's law**. It helps to determine the atomic number Z of an atom. Here, b is the screening constant.



Moseley's observations can be mathematically expressed as $\sqrt{\nu} = a(Z - b)$, where a and b are positive constants for one type of X-rays and for all elements (independent of Z).

Moseley's law can be derived on the basis of Bohr's theory of atom.

$$\text{Frequency of X-rays is given by } \sqrt{\nu} = \sqrt{cR \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)} (Z - b)$$

(For multielectron system)

b is known as the screening constant or shielding constant, and $(Z - b)$ is the effective nuclear charge.

$$\text{For } K_\alpha \text{ line: } n_1 = 1, n_2 = 2, \text{ therefore } \sqrt{\nu} = \sqrt{\frac{3RC}{4}} (Z - b) = a(Z - b)$$

$$\text{Here } a = \sqrt{\frac{3RC}{4}}, \quad [b = 1 \text{ for } K_\alpha \text{ lines}]$$

ILLUSTRATION 6.50

If the K_α radiation of Mo ($Z = 42$) has a wavelength of $0.71 \text{ \AA}$, calculate wavelength of the corresponding radiation of Cu, i.e., K_α for Cu ($Z = 29$) assuming $b = 1$.

Sol. According to Moseley's law: $\sqrt{\nu} = a(Z - 1)$

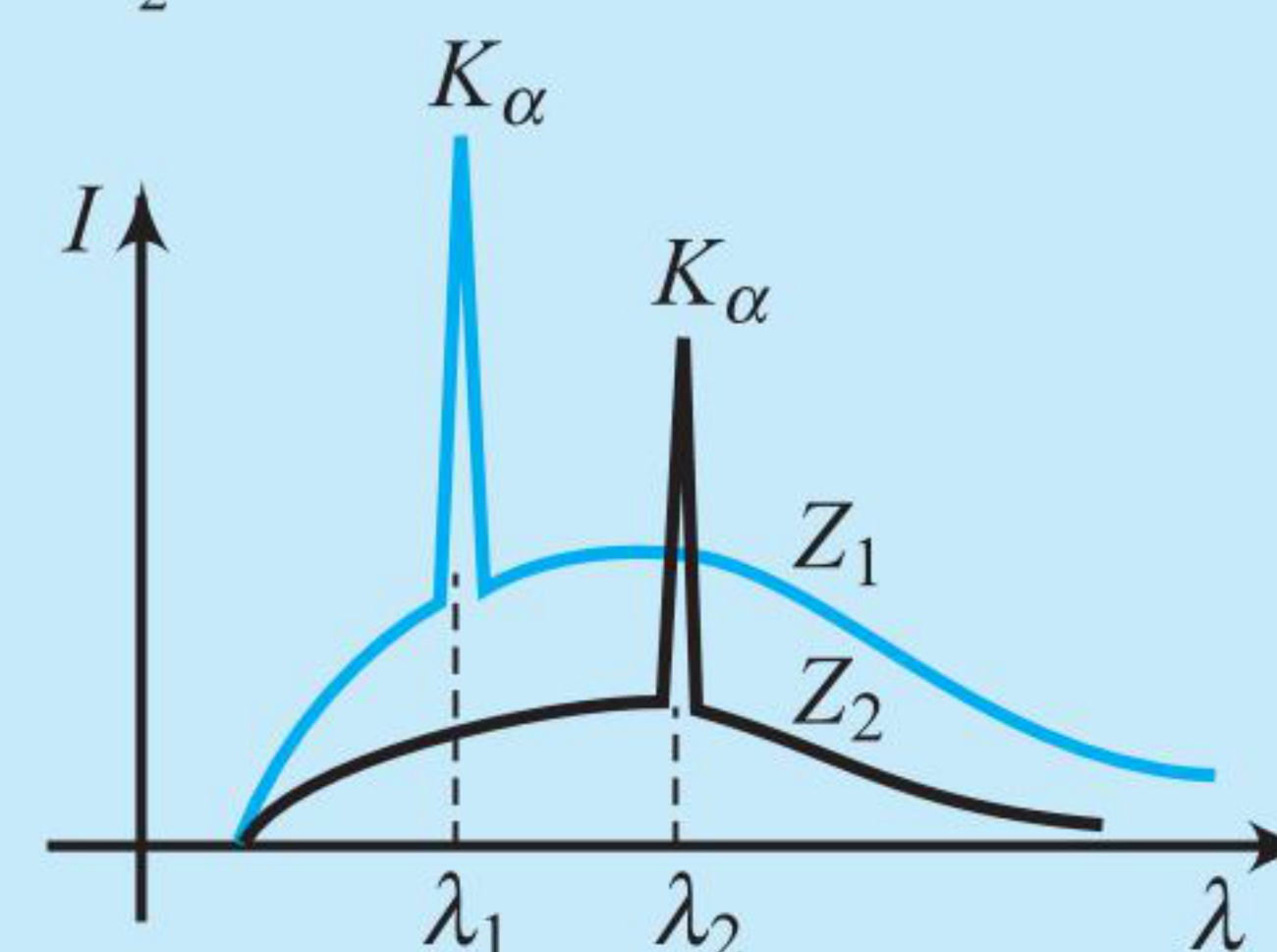
$$\Rightarrow (Z - 1)^2 \propto \nu \text{ or } (Z - 1)^2 \propto 1/\lambda$$

$$\Rightarrow \frac{(Z_{\text{Mo}} - 1)^2}{(Z_{\text{Cu}} - 1)^2} = \frac{\lambda_{\text{Cu}}}{\lambda_{\text{Mo}}}$$

$$\text{or } \lambda_{\text{Cu}} = \lambda_{\text{Mo}} \frac{(Z_{\text{Mo}} - 1)^2}{(Z_{\text{Cu}} - 1)^2} = 0.71 \times \left(\frac{41}{28} \right)^2 = 1.52 \text{ \AA}$$

ILLUSTRATION 6.51

Compare Z_1 and Z_2 .



$$\text{Sol. } \sqrt{\nu} = \sqrt{cR \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)} \cdot (Z - b)$$

If Z is greater, then ν will be greater and hence λ will be less. Since $\lambda_1 < \lambda_2$, $Z_1 > Z_2$.

ILLUSTRATION 6.52

A cobalt target ($Z = 27$) is bombarded with electrons and the wavelengths of its characteristic spectrum are measured. A second, fainter, characteristic spectrum is also found because of an impurity in the target. The wavelength of the K_α lines are 178.9 pm (cobalt) and 143.5 pm (impurity). What is the impurity?

Sol. Using Moseley's law and putting c/λ for ν (and assuming $b = 1$), we obtain

$$\sqrt{\frac{c}{\lambda_{\text{Co}}}} = aZ_{\text{Co}} - a \quad \text{and} \quad \sqrt{\frac{c}{\lambda_x}} = aZ_x - a$$

$$\text{Dividing yields } \sqrt{\frac{\lambda_{\text{Co}}}{\lambda_x}} = \frac{Z_x - 1}{Z_{\text{Co}} - 1}$$

Substituting gives us $\sqrt{\frac{178.9 \text{ pm}}{143.5 \text{ pm}}} = \frac{Z_x - 1}{27 - 1}$

Solving for the unknown, we find $Z_x = 30.0$; the impurity is zinc.

ILLUSTRATION 6.53

Find the constants a and b in Moseley's equation $\sqrt{\nu} = a(Z - b)$ from the following data.

Element	Z	Wavelength of K_α X-ray
Mo	42	71 pm
Co	27	178.5 pm

Sol. Moseley's equation is $\sqrt{\nu} = a(Z - b)$

$$\text{Thus, } \sqrt{\frac{c}{\lambda_1}} = a(Z_1 - b) \quad \dots(i)$$

$$\text{and } \sqrt{\frac{c}{\lambda_2}} = a(Z_2 - b) \quad \dots(ii)$$

$$\text{From Eqs. (i) and (ii), } \sqrt{c} \left(\frac{1}{\sqrt{\lambda_1}} - \frac{1}{\sqrt{\lambda_2}} \right) = a(Z_1 - Z_2)$$

$$\begin{aligned} \text{or } a &= \frac{\sqrt{c}}{(Z_1 - Z_2)} \left(\frac{1}{\sqrt{\lambda_1}} - \frac{1}{\sqrt{\lambda_2}} \right) \\ &= \frac{(3 \times 10^8)^{1/2}}{42 - 27} \left[\frac{1}{(71 \times 10^{-12})^{1/2}} - \frac{1}{(178.5 \times 10^{-12})^{1/2}} \right] \\ &= 5.0 \times 10^7 \text{ (Hz)}^{1/2} \end{aligned}$$

$$\text{Dividing Eq. (i) by Eq. (ii), } \sqrt{\frac{\lambda_2}{\lambda_1}} = \frac{Z_1 - b}{Z_2 - b} \text{ or } \sqrt{\frac{178.5}{71}} = \frac{42 - b}{27 - b}$$

$$\text{or } b = 1.37$$

ILLUSTRATION 6.54

An X-ray tube with a copper target is found to be emitting lines other than those due to copper. The K_α line of copper is known to have a wavelength $1.5405 \text{ \AA}$ and the other two K_α lines observed have wavelengths $0.7090 \text{ \AA}$ and $1.6578 \text{ \AA}$. Identify the impurities.

Sol. According to Moseley's equation for K_α radiation,

$$\frac{1}{\lambda} = R(Z - 1)^2 \left[\frac{1}{1^2} - \frac{1}{2^2} \right] \quad (\lambda = \text{wavelength corresponding to Cu})$$

Let λ_1 and λ_2 be the two other unknown wavelengths, then

$$\frac{\lambda_1}{\lambda} = \frac{(Z - 1)^2}{(Z_1 - 1)^2} = \frac{0.7090}{1.5405}$$

$$\text{For copper, } Z = 29, \text{ therefore, } (Z_1 - 1) = 28 \sqrt{\frac{1.5405}{0.7092}} = 41$$

$$\text{or } Z_1 = 42 \text{ (Molybdenum)}$$

$$\text{Similarly, } \frac{\lambda_2}{\lambda} = \frac{(28)^2}{(Z_2 - 1)^2} = \frac{1.6578}{1.5405}$$

$$(Z_2 - 1) = 28 \sqrt{\left(\frac{1.5405}{1.6578} \right)}$$

$$\text{or } (Z_2 - 1) = 27 \text{ or } Z_2 = 28 \text{ (Nickel)}$$

So, the impurities are molybdenum and nickel.

ILLUSTRATION 6.55

The K -absorption edge of an unknown element is $0.171 \text{ \AA}$.

- Identify the element.
- Find the average wavelengths of the K_α , K_β , and K_γ lines.
- If a 100 eV electron strikes the target of this element, what is the minimum wavelength of the X-ray emitted?

Sol. From Moseley's law, the wavelength of K series of X-rays is given by taking modified Rydberg's formula given as

$$\frac{1}{\lambda} = R(Z - 1)^2 \left(1 - \frac{1}{n^2} \right) \text{ for } K \text{ lines, where } n = 2, 3, 4, \dots$$

- For K -absorption edge, we put $n = \infty$ in above expression to get

$$(Z - 1) = \sqrt{\frac{1}{\lambda R}}$$

$$\text{or } Z = \sqrt{\frac{1}{(0.171 \times 10^{-10})(1.097 \times 10^7)}} + 1 = 74$$

The element is tungsten.

- For K_α line: $\frac{1}{\lambda_{K_\alpha}} = R(74 - 1)^2 \left[1 - \frac{1}{2^2} \right]$

$$\Rightarrow \lambda_{K_\alpha} = 0.228 \text{ \AA}$$

$$\text{For } K_\beta \text{ line: } \frac{1}{\lambda_{K_\beta}} = R(74 - 1)^2 \left[1 - \frac{1}{3^2} \right]$$

$$\Rightarrow \lambda_{K_\beta} = 0.192 \text{ \AA}$$

$$\text{For } K_\gamma \text{ line: } \frac{1}{\lambda_{K_\gamma}} = R(74 - 1)^2 \left[1 - \frac{1}{4^2} \right]$$

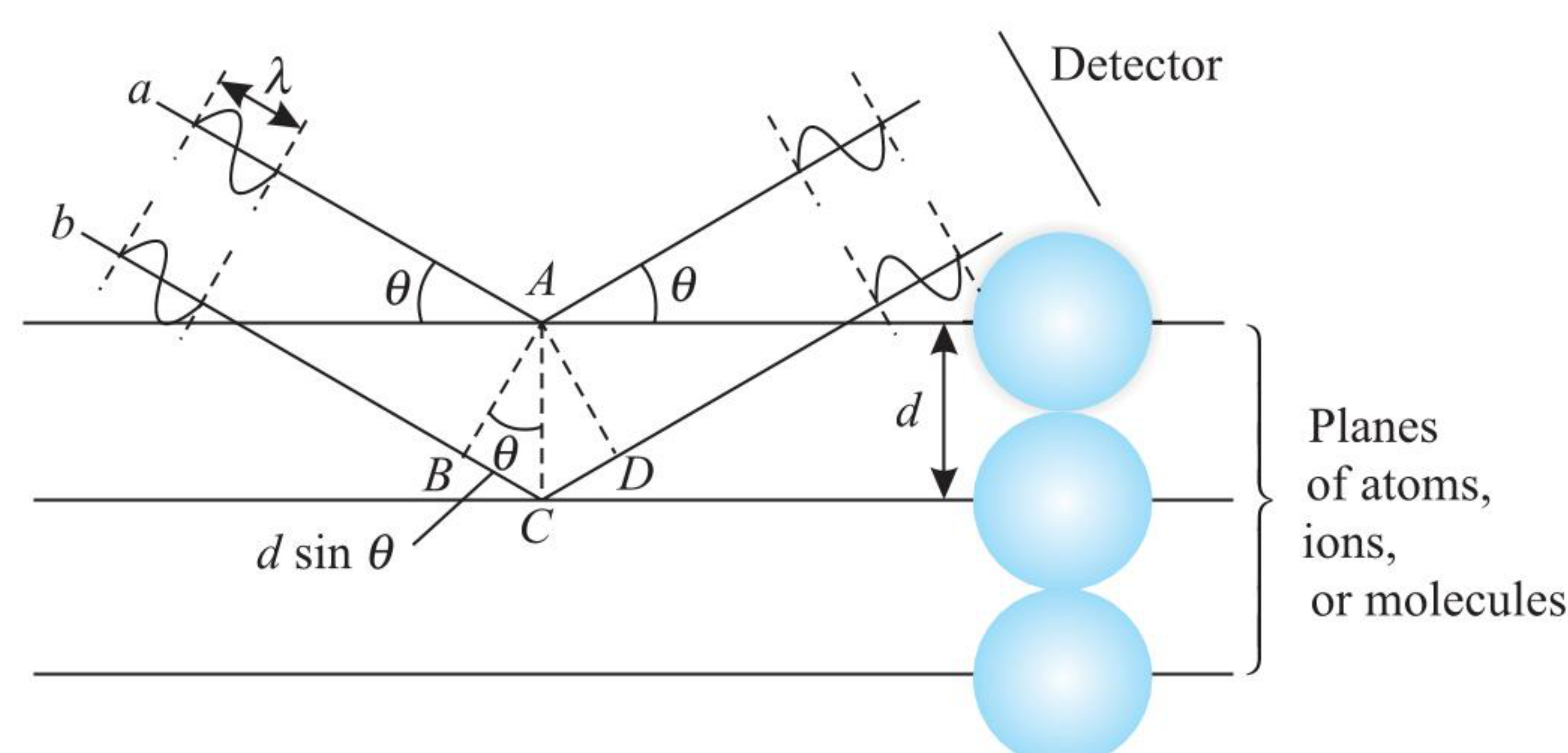
$$\Rightarrow \lambda_{K_\gamma} = 0.182 \text{ \AA}$$

- The shortest wavelength corresponding to an electron with kinetic energy 100 eV is given by

$$\lambda_c = \frac{hc}{E} = \frac{12431}{100} \text{ \AA} = 124.31 \text{ \AA}$$

BRAGG'S LAW

X-rays are electromagnetic waves of high energy with short wavelengths and may be diffracted by suitable diffracting obstacle. However, the diffraction effects are appreciable only when the diffracting apertures are of the order of the wavelength, i.e., of the order of $1 \text{ \AA}$. This is almost the size of an atom and it is difficult to construct slits with such small gaps so that X-rays can be appreciably diffracted.



In case of solid crystals, atoms are arranged in fairly regular pattern with interatomic gaps of the order of $1 \text{ \AA}$. Almost all the metals at ordinary temperature are crystalline. These metals may act as natural three-dimensional gratings for the diffraction of X-rays.

The structure of a solid can be considered as a series of parallel planes of atoms separated by a inter planer distance d . Suppose, an X-ray beam is incident on a solid, making an angle θ with the planes of the atoms. These X-rays are diffracted by different atoms and the diffracted rays interfere. In certain directions, the interference is constructive and we obtain strong reflected X-rays. The analysis shows that there will be a strong reflected X-ray beam only if

$$2d \sin \theta = n\lambda \quad \dots(i)$$

where n is an integer. If the X-rays are incident at one of these angles, they are reflected; otherwise they are absorbed. When they are reflected, the laws of reflection are obeyed, i.e., (a) the angle of incidence is equal to the angle of reflection and (b) the incident ray, the reflected ray and the normal to the reflecting plane are coplanar. Equation (i) is known as Bragg's law.

By using a monochromatic X-rays beam and noting the angles of strong reflection, the interplanar spacing d and several informations about the structure of the solid can be obtained.

CONCEPT APPLICATION EXERCISE 6.4

- State the following statements as True or False.
 - X-rays are electromagnetic waves because these are produced by the deceleration of fast moving electrons.
 - X-rays are electromagnetic waves because these produce line spectrum.
 - The cut-off wavelength depends on the target material.
 - The intensity of K_α radiation is more than that of K_β .
 - The frequency of K_α radiation is more than that of K_β .
- Calculate the cut-off wavelength of the X-ray emitted by a Coolidge tube operating at 40 kV.
- Calculate the wavelength of K_α line for the target made of tungsten ($Z = 74$).

- Obtain a relation between the frequencies of K_α , K_β , and L_α lines for a target material.
- An X-rays tube operates at 20 kV. Find the maximum speed of the electrons striking the anticathode, given the charge of electron = 1.6×10^{-19} coulomb and mass of electron = 9×10^{-31} kg.
- (a) An X-ray tube produces a continuous spectrum of radiation with its short wavelength end at $0.45 \text{ \AA}$. What is the maximum energy of a photon in the radiation?
(b) From your answer to (a), guess what order of accelerating voltage (for electrons) is required in such a tube?
- The wavelength of characteristic X-ray K_α line emitted from Zinc ($Z = 30$) is $1.415 \text{ \AA}$. Find the wavelength of the K_α line emitted from molybdenum ($Z = 42$).
- If the short series limit of the Balmer series for hydrogen is $3644 \text{ \AA}$, find the atomic number of the element which gives X-ray wavelengths down to $1 \text{ \AA}$. Identify the element.
- A material whose K-absorption edge is $0.2 \text{ \AA}$ is irradiated by X-rays of wavelength $0.15 \text{ \AA}$. Find the maximum energy of the photoelectrons that are emitted from the K shell.
- Calculate the wavelength of the emitted characteristic X-ray from a tungsten ($Z = 74$) target when an electron drops from an M shell to a vacancy in the shell.
- A potential difference of 20 kV is applied to an X-ray tube. Find the minimum wavelength of X-rays generated.
- The wavelength of X-rays produced by an X-ray tube is $0.76 \text{ \AA}$. What is the atomic number of the anode material of the tube?

ANSWERS

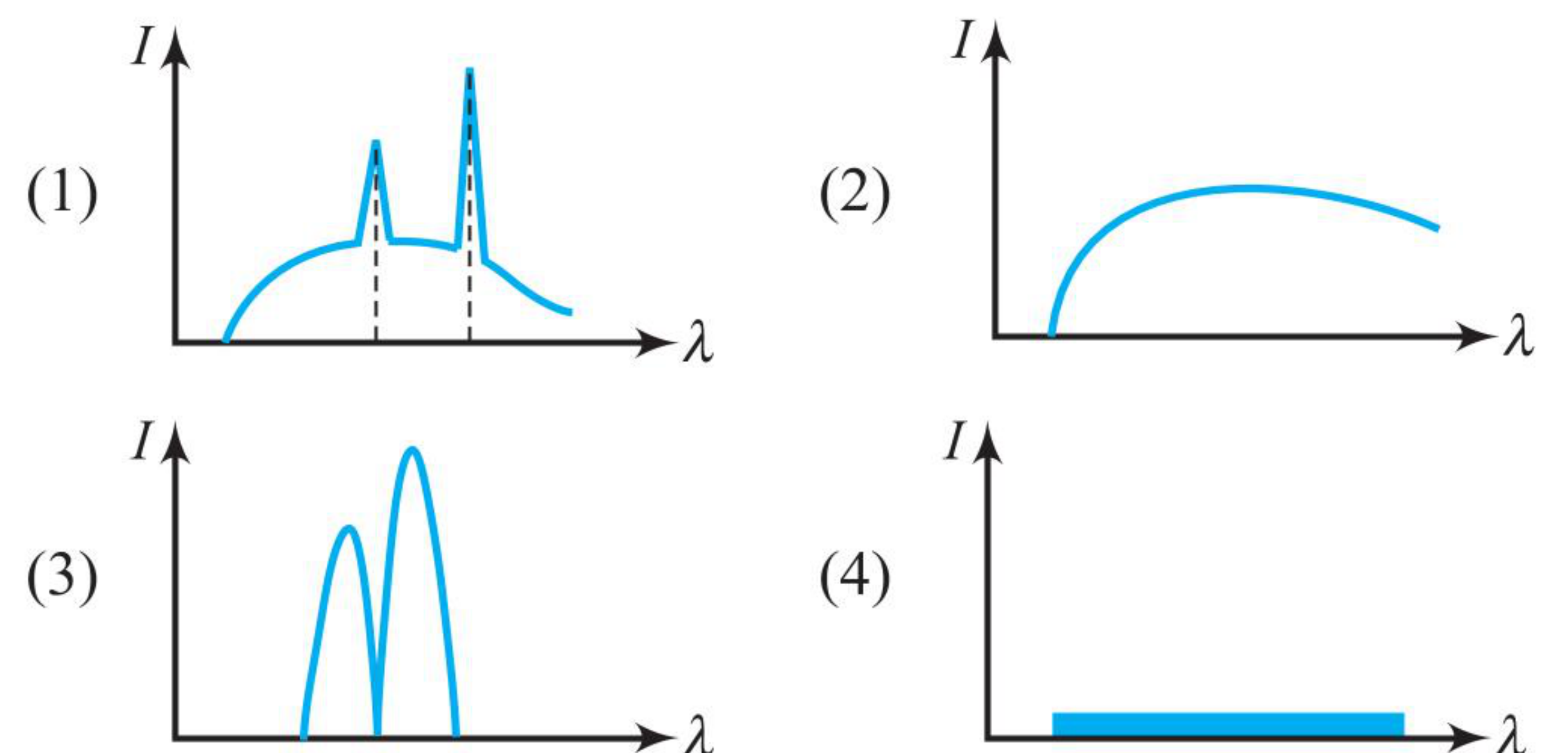
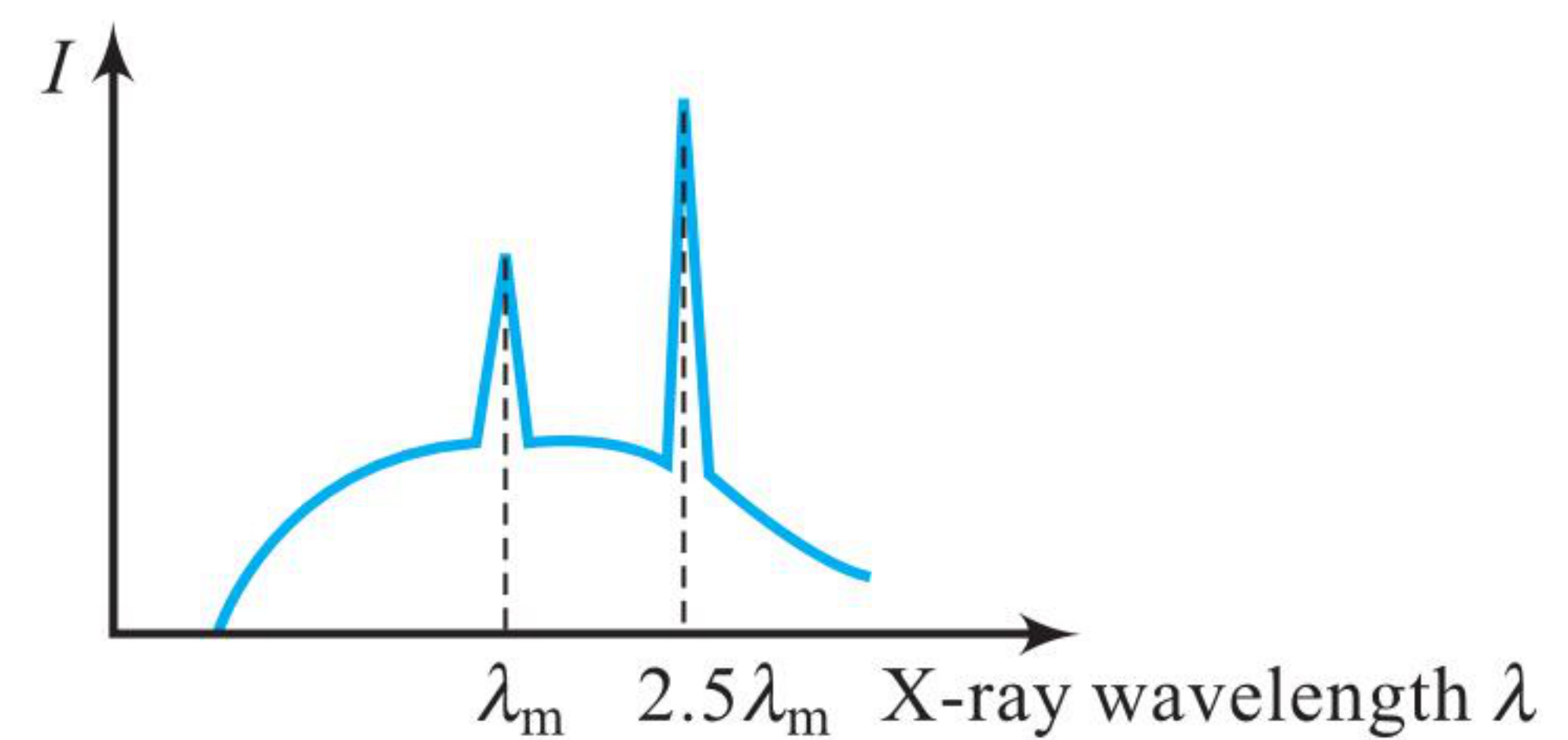
- (a) True (b) True (c) False (d) True (e) False
- $0.31 \text{ \AA}$ 3. $0.228 \text{ \AA}$ 5. $8.4 \times 10^7 \text{ m s}^{-1}$
- (a) 27.624 keV (b) 30 kV 7. $0.708 \text{ \AA}$
- Gallium 9. 20.718 keV 11. $0.6215 \text{ \AA}$ 12. 41

Exercises

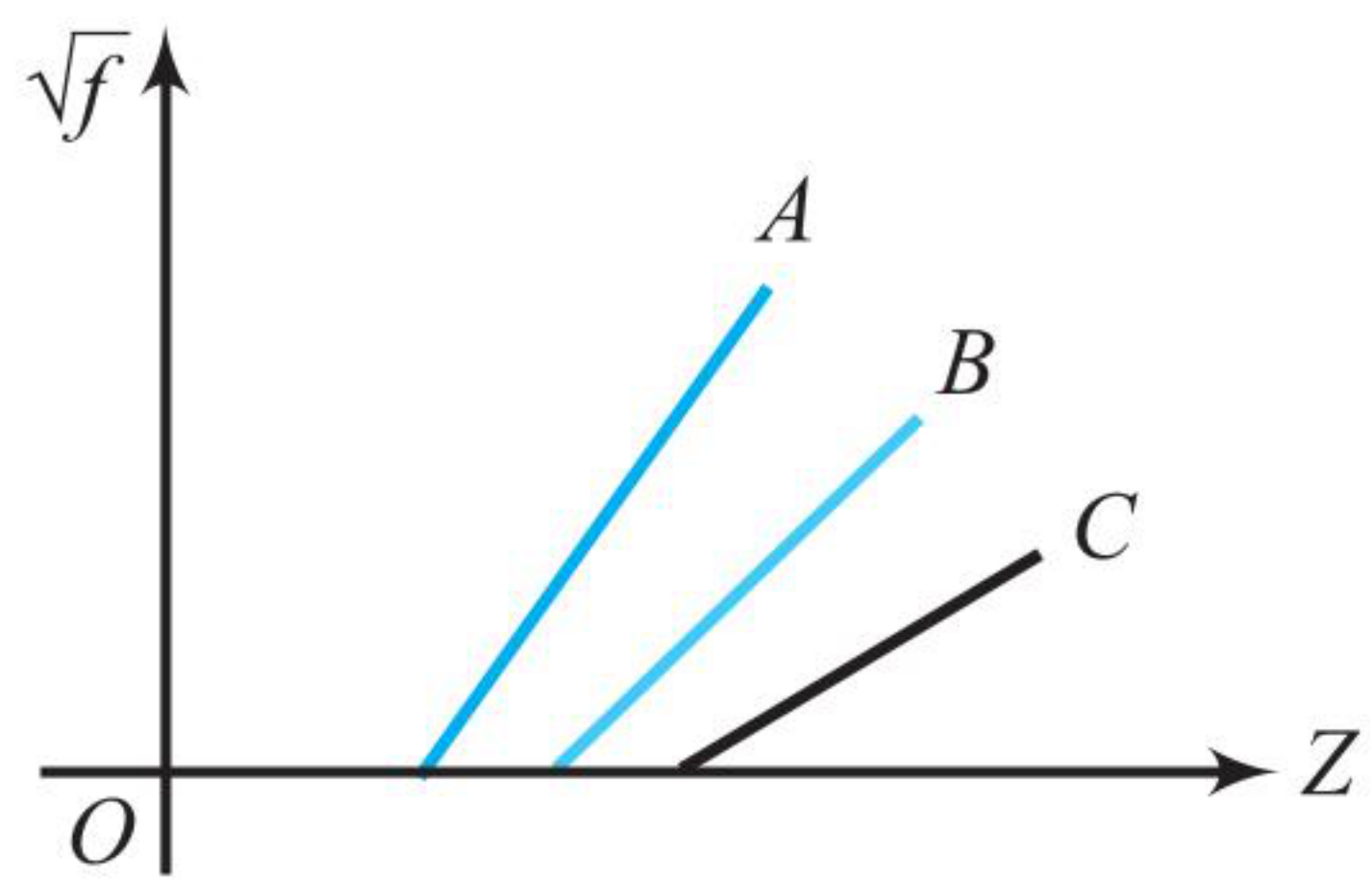
Single Correct Answer Type

- An electron jumps from the fourth orbit to the second orbit of hydrogen atom. Given: the Rydberg's constant $R = 10^5 \text{ cm}^{-1}$. The frequency, in Hz, of the emitted radiation will be
 - $\frac{3}{16} \times 10^5$
 - $\frac{3}{6} \times 10^{15}$
 - $\frac{9}{16} \times 10^5$
 - $\frac{9}{16} \times 10^{15}$
- Given: mass number of gold = 197, density of gold = 19.7 g cm^{-3} , Avogadro's number = 6×10^{23} . The radius of the gold atom is approximately:
 - $1.5 \times 10^{-8} \text{ m}$
 - $1.5 \times 10^{-9} \text{ m}$
 - $1.5 \times 10^{-10} \text{ m}$
 - $1.5 \times 10^{-12} \text{ m}$
- Transitions between three energy levels in a particular atom give rise to three spectral lines of wavelengths, in increasing magnitudes, λ_1 , λ_2 , and λ_3 . Which one of the following equations correctly relates λ_1 , λ_2 , and λ_3 ?
 - $\lambda_1 = \lambda_2 - \lambda_3$
 - $\lambda_1 = \lambda_3 - \lambda_2$
 - $\frac{1}{\lambda_1} = \frac{1}{\lambda_2} + \frac{1}{\lambda_3}$
 - $\frac{1}{\lambda_1} = \frac{1}{\lambda_3} - \frac{1}{\lambda_2}$
- The ratio of the speed of the electron in the first Bohr orbit of hydrogen and the speed of light is equal to (where e , h , and c have their usual meanings in cgs system)
 - $2\pi hc/e^2$
 - $er^2 h/2\pi c$
 - $e^2 c/2\pi h$
 - $2\pi e^2/hc$
- Suppose two deuterons must get as close as 10^{-14} m in order for the nuclear force to overcome the repulsive electrostatic force. The height of the electrostatic barrier is nearest to
 - 0.14 MeV
 - 2.3 MeV
 - 1.8 MeV
 - 0.56 MeV
- An electron in H atom makes a transition from $n = 3$ to $n = 1$. The recoil momentum of H atom will be
 - $6.45 \times 10^{-27} \text{ N s}$
 - $6.8 \times 10^{-27} \text{ N s}$
 - $6.45 \times 10^{-24} \text{ N s}$
 - $6.8 \times 10^{-24} \text{ N s}$
- A hydrogen-like atom emits radiations of frequency $2.7 \times 10^{15} \text{ Hz}$ when it makes a transition from $n = 2$ to $n = 1$. The frequency emitted in a transition from $n = 3$ to $n = 1$ will be
 - $1.8 \times 10^{15} \text{ Hz}$
 - $3.2 \times 10^{15} \text{ Hz}$
 - $4.7 \times 10^5 \text{ Hz}$
 - $6.9 \times 10^{15} \text{ Hz}$
- The total energy of an electron in the ground state of hydrogen atom is -13.6 eV . The potential energy of an electron in the ground state of Li^{2+} ion will be
 - 122.4 eV
 - 122.4 eV
 - 244.8 eV
 - 244.8 eV
- The wavelength of radiation required to excite the electron from the first orbit to the third orbit in a doubly ionized lithium atom will be
 - 134.25 Å
 - 125.5 Å
 - 113.7 Å
 - 110 Å
- The force acting on the electron in a hydrogen atom depends on the principal quantum number as
 - $F \propto n^2$
 - $F \propto \frac{1}{n^2}$
 - $F \propto n^4$
 - $F \propto \frac{1}{n^4}$
- If potential energy between a proton and an electron is given by $|U| = ke^2/2R^3$, where e is the charge of electron and R is the radius of atom, then radius of Bohr's orbit is given by (h = Planck's constant, k = constant)
 - $\frac{ke^2 m}{h^2}$
 - $\frac{6\pi^2 ke^2 m}{n^2 h^2}$
 - $\frac{2\pi ke^2 m}{n h^2}$
 - $\frac{4\pi^2 ke^2 m}{n^2 h^2}$
- How many revolutions does an electron complete in one second in the first orbit of hydrogen atom?
 - 6.62×10^{15}
 - 100
 - 1000
 - 1
- The radius of hydrogen atom in the ground state is $5.3 \times 10^{-11} \text{ m}$. When struck by an electron, its radius is found to be $21.2 \times 10^{-11} \text{ m}$. The principal quantum number of the final state will be
 - 1
 - 2
 - 3
 - 4
- The wavelength of the first line of Balmer series is 6563 Å. The Rydberg's constant is
 - $1.09 \times 10^5 \text{ m}^{-1}$
 - $1.09 \times 10^6 \text{ m}^{-1}$
 - $1.097 \times 10^7 \text{ m}^{-1}$
 - $1.09 \times 10^8 \text{ m}^{-1}$
- A hydrogen atom and a Li^{++} ion are both in the second excited state. If l_{H} and l_{Li} are their respective electronic angular momenta, and E_{H} and E_{Li} their respective energies, then
 - $l_{\text{H}} < l_{\text{Li}}$ and $|E_{\text{H}}| > |E_{\text{Li}}|$
 - $l_{\text{H}} = l_{\text{Li}}$ and $|E_{\text{H}}| < |E_{\text{Li}}|$
 - $l_{\text{H}} = l_{\text{Li}}$ and $|E_{\text{H}}| > |E_{\text{Li}}|$
 - $l_{\text{H}} < l_{\text{Li}}$ and $|E_{\text{H}}| < |E_{\text{Li}}|$
- Imagine an atom made of a proton and a hypothetical particle of double the mass of the electron but having the same charge as the electron. Apply the Bohr atom model and consider all possible transitions of this hypothetical particle to the first excited level. The longest wavelength photon that will be emitted has wavelength (given in terms of the Rydberg constant R for the hydrogen atom) equal to
 - $\frac{9}{5R}$
 - $\frac{36}{5R}$
 - $\frac{18}{5R}$
 - $\frac{4}{R}$
- The electric potential energy between a proton and an electron is given by $U = U_0 \ln r/r_0$, where r_0 is a constant. Assuming Bohr's model to be applicable, write the variation of r with n , n being the principal quantum number.

- (1) $r_n \propto n$ (2) $r_n \propto 1/n$
 (3) $r_n \propto n^2$ (4) $r_n \propto 1/n^2$
18. The longest wavelength that a singly ionized helium atom in its ground state will absorb is
 (1) 912 Å (2) 304 Å
 (3) 606 Å (4) 1216 Å
19. The electron in a hydrogen atom makes a transition $n_1 \rightarrow n_2$, where n_1 and n_2 are the principal quantum numbers of the two states. Assume the Bohr model as valid in this case. The frequency of the orbital motion of the electron in the initial state is $1/27$ of that in the final state. The possible values of n_1 and n_2 are
 (1) $n_1 = 6, n_2 = 3$ (2) $n_1 = 4, n_2 = 2$
 (3) $n_1 = 8, n_2 = 1$ (4) $n_1 = 3, n_2 = 1$
20. A photon collides with a stationary hydrogen atom in the ground state inelastically. Energy of the colliding photon is 10.2 eV. Almost instantaneously, another photon collides with same hydrogen atom inelastically with an energy of 15 eV. What will be observed by the detector?
 (1) Two photons of energy 10.2 eV
 (2) Two photons of energy 1.4 eV
 (3) One photon of energy 10.2 eV and one electron of energy 1.4 eV
 (4) One electron having kinetic energy nearly 11.6 eV
21. The ratio (in SI units) of magnetic dipole moment to that of the angular momentum of an electron of mass m kg and charge e coulomb in Bohr's orbit of hydrogen atom is
 (1) $\frac{e}{2m}$ (2) $\frac{e}{m}$
 (3) $\frac{2e}{m}$ (4) none of these
22. If an electron in $n = 3$ orbit of hydrogen atom jumps down to $n = 2$ orbit, the amount of energy released and the wavelength of radiation emitted are
 (1) 0.85 eV, 6566 Å (2) 1.89 eV, 1240 Å
 (3) 1.89 eV, 6566 Å (4) 1.5 eV, 6566 Å
23. In hydrogen spectrum, the shortest wavelength in Balmer series is the λ . The shortest wavelength in the Brackett series will be
 (1) 2λ (2) 4λ
 (3) 9λ (4) 16λ
24. If the first excitation potential of a hydrogen-like atom is V electron volt, then the ionization energy of this atom will be
 (1) V electron volt
 (2) $\frac{3V}{4}$ electron volt
 (3) $\frac{4V}{3}$ electron volt
 (4) cannot be calculated by given information
25. Two hydrogen atoms are in excited state with electrons residing in $n = 2$. The first one is moving toward left and emits a photon of energy E_1 toward right. The second one is moving toward right with the same speed and emits a photon of energy E_2 toward left. Taking recoil of nucleus into account, during the emission process
 (1) $E_1 > E_2$ (2) $E_1 < E_2$
 (3) $E_1 = E_2$ (4) information insufficient
26. In a hydrogen atom following Bohr's postulates, the product of linear momentum and angular momentum is proportional to $(n)^x$ where n is the orbit number. Then x is
 (1) 0 (2) 2
 (3) -2 (4) 1
27. The voltage applied to an X-ray tube is 18 kV. The maximum mass of photon emitted by the X-ray tube will be
 (1) 2×10^{-13} kg (2) 3.2×10^{-36} kg
 (3) 3.2×10^{-32} kg (4) 9.1×10^{-31} kg
28. The wavelength of K_α X-rays of two metals A and B are $4/1875R$ and $1/675R$, respectively, where ' R ' is Rydberg's constant. The number of elements lying between A and B according to their atomic numbers is
 (1) 3 (2) 6
 (3) 5 (4) 4
29. One of the lines in the emission spectrum of Li^{2+} has the same wavelength as that of the second line of Balmer series in hydrogen spectrum. The electronic transition corresponding to this line is
 (1) $n = 4 \rightarrow n = 2$ (2) $n = 8 \rightarrow n = 2$
 (3) $n = 8 \rightarrow n = 4$ (4) $n = 12 \rightarrow n = 6$
30. The photon radiated from hydrogen corresponding to the second line of Lyman series is absorbed by a hydrogen-like atom X in the second excited state. Then, the hydrogen-like atom X makes a transition of n th orbit.
 (1) $X = \text{He}^+, n = 4$ (2) $X = \text{Li}^{++}, n = 6$
 (3) $X = \text{He}^+, n = 6$ (4) $X = \text{Li}^{++}, n = 9$
31. The element which has a K_α X-rays line of wavelength 1.8 Å is
 ($R = 1.1 \times 10^7 \text{ m}^{-1}$, $b = 1$ and $\sqrt{5/33} = 0.39$)
 (1) Co, $Z = 27$ (2) Fe, $Z = 26$
 (3) Mn, $Z = 25$ (4) Ni, $Z = 28$
32. When an electron accelerated by potential difference U is bombarded on a specific metal, the emitted X-ray spectrum obtained is shown in figure. If the potential difference is reduced to $U/3$, the correct spectrum is



33. In a hydrogen atom, the electron makes a transition from $n = 2$ to $n = 1$. The magnetic field produced by the circulating electron at the nucleus
- decreases 16 times
 - increases 4 times
 - decreases 4 times
 - increases 32 times
34. An electron in a hydrogen atom makes a transition from first excited state to ground state. The equivalent current due to circulating electron:
- increases 2 times
 - increases 4 times
 - increases 8 times
 - remains the same
35. When the voltage applied to an X-ray tube increases from $V_1 = 10$ kV to $V_2 = 20$ kV, the wavelength interval between K_α line and cut-off wavelength of continuous spectrum increases by a factor of 3. Atomic number of the metallic target is
- 28
 - 29
 - 65
 - 66
36. Monochromatic radiations of wavelength λ are incident on a hydrogen sample in the ground state. Hydrogen atom absorbs the light and subsequently emits radiations of 10 different wavelengths. The value of λ is nearly
- 203 nm
 - 95 nm
 - 80 nm
 - 73 nm
37. The power of an X-ray tube is 16 W. If the potential difference applied across the tube is 5 kV, then the number of electrons striking the target per second is
- 8.4×10^{16}
 - 5×10^{17}
 - 2×10^{16}
 - 2×10^{19}
38. A hydrogen-like atom (atomic number Z) is in a higher excited state of quantum number n . This excited atom can make a transition to the first excited state by successively emitting two photons of energies 10.2 eV and 17.0 eV, respectively. Alternatively, the atom from the same excited state can make a transition to the second excited state by successively emitting two photons of energies 4.25 eV and 5.95 eV, respectively. The values of n and Z are, respectively,
- 6 and 6
 - 3 and 3
 - 6 and 3
 - 3 and 6
39. Hydrogen atoms in a sample are excited to $n = 5$ state and it is found that photons of all possible wavelengths are present in the emission spectra. The minimum number of hydrogen atoms in the sample would be
- 5
 - 6
 - 10
 - infinite
40. A sample of hydrogen atoms is in excited state (all the atoms). The photons emitted from this sample are made to pass through a filter through which light having wavelength greater than 800 nm can only pass. Only one type of photons are found to pass the filter. The sample's excited state initially is [Take $hc = 1240$ eV-nm, ground state energy of hydrogen atom = -13.6 eV.]
- fifth excited state
 - fourth excited state
 - third excited state
 - second excited state
41. Mark out the correct statement regarding X-rays.
- When fast moving electrons strike the metal target, they enter the metal target and in a very short time span come to rest, and thus an accelerated charged electron produces electromagnetic waves (X-rays).
 - Characteristic X-rays are produced due to transition of an electron from higher energy levels to vacant lower energy levels.
 - X-rays spectrum is a discrete spectra just like hydrogen spectra.
 - Both (1) and (2) are correct.
42. An electron collides with a hydrogen atom in its ground state and excites it to $n = 3$. The energy given to hydrogen atom in this inelastic collision is [Neglect the recoiling of hydrogen atom]
- 10.2 eV
 - 12.1 eV
 - 12.5 eV
 - None of these
43. Electrons in a hydrogen-like atom ($Z = 3$) make transitions from fourth excited state to third excited state and from third to second excited state. The resulting radiations are incident on a metal plate to eject photoelectrons. The stopping potential for photoelectrons ejected by the shorter wavelength is 3.95 V. The stopping potential for the photoelectrons ejected by the longer wavelength is
- 2.0 V
 - 0.75 V
 - 0.6 V
 - none of the above
44. In which of the following transitions will the wavelength be minimum?
- $n_1 = 5$ to $n_2 = 4$
 - $n_1 = 4$ to $n_2 = 3$
 - $n_1 = 3$ to $n_2 = 2$
 - $n_1 = 2$ to $n_2 = 1$
45. A neutron having kinetic energy 5 eV is incident on a hydrogen atom in its ground state. The collision
- must be elastic
 - must be completely inelastic
 - may be partially elastic
 - information is insufficient
46. If the average life time of an excited state of hydrogen is of the order of 10^{-8} s, then the number of revolutions an electron will make when it is in $n = 2$ state before coming to ground state will be [Take $a_0 = 0.53$ Å and all standard data if required]
- 10^7
 - 8×10^6
 - 2×10^5
 - none of these
47. An electron of energy 11.2 eV undergoes an inelastic collision with a hydrogen atom in its ground state. [Neglect recoiling of atom as $m_H \gg m_e$]. Then in this case
- the outgoing electron has energy 11.2 eV
 - the entire energy is absorbed by the H atom and the electron stops
 - 10.2 eV of the incident electron energy is absorbed by the H atom and the electron would come out with 1.0 eV energy
 - none of these

48. The recoil speed of hydrogen atom after it emits a photon in going from $n = 2$ state to $n = 1$ state is nearly [Take $R_\infty = 1.1 \times 10^7 \text{ m}^{-1}$ and $h = 6.63 \times 10^{-34} \text{ Js}$]
- (1) 1.5 m s^{-1} (2) 3.3 m s^{-1}
(3) 4.5 m s^{-1} (4) 6.6 m s^{-1}
49. A beam of 13.0 eV electrons is used to bombard gaseous hydrogen. The series obtained in emission spectra is/are
- (1) Lyman series (2) Balmer series
(3) Brackett series (4) All of these
50. The wavelength of the spectral line that corresponds to a transition in hydrogen atom from $n = 10$ to ground state would be [In which part of electromagnetic spectrum this line lies?]
- (1) 92.25 nm , ultraviolet (2) 92.25 nm , infrared
(3) 86.95 nm , ultraviolet (4) 97.65 nm , ultraviolet
51. The angular momentum of an electron in hydrogen atom is $4h/2\pi$. Kinetic energy of this electron is
- (1) 4.35 eV (2) 1.51 eV
(3) 0.85 eV (4) 13.6 eV
52. Difference between n th and $(n + 1)$ th Bohr's radius of hydrogen atom is equal to $(n - 1)$ th Bohr's radius. The value of n is
- (1) 1 (2) 2
(3) 3 (4) 4
53. K_α wavelength emitted by an atom of atomic number $Z = 11$ is λ . The atomic number for an atom that emits K_α radiation with wavelength 4λ is
- (1) 6 (2) 4
(3) 11 (4) 44
54. A proton of mass m moving with a speed v_0 approaches a stationary proton that is free to move. Assume impact parameter to be zero, i.e., head-on collision. How close will the incident proton go to other proton?
- (1) $\frac{e^3}{\pi \epsilon_0 m^2 v_0}$ (2) $\frac{e^3}{\pi \epsilon_0 m v_0}$
(3) $\frac{e^2}{\pi \epsilon_0 m v_0^2}$ (4) None of these
55. If an X-ray tube operates at the voltage of 10 kV , find the ratio of the de Broglie wavelength of the incident electrons to the shortest wavelength of X-rays produced. The specific charge of electron is $1.8 \times 10^{11} \text{ C kg}^{-1}$.
- (1) 1 (2) 0.1
(3) 1.8 (4) 1.2
56. The potential difference across the Coolidge tube is 20 kV and 10 mA current flows through the voltage supply. Only 0.5% of the energy carried by the electrons striking the target is converted into X-rays. The power carried by the X-ray beam is P . Then
- (1) $P = 0.1 \text{ W}$ (2) $P = 1 \text{ W}$
(3) $P = 2 \text{ W}$ (4) $P = 10 \text{ W}$
57. Electrons with energy 80 keV are incident on the tungsten target of an X-ray tube. K shell electrons of tungsten have -72.5 keV energy. X-rays emitted by the tube contain only
- (1) a continuous X-ray spectrum (Bremsstrahlung) with a minimum wavelength of $0.155 \text{ \AA}$
(2) a continuous X-ray spectrum (Bremsstrahlung) with all wavelengths
(3) a continuous X-ray spectrum of tungsten
(4) a continuous X-ray spectrum (Bremsstrahlung) with a minimum wavelength of $0.155 \text{ \AA}$ and the characteristic X-ray spectrum of tungsten
58. Consider a hypothetical annihilation of a stationary electron with a stationary positron. What is the wavelength of the resulting radiation?
- (1) $\lambda = \frac{h}{m_0 c}$ (2) $\lambda = \frac{2h}{m_0 c^2}$
(3) $\lambda = \frac{h}{2m_0 c^2}$ (4) None of these
59. If $10,000 \text{ V}$ are applied across an X-ray tube, find the ratio of wavelength of the incident electrons and the shortest wavelength of X-ray coming out of the X-ray tube, given e/m of electron $= 1.8 \times 10^{11} \text{ C kg}^{-1}$
- (1) 1:10 (2) 10:1
(3) 5:1 (4) 1:5
60. Figure shows Moseley's plot between $\sqrt{f}$ and Z , where f is the frequency and Z is the atomic number. Three lines A , B , and C shown in the graph may represent
- 
- (1) K_α , K_β , and K_γ lines, respectively
(2) K_γ , K_β , and K_α lines, respectively
(3) K_α , L_α , and M_α lines, respectively
(4) Nothing
61. An electron in the ground state of hydrogen has an angular momentum L_1 and an electron in the first excited state of lithium has an angular momentum L_2 . Then
- (1) $L_1 = L_2$ (2) $L_1 = 4L_2$
(3) $L_2 = 2L_1$ (4) $L_1 = 2L_2$
62. The approximate value of quantum number n for the circular orbit of hydrogen of 0.0001 mm in diameter is
- (1) 1000 (2) 60
(3) 10000 (4) 31
63. If elements of quantum number greater than n were not allowed, the number of possible elements in nature would have been
- (1) $\frac{1}{2}n(n+1)$ (2) $\left\{ \frac{n(n+1)}{2} \right\}^2$
(3) $\frac{1}{6}n(n+1)(2n+1)$ (4) $\frac{1}{3}n(n+1)(2n+1)$

64. The velocity of an electron in the first orbit of H atom is v . The velocity of an electron in the second orbit of He^+ is
 (1) $2v$ (2) v
 (3) $\frac{v}{2}$ (4) $\frac{v}{4}$
65. The recoil speed of a hydrogen atom after it emits a photon in going from $n = 5$ state to $n = 1$ state is
 (1) 4.718 m s^{-1} (2) 7.418 m s^{-1}
 (3) 4.178 m s^{-1} (4) 7.148 m s^{-1}
66. If a hydrogen atom emits a photon of energy 12.1 eV , its orbital angular momentum changes by ΔL . Then, ΔL equals
 (1) $1.05 \times 10^{-34} \text{ J s}$ (2) $2.11 \times 10^{-34} \text{ J s}$
 (3) $3.16 \times 10^{-34} \text{ J s}$ (4) 4.22×10^{-34}
67. An electron with kinetic energy $E \text{ eV}$ collides with a hydrogen atom in the ground state. The collision is observed to be elastic for
 (1) $0 < E < \infty$ (2) $0 < E < 10.2 \text{ eV}$
 (3) $0 < E < 13.6 \text{ eV}$ (4) $0 < E < 3.4 \text{ eV}$
68. The wavelength K_α of X-rays produced by the X-ray tube is $0.76 \text{ \AA}$. The atomic number of the node material of the tube is
 (1) 30 (2) 40
 (3) 50 (4) 60
69. Let the potential energy of a hydrogen atom in the ground state be zero. Then, its energy in the first excited state will be
 (1) 10.2 eV (2) 13.6 eV
 (3) 23.8 eV (4) 27.2 eV
70. When photons of wavelength λ_1 are incident on an isolated sphere suspended by an insulated thread, the corresponding stopping potential is found to be V . When photons of wavelength λ_2 are used, the corresponding stopping potential was thrice the above value. If light of wavelength λ_3 is used, calculate the stopping potential for this case.
 (1) $\frac{hc}{e} \left[\frac{1}{\lambda_3} + \frac{1}{2\lambda_2} - \frac{1}{\lambda_1} \right]$ (2) $\frac{hc}{e} \left[\frac{1}{\lambda_3} + \frac{1}{2\lambda_2} - \frac{3}{2\lambda_1} \right]$
 (3) $\frac{hc}{e} \left[\frac{1}{\lambda_3} + \frac{1}{\lambda_2} - \frac{1}{\lambda_1} \right]$ (4) $\frac{hc}{e} \left[\frac{1}{\lambda_3} - \frac{1}{\lambda_2} - \frac{1}{\lambda_1} \right]$
71. The ionization energy of the ionized sodium atom Na^{10+} is
 (1) 13.6 eV (2) $13.6 \times 11 \text{ eV}$
 (3) $(13.6/11) \text{ eV}$ (4) $13.6 \times (11^2) \text{ eV}$
72. The shortest wavelength produced in an X-ray tube operating at 0.5 million volt is
 (1) dependent on the target element
 (2) about $2.5 \times 10^{-12} \text{ m}$
 (3) double of the shortest wavelength produced at 1 million volt
 (4) dependent only on the target material
73. If the K_α radiation of Mo ($Z = 42$) has a wavelength of $0.71 \text{ \AA}$ find the wavelength of the corresponding radiation of Cu ($Z = 29$)
 (1) $1 \text{ \AA}$ (2) $2 \text{ \AA}$
 (3) $1.52 \text{ \AA}$ (4) $1.25 \text{ \AA}$
74. A beam of electron accelerated by a large potential difference V is made to strike a metal target to produce X-rays. For which of the following values of V , will the resulting X-rays have the lowest minimum wavelength?
 (1) 10 kV (2) 20 kV
 (3) 30 kV (4) 40 kV
75. The minimum kinetic energy required for ionization of a hydrogen atom is E_1 in case electron is collided with hydrogen atom. It is E_2 if the hydrogen ion is collided and E_3 when helium ion is collided. Then,
 (1) $E_1 = E_2 = E_3$ (2) $E_1 > E_2 > E_3$
 (3) $E_1 < E_2 < E_3$ (4) $E_1 > E_3 > E_2$
76. The ratio between total acceleration of the electron in singly ionized helium atom and hydrogen atom (both in ground state) is
 (1) 1 (2) 8
 (3) 4 (4) 16
77. The shortest wavelength of the Brackett series of a hydrogen-like atom (atomic number = Z) is the same as the shortest wavelength of the Balmer series of hydrogen atom. The value of Z is
 (1) 2 (2) 3
 (3) 4 (4) 6
78. A neutron moving with a speed v makes a head-on collision with a hydrogen atom in ground state kept at rest. The minimum kinetic energy of the neutron for which inelastic collision will take place is (assume that mass of proton is nearly equal to the mass of neutron)
 (1) 10.2 eV (2) 20.4 eV
 (3) 12.1 eV (4) 16.8 eV
79. Angular momentum (L) and radius (r) of a hydrogen atom are related as
 (1) $Lr = \text{constant}$ (2) $Lr^2 = \text{constant}$
 (3) $Lr^4 = \text{constant}$ (4) none of these
80. In hydrogen and hydrogen-like atoms, the ratio of $E_{4n} - E_{2n}$ and $E_{2n} - E_n$ varies with atomic number Z and principal quantum number n as
 (1) $\frac{Z^2}{n^2}$ (2) $\frac{Z^4}{n^4}$
 (3) $\frac{Z}{n}$ (4) none of these
81. According to Bohr's theory of hydrogen atom, the product of the binding energy of the electron in the n^{th} orbit and its radius in the n^{th} orbit
 (1) is proportional to n^2
 (2) is inversely proportional to n^3
 (3) has a constant value of $10.2 \text{ eV-\AA}$
 (4) has a constant value of $7.2 \text{ eV-\AA}$
82. In X-ray tube, when the accelerating voltage V is halved, the difference between the wavelength of K_α line and minimum wavelength of continuous X-ray spectrum
 (1) remains constant
 (2) becomes more than two times

- (3) becomes half
(4) becomes less than two times
83. An electron is in an excited state in a hydrogen like atom. It has a total energy of -3.4 eV. The kinetic energy of electron is E and its de Broglie wavelength is λ
(1) $E = 6.8$ eV; $\lambda = 6.6 \times 10^{-10}$ m
(2) $E = 3.4$ eV; $\lambda = 6.6 \times 10^{-10}$ m
(3) $E = 3.4$ eV; $\lambda = 6.6 \times 10^{-11}$ m
(4) $E = 6.8$ eV; $\lambda = 6.6 \times 10^{-11}$ m
84. The circumference of the second Bohr orbit of electron in hydrogen atom is 600 nm. The potential difference that must be applied between the plates so that the electrons have the de Broglie wavelength corresponding in this circumference is
(1) 10^{-5} V (2) $\frac{5}{3} \times 10^{-5}$ V
(3) 5×10^{-5} V (4) 3×10^{-5} V
85. X-rays emitted from a copper target and a molybdenum target are found to contain a line of wavelength 22.85 nm attributed to the K_α line of an impurity element. The K_α lines of copper ($Z = 29$) and molybdenum ($Z = 42$) have wavelengths 15.42 nm and 7.12 nm, respectively. The atomic number of the impurity element is
(1) 22 (2) 23
(3) 24 (4) 25
86. An electron in a Bohr orbit of hydrogen atom with the quantum number n_2 has an angular momentum 4.2176×10^{-34} kg m² s⁻¹. If the electron drops from this level to the next lower level, the wavelength of this line is
(1) 18 nm (2) 187.6 pm
(3) 1876 Å (4) 1.876×10^4 Å
87. In the Bohr model of a π -mesic atom, a π -meson of mass m_π and of the same charge as the electron is in a circular orbit of radius r about the nucleus with an orbital angular momentum $h/2\pi$. If the radius of a nucleus of atomic number Z is given by $R = 1.6 \times 10^{-15} Z^{\frac{1}{3}}$ m, then the limit on Z for which ($\epsilon_0 h^2 / \pi m e^2 = 0.53$ Å and $m_\pi / m_e = 264$) π -mesic atoms might exist is
(1) < 105 (2) > 105
(3) < 37 (4) > 37
88. In the spectrum of singly ionized helium, the wavelength of a line observed is almost the same as the first line of Balmer series of hydrogen. It is due to transition of electron
(1) from $n_1 = 6$ to $n_2 = 4$ (2) from $n_1 = 5$ to $n_2 = 3$
(3) from $n_1 = 4$ to $n_2 = 2$ (4) from $n_1 = 3$ to $n_2 = 2$
89. A K_α X-ray emitted from a sample has an energy of 7.46 keV. Of which element is the sample made?
(1) Calcium (Ca, $Z = 20$) (2) Cobalt (Co, $Z = 27$)
(3) Cadmium (Cd, $Z = 48$) (4) Nickel (Ni, $Z = 28$)
90. If elements with principal quantum number $n > 4$ were not allowed in nature, the number of possible elements would be
(1) 60 (2) 32
(3) 4 (4) 64
91. Consider the spectral line resulting from the transition $n = 2 \rightarrow n = 1$ in the atoms and ions given below. The shortest wavelength is produced by
(1) hydrogen atom (2) deuterium atom
(3) singly ionized helium (4) doubly ionized lithium
92. As per the Bohr model, the minimum energy (in eV) required to remove an electron from the ground state of doubly ionized Li atom ($Z = 3$) is
(1) 1.51 (2) 13.6
(3) 40.8 (4) 122.4
93. If the atom $^{257}_{100}\text{Fm}$ follows the Bohr model and the radius of $^{257}_{100}\text{Fm}$ is n times the Bohr radius, then find n .
(1) 100 (2) 200
(3) 4 (4) $1/4$
94. Electrons with de Broglie wavelength λ fall on the target in an X-ray tube. The cut-off wavelength of the emitted X-rays is
(1) $\lambda_0 = \frac{2mc\lambda^2}{h}$ (2) $\lambda_0 = \frac{2h}{mc}$
(3) $\lambda_0 = \frac{2m^2c^2\lambda^3}{h^2}$ (4) $\lambda_0 = \lambda$

Multiple Correct Answers Type

1. Hydrogen atoms absorb radiation of wavelength λ_0 and consequently emit radiations of 6 different wavelengths of which two wavelengths are shorter than λ_0 . Then,
(1) the final excited state of the atoms is $n = 4$
(2) the initial state of the atoms may be $n = 2$
(3) the initial state of the atoms may be $n = 3$
(4) there are three transitions belonging to Lyman series
2. Energy liberated in the de-excitation of hydrogen atom from the third level to the first level falls on a photo-cathode. Later when the same photo-cathode is exposed to a spectrum of some unknown hydrogen-like gas, excited to the second energy level, it is found that the de Broglie wavelength of the fastest photoelectrons now ejected has decreased by a factor of 3. For this new gas, difference of energies of the second Lyman line and the first Balmer line is found to be 3 times the ionization potential of the hydrogen atom. Select the correct statement(s).
(1) The gas is lithium.
(2) The gas is helium.
(3) The work function of photo-cathode is 8.5 eV.
(4) The work function of photo-cathode is 5.5 eV.
3. According to Bohr's theory of hydrogen atom, for the electron in the n th permissible orbit
(1) linear momentum $\propto \frac{1}{n}$ (2) radius of orbit $\propto n$
(3) kinetic energy $\propto \frac{1}{n^2}$ (4) angular momentum $\propto n$
4. When a hydrogen atom is excited from ground state to first excited state, then
(1) its kinetic energy increases by 20 eV
(2) its kinetic energy decreases by 10.2 eV
(3) its potential energy increases by 20.4 eV
(4) its angular momentum increases by 1.05×10^{-34} J s

5. In an X-ray tube, the voltage applied is 20 kV. The energy required to remove an electron from L shell is 19.9 keV. In the X-rays emitted by the tube,
 - (1) minimum wavelength will be 62.1 nm.
 - (2) energy of the characteristic X-rays will be equal to or less than 19.9 keV.
 - (3) L_α X-ray may be emitted.
 - (4) L_α X-ray will have energy 19.9 keV.
6. Suppose the potential energy between an electron and a proton at a distance r is given by $Ke^2/3r^3$. Application of Bohr's theory to hydrogen atom in this case shows that
 - (1) energy in the n th orbit is proportional to n^6
 - (2) energy is proportional to m^{-3} (m = mass of electron)
 - (3) energy in the n th orbit is proportional to n^{-2}
 - (4) energy is proportional to m^3 (m = mass of electron)
7. Let A_n be the area enclosed by the n th orbit in a hydrogen atom. The graph of $\ln(A_n/A_1)$ against $\ln(n)$
 - (1) will pass through origin
 - (2) will be a straight line with slope 4
 - (3) will be a monotonically increasing nonlinear curve
 - (4) will be a circle
8. Mark out the correct statement(s).
 - (1) Line spectra contain information about atoms only.
 - (2) Line spectra contain information about both atoms and molecules.
 - (3) Band spectra contain information about both atoms and molecules.
 - (4) Band spectra contain information about molecules only.
9. A hydrogen atom having kinetic energy E collides with a stationary hydrogen atom. Assume all motions are taking place along the line of motion of the moving hydrogen atom. For this situation, mark out the correct statement(s).
 - (1) For $E \geq 20.4$ eV only, collision would be elastic.
 - (2) For $E \geq 20.4$ eV only, collision would be inelastic.
 - (3) For $E = 2.4$ eV, collision would be perfectly inelastic.
 - (4) For $E = 18$ eV, the KE of initially moving hydrogen atom after collision is zero.
10. In Bohr's model of hydrogen atom,
 - (1) the radius of n th orbit is proportional to n^2
 - (2) the total energy of electron in n th orbit is proportional to n
 - (3) the angular momentum of the electron in an orbit is an integral multiple of $h/2\pi$
 - (4) the magnitude of the potential energy of an electron in any orbit is greater than its kinetic energy
11. Which of the following are in the ascending order of wavelength?
 - (1) $H_\alpha, H_\beta, H_\gamma, \dots$ lines in the Balmer series of hydrogen atom.
 - (2) Lyman limit, Balmer limit, and Paschen limit in the hydrogen spectrum
 - (3) Violet, blue, yellow, and red colors in solar spectrum.
 - (4) None of the above.
12. Continuous spectrum is produced by
 - (1) incandescent electric bulb
 - (2) sun
 - (3) hydrogen molecules
 - (4) sodium vapor lamp
13. A gas of monoatomic hydrogen is bombarded with a stream of electrons that have been accelerated from rest through a potential difference of 12.75 V. In the emission spectrum, one can observe lines of
 - (1) Lyman series
 - (2) Balmer series
 - (3) Paschen series
 - (4) Pfund series
14. If the potential energy of the electron in the first allowed orbit in hydrogen atom is E , its
 - (1) ionization potential is $-E/2$
 - (2) kinetic energy is $-E/2$
 - (3) total energy is $E/2$
 - (4) none of these
15. According to Bohr's theory of the hydrogen atom, for the electron in the n th allowed orbit
 - (1) the linear momentum is proportional to $(1/n)$
 - (2) the radius is proportional to n
 - (3) the kinetic energy is proportional to $(1/n^2)$
 - (4) the angular momentum is proportional to n
16. Whenever a hydrogen atom emits a photon in the Balmer series,
 - (1) it may emit another photon in the Balmer series
 - (2) it must emit another photon in the Lyman series
 - (3) the second photon, if emitted, will have a wavelength of about 122 nm
 - (4) it may emit a second photon, but the wavelength of this photon cannot be predicted
17. Let $\lambda_\alpha, \lambda_\beta$ and λ'_α denote the wavelengths of the X-rays of the K_α, K_β and L_α lines in the characteristic X-rays for a metal. Then,
 - (1) $\lambda'_\alpha > \lambda_\alpha > \lambda_\beta$
 - (2) $\lambda'_\alpha > \lambda_\beta > \lambda_\alpha$
 - (3) $\frac{1}{\lambda_\beta} = \frac{1}{\lambda_\alpha} + \frac{1}{\lambda'_\alpha}$
 - (4) $\frac{1}{\lambda_\alpha} = \frac{1}{\lambda_\beta} + \frac{1}{\lambda'_\alpha}$
18. Which of the following statements are correct for an X-ray tube?
 - (1) On increasing potential difference between the filament and target, photon flux of X-rays increases.
 - (2) On increasing potential difference between the filament and target, frequency of X-rays increases.
 - (3) On increasing filament current, cut-off wavelength increases.
 - (4) On increasing filament current, intensity of X-rays increases.
19. Which of the following statements about hydrogen spectrum are correct?
 - (1) All the lines of Lyman series lie in ultraviolet region.
 - (2) All the lines of Balmer series lie in visible region.
 - (3) All the lines of Paschen series lie in infrared region.
 - (4) None of these.

20. Which of the following statements are true?

- (1) The shortest wavelength of X-rays emitted from an X-ray tube depends on the current in the tube.
- (2) Characteristic X-ray spectra is simple as compared to optical spectra.
- (3) X-rays cannot be diffracted by means of an ordinary grating.
- (4) There exists a sharp limit on the short wavelength side for each continuous X-ray spectrum.

21. An X-ray tube is operated at 6.6 kV. In the continuous spectrum of the emitted X-rays, which of the following frequencies will be missing?

- (1) 10^{18} Hz
- (2) 1.5×10^{18} Hz
- (3) 2×10^{18} Hz
- (4) 2.5×10^{18} Hz

22. Which of the following statements are correct?

- (1) If angular momentum of the Earth due to its motion around the Sun were quantized according to Bohr's relation $L = nh/2\pi$, then the quantum number n would be of the order of 10^{74} .
- (2) If elements with principal quantum number > 4 were not allowed in nature, then the number of possible elements would be 64.
- (3) Rydberg's constant varies with mass number of the element.
- (4) The ratio of the wave number of K_α line of Balmer series for hydrogen and that of K_α line of Balmer series for singly ionized helium is exactly 4.

23. According to Einstein's theory of relativity, mass can be converted into energy and vice-versa. The lightest elementary particle, taken to be the electron, has a mass equivalent to 0.51 MeV of energy. Then, we can say that

- (1) the minimum amount of energy available through conversion of mass into energy is 1.2 MeV
- (2) the least energy of a γ -ray photon that can be converted into mass is 1.02 MeV
- (3) whereas the minimum energy released by conversion of mass into energy is 1.02 MeV, it is only a γ -ray photon of energy 0.51 MeV and above that can be converted into mass
- (4) whereas the minimum energy released by conversion of mass into energy is 0.51 MeV, it is only a γ -ray photon of energy 1.02 MeV and above that can be converted into mass

24. X-ray from a tube with a target A of atomic number Z shows strong K lines for target A and weak K lines for impurities. The wavelength of K_α lines is λ_z for target A and λ_1 and λ_2 for two impurities.

$$\frac{\lambda_z}{\lambda_1} = 4 \quad \text{and} \quad \frac{\lambda_z}{\lambda_2} = \frac{1}{4}$$

Assuming the screening constant of K_α lines to be unity, select the correct statement(s).

- (1) The atomic number of first impurity is $2Z - 1$.
- (2) The atomic number of first impurity is $2Z + 1$.

(3) The atomic number of second impurity is $\frac{(z+1)}{2}$.

(4) The atomic number of second impurity is $\frac{z}{2} + 1$.

25. In Bohr's model of the hydrogen atom, let R , V , T , and E represent the radius of the orbit, speed of the electron, time period of revolution of electron, and the total energy of the electron, respectively. The quantities proportional to the quantum number n are

- (1) VR
- (2) RE
- (3) $\frac{T}{R}$
- (4) $\frac{V}{E}$

26. An X-ray tube is operating at 50 kV and 20 mA. The target material of the tube has mass of 1 kg and specific heat $495 \text{ J kg}^{-1} \text{ }^\circ\text{C}^{-1}$. One percent of applied electric power is converted into X-rays and the remaining energy goes into heating the target. Then,

- (1) a suitable target material must have high melting temperature
- (2) a suitable target material must have low thermal conductivity
- (3) the average rate of rise of temperature of the target would be 2°C s^{-1}
- (4) the minimum wavelength of X-rays emitted is about $0.25 \times 10^{-10} \text{ m}$

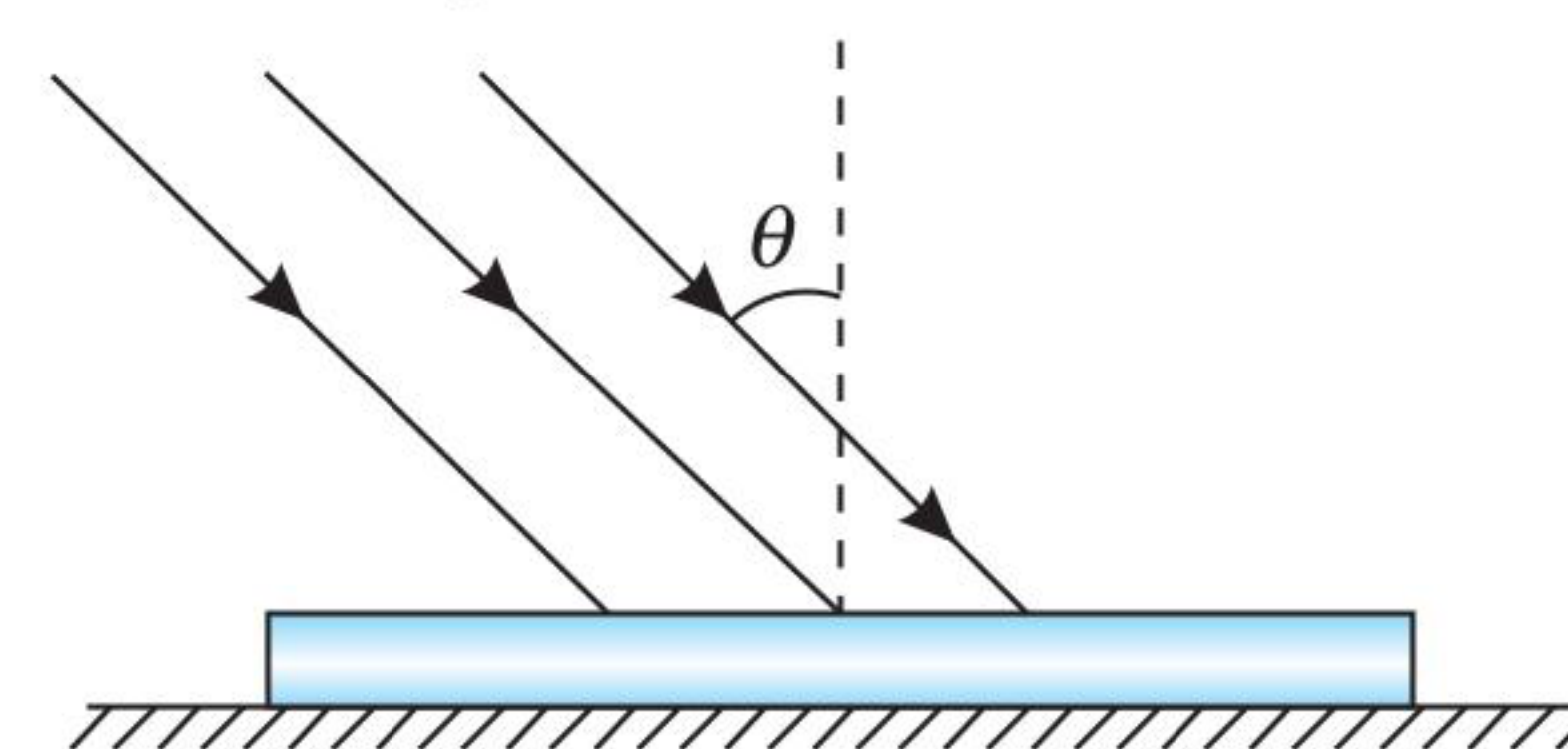
27. For a certain metal, the K absorption edge is at $0.172 \text{ \AA}$. The wavelength of K_α , K_β , and K_γ lines of K series are $0.210 \text{ \AA}$, $0.192 \text{ \AA}$, and $0.180 \text{ \AA}$, respectively. The energies of K , L , and M orbits are E_K , E_L and E_M , respectively. Then

- (1) $E_K = -13.04 \text{ keV}$
- (2) $E_L = -7.52 \text{ keV}$
- (3) $E_M = -3.21 \text{ keV}$
- (4) $E_K = 13.04 \text{ keV}$

28. The third line of the Balmer series spectrum of a hydrogen-like ion of atomic number Z equals to 108.5 nm . The binding energy of the electron in the ground state of these ions is E_B . Then

- (1) $Z = 2$
- (2) $E_B = 54.4 \text{ eV}$
- (3) $Z = 3$
- (4) $E_B = 122.4 \text{ eV}$

29. A hydrogen like atom is in higher excited state of quantum number n . This excited atom can make transition to the ground state by successively emitting two photon of energies 28.3 eV and 302.2 eV respectively, when this element is used in X-ray experiment K_α peak occurs at $\lambda = 7.64 \text{ nm}$ ($R = 1.09 \times 10^7 \text{ m}^{-1}$)



- (1) Atomic number of atom $Z = 5$
- (2) The value of $n = 6$
- (3) n can't be calculated until state of atom after first emission is known.
- (4) Atom is in 3^{rd} excited state after emitting first photon

30. Consider an atom made up of a proton and a hypothetical particle of triple the mass of electron but having same charge as the electron. Apply the Bohr model and consider all possible transitions of this hypothetical particle from the second excited state to the lower states. The possible wavelengths emitted is (are) (given in terms of the Rydberg constant R for the hydrogen atom)

- (1) $\frac{8}{5R}$ (2) $\frac{3}{8R}$
 (3) $\frac{4}{9R}$ (4) $\frac{12}{5R}$

31. The mass number of a nucleus is

- (1) always less than its atomic number
 (2) always more than its atomic number
 (3) sometimes equal to its atomic number
 (4) sometimes more than and sometimes equal to its atomic number

32. The electron in a hydrogen atom makes a transition $n_1 \rightarrow n_2$, where n_1 and n_2 are the principal quantum numbers of the two states. Assume the Bohr model to be valid. The time period of the electron in the initial state is eight times that in the final state. The possible values of n_1 and n_2 are

- (1) $n_1 = 4, n_2 = 2$ (2) $n_1 = 8, n_2 = 2$
 (3) $n_1 = 8, n_2 = 1$ (4) $n_1 = 6, n_2 = 3$

Linked Comprehension Type

For Problems 1–3

Hydrogen is the simplest atom of nature. There is one proton in its nucleus and an electron moves around the nucleus in a circular orbit. According to Niels Bohr, this electron moves in a stationary orbit. When this electron is in the stationary orbit, it emits no electromagnetic radiation. The angular momentum of the electron is quantized, i.e., $mvr = (nh/2\pi)$, where m = mass of the electron, v = velocity of the electron in the orbit, r = radius of the orbit, and $n = 1, 2, 3, \dots$. When transition takes place from K th orbit to J th orbit, energy photon is emitted. If the wavelength of the emitted photon is λ , we

find that $\frac{1}{\lambda} = R \left[\frac{1}{J^2} - \frac{1}{K^2} \right]$, where R is

Rydberg's constant.

On a different planet, the hydrogen atom's structure was somewhat different from ours. The angular momentum of electron was $P = 2n(h/2\pi)$, i.e., an even multiple of $(h/2\pi)$.

Answer the following questions regarding the other planet based on above passage:

1. The minimum permissible radius of the orbit will be

- (1) $\frac{2\varepsilon_0 h^2}{m\pi e^2}$ (2) $\frac{4\varepsilon_0 h^2}{m\pi e^2}$
 (3) $\frac{\varepsilon_0 h^2}{m\pi e^2}$ (4) $\frac{\varepsilon_0 h^2}{2m\pi e^2}$

2. In our world, the velocity of electron is v_0 when the hydrogen atom is in the ground state. The velocity of electron in this state on the other planet should be

- (1) v_0 (2) $v_0/2$
 (3) $v_0/4$ (4) $v_0/8$

3. In our world, the ionization potential energy of a hydrogen atom is 13.6 eV. On the other planet, this ionization potential energy will be

- (1) 13.6 eV (2) 3.4 eV
 (3) 1.5 eV (4) 0.85 eV

For Problems 4–6

The electrons in a H-atom kept at rest, jumps from the m th shell to the n th shell ($m > n$). Suppose instead of emitting electromagnetic wave, the energy released is converted into kinetic energy of the atom. Assume Bohr's model and conservation of angular momentum are valid. Now, answer the following questions.

4. What principle is violated here?

- (1) Laws of motion (2) Energy conservation
 (3) Nothing is violated (4) Cannot be decided

5. Calculate the angular velocity of the atom about the nucleus if I is the moment of inertia.

- (1) $\frac{(m+n)h}{6.28 I}$ (2) $\frac{(m+n)h}{1.57 I}$
 (3) $\frac{(m-n)h}{6.28 I}$ (4) $\frac{(m-n)h}{1.57 I}$

6. If the above comprehension be true, what is not valid here?

- (1) $F = ma$ (2) $\tau = I\alpha$
 (3) $F = dp/dt$ (4) All of them

For Problems 7–12

The energy levels of a hypothetical one electron atom are shown in figure.

$n = \infty$	0 eV
$n = 5$	-0.80 eV
$n = 4$	-1.45 eV
$n = 3$	-3.08 eV
$n = 2$	-5.30 eV
$n = 1$	-15.6 eV

7. Find the ionization potential of the atom.

- (1) 11.2 eV (2) 13.5 eV
 (3) 15.6 eV (4) 12.6 eV

8. Find the short wavelength limit of the series terminating at $n = 2$.

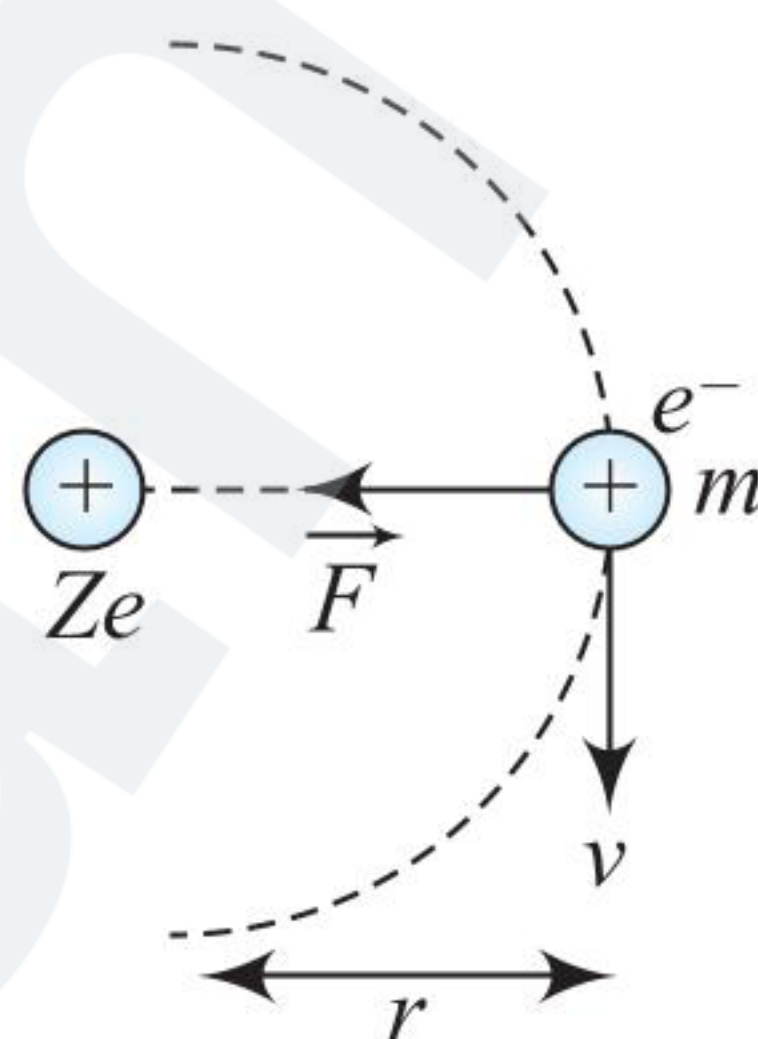
- (1) 3256 Å (2) 2339 Å
 (3) 2509 Å (4) 3494 Å

9. Find the excitation potential for the state $n = 3$.

- (1) 14.64 eV (2) 9.93 eV
 (3) 12.52 eV (4) 10.04 eV

10. Find the wave number of the photon emitted for the transition $n = 3$ to $n = 1$

- (1) $2.23 \times 10^7 \text{ m}^{-1}$ (2) $1.009 \times 10^7 \text{ m}^{-1}$
 (3) $3.005 \times 10^6 \text{ m}^{-1}$ (4) $0.432 \times 10^6 \text{ m}^{-1}$



11. If an electron with initial kinetic energy 6 eV is to interact with this hypothetical atom, what minimum energy will this electron carry after interaction?

- (1) 2 eV (2) 3 eV
(3) 6 eV (4) 0 eV

12. The initial kinetic energy of an electron is 11 eV and it interacts with the above said hypothetical one electron atom, the minimum energy carried by the electron after interaction is

- (1) 0.7 eV (2) 0.3 eV
(3) 0.9 eV (4) 1 eV

For Problems 13–14

The electron in a hydrogen atom at rest makes a transition from $n = 2$ energy state to the $n = 1$ ground state.

13. Find the energy (eV) of the emitted photon.

- (1) 5.8 eV (2) 8.3 eV
(3) 10.2 eV (4) 12.7 eV

14. Assume that all of the energy corresponding to transition from $n = 2$ to $n = 1$ is carried off by the photon. By setting the momentum of the system (atom + photon) equal to zero after the emission and assuming that the recoil energy of the atom is smaller compared with the $n = 2$ to $n = 1$ energy level separation, find the energy of the recoiling hydrogen atom.

- (1) 2.75×10^{-7} eV (2) 5.54×10^{-8} eV
(3) 8.11×10^{-8} eV (4) 10.36×10^{-7} eV

For Problems 15–16

Consider a hypothetical hydrogen-like atom. The wavelength, in Å, for the spectral lines for transitions from $n = p$ to $n = 1$ are given by

$$\lambda = \frac{1500p^2}{p^2 - 1} \quad \text{where } p = 1, 2, 3, 4, \dots$$

15. Find the wavelength of the most energetic photons in this series.

- (1) 1800 Å (2) 1500 Å
(3) 1300 Å (4) 1650 Å

16. What is the ionization potential of this element?

- (1) 3.96 V (2) 9.23 V
(3) 6.34 V (4) 8.28 V

For Problems 17–20

A sample of hydrogen gas in its ground state is irradiated with photons of 10.2 eV energies. The radiation from the above sample is used to irradiate two other samples of excited ionized He^+ and excited ionized Li^{2+} , respectively. Both the ionized samples absorb the incident radiation.

17. How many spectral lines are obtained in the spectra of Li^{2+} ?

- (1) 10 (2) 15
(3) 20 (4) 17

18. Which is the smallest wavelength that will be observed in spectra of He^+ ion?

- (1) 24.4 nm (2) 28.8 nm
(3) 22.2 nm (4) 30.6 nm

19. How many spectral lines are observed in spectra of He^+ ion?

- (1) 2 (2) 4
(3) 6 (4) 8

20. Which is the smallest wavelength observed in the spectra of Li^{2+} ?

- (1) 8.6 nm (2) 10.4 nm
(3) 12.8 nm (4) 14.6 nm

For Problems 21–22

An electron and a proton are separated by a distance r so that the potential energy between them is $u = k \log r$, where k is a constant.

21. In such an atom, radius of n th Bohr's orbit is

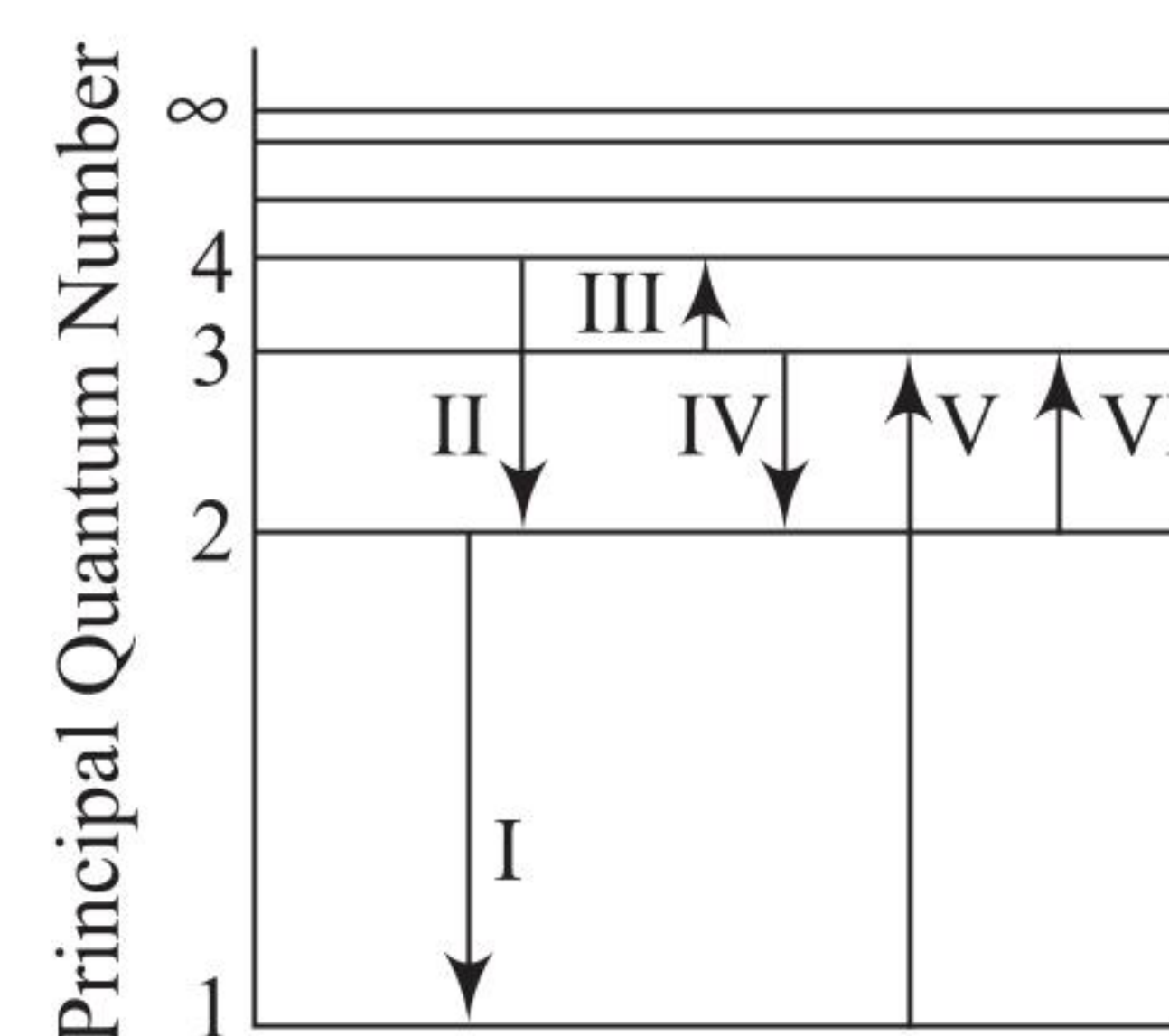
- (1) $\frac{2nh}{\pi\sqrt{mk}}$ (2) $\frac{nh}{2\pi\sqrt{2mk}}$
(3) $\frac{nh}{2\pi\sqrt{mk}}$ (4) $\frac{nh}{4\pi\sqrt{mk}}$

22. The expression for various energy levels of the above said hypothetical atom is

- (1) $\frac{k}{2} \left[1 + \log \frac{n^2 h^2}{4\pi^2 mk} \right]$ (2) $2k \left[2 + \log \frac{n^2 h^2}{4\pi^2 mk} \right]$
(3) $k \left[2 + \log \frac{n^2 h^2}{4\pi^2 mk} \right]$ (4) $\frac{k}{2} \left[1 + \log \frac{n^2 h^2}{2\pi^2 mk} \right]$

For Problems 23–25

Pertain to the following statement and figure.



The figure above shows an energy level diagram of the hydrogen atom. Several transitions are marked as I, II, III, ... The diagram is only indicative and not to scale.

23. In which transition is a Balmer series photon absorbed?

- (1) II (2) III
(3) IV (4) VI

24. The wavelength of the radiation involved in transition II is

- (1) 291 nm (2) 364 nm
(3) 487 nm (4) 652 nm

25. Which transition will occur when a hydrogen atom is irradiated with radiation of wavelength 103 nm?

- (1) I (2) II
(3) IV (4) V

For Problems 26–28

A certain species of ionized atoms produces emission line spectrum according to the Bohr model. A group of lines in the spectrum is forming a series in which the shortest wavelength is

22.79 nm and the longest wavelength is 41.02 nm. The atomic number of atom is Z .

Based on above information, answer the following questions:

26. The value of Z is

- | | |
|-------|-------|
| (1) 2 | (2) 3 |
| (3) 4 | (4) 5 |

27. The series belongs to

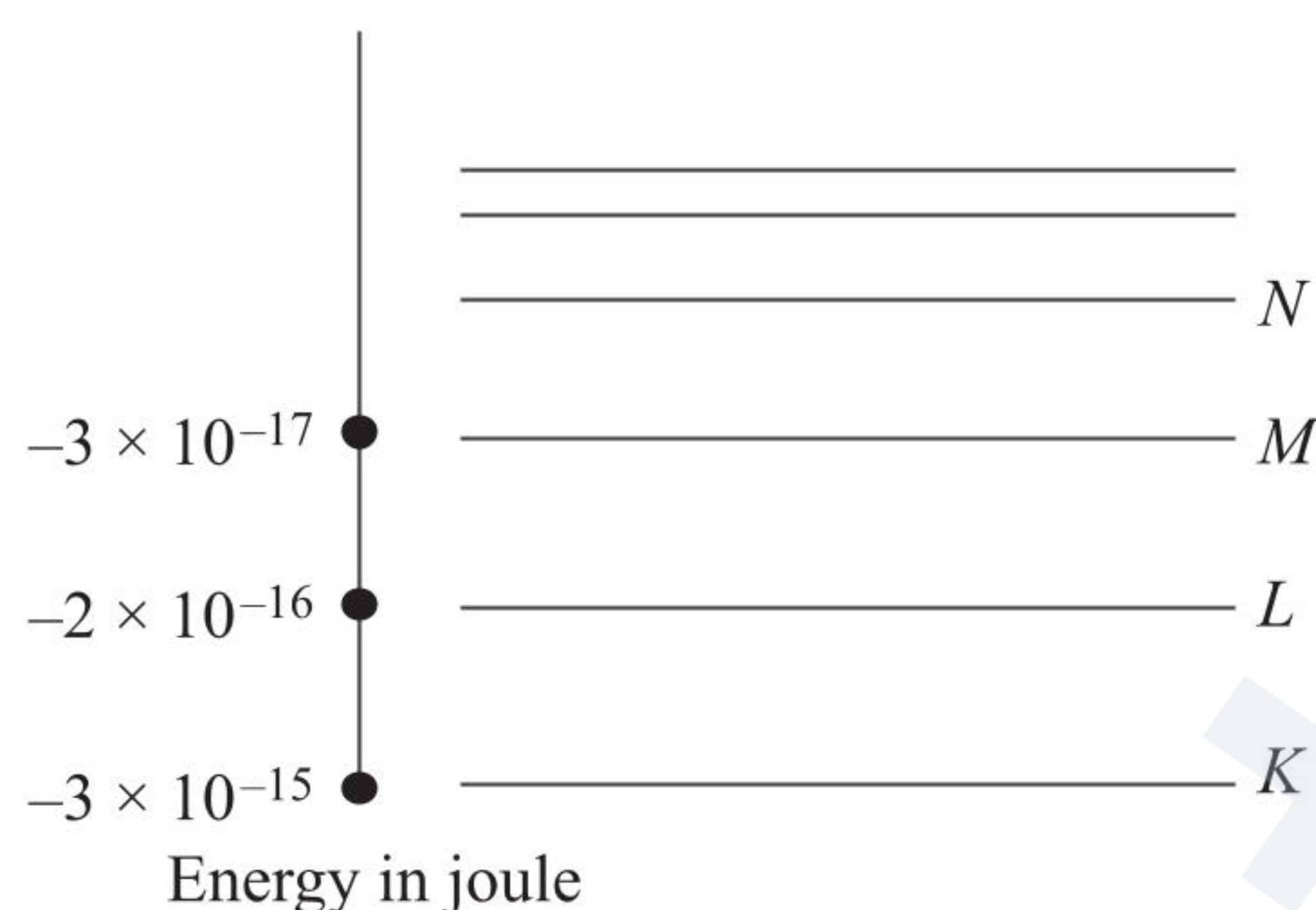
- | | |
|-------------|--------------|
| (1) Lyman | (2) Balmer |
| (3) Paschen | (4) Brackett |

28. The next to longest wavelength in the series of lines is

- | | |
|--------------|--------------|
| (1) 35.62 nm | (2) 30.47 nm |
| (3) 25.68 nm | (4) 12.64 nm |

For Problems 29–31

Simplified model of electron energy levels for a certain atom is shown in figure. The atom is bombarded with fast moving electrons. The impact of one of these electrons can cause the removal of electron from K -level, thus, creating a vacancy in the K -level. This vacancy in K -level is filled by an electron from L -level and the energy released in this transition can either appear as electromagnetic waves or may all be used to knock out an electron from M -level of the atom.



Based on the above information, answer the following questions:

29. The minimum potential difference through which bombarding electron beam must be accelerated from rest to cause the ejection of electron from K -level is

- | | |
|-------------|-------------|
| (1) 18750 V | (2) 400 kV |
| (3) 2.16 kV | (4) 21.6 kV |

30. Wavelength of the electromagnetic waves emitted due to transition from L to K -level is

- | | |
|---------------------------|-------------------------------|
| (1) 8×10^{-10} m | (2) 7.104×10^{-11} m |
| (3) 22.46 nm | (4) 142.6×10^{-8} m |

31. The KE of the emitted electron from M -level is

- | | |
|-----------------------------|-----------------------------|
| (1) 260×10^{-17} J | (2) 2×10^{-18} J |
| (3) 240×10^{-17} J | (4) 277×10^{-17} J |

For Problems 32–34

A monochromatic beam of light having photon energy 12.5 eV is incident on a sample A of atomic hydrogen gas in which all atoms are in the ground state. The emission spectra obtained from this sample is incident on another sample B of atomic hydrogen gas in which all atoms are in the first excited state.

Based on the above information, answer the following questions:

32. The atoms of sample A after passing of light through it

- (1) may be in the first excited state
- (2) may be in the second excited state
- (3) may be in both first and second excited states
- (4) none of the above

33. The emission spectra of sample A

- | | |
|-----------------------|-----------------------|
| (1) must have 3 lines | (2) must have 2 lines |
| (3) may have 2 lines | (4) it is not formed |

34. The atoms of sample B

- (1) will ionize when emission spectra of A is incident on B
- (2) may ionize when emission spectra of A is incident on B
- (3) will be excited to some higher energy state but won't ionize
- (4) none of the above

For Problems 35–37

An electron orbits a stationary nucleus of charge $+Ze$, where Z is a constant and e is the magnitude of electronic charge. It requires 47.2 eV to excite the electron from the second Bohr orbit to the third Bohr orbit.

35. The value of Z is

- | | |
|-------|-------|
| (1) 5 | (2) 4 |
| (3) 3 | (4) 2 |

36. The radius of the first Bohr orbit is

- | | |
|-------------------------------|-------------------------------|
| (1) 0.529×10^{-10} m | (2) 0.106×10^{-10} m |
| (3) 0.318×10^{-10} m | (4) none of these |

37. Angular momentum of electron in the first Bohr orbit is

- | | |
|--------------------------------|--------------------------------|
| (1) 1.05×10^{-13} J s | (2) 2.10×10^{-13} J s |
| (3) 3.15×10^{-13} J s | (4) Can be (1) or (2) |

For Problems 38–39

When high energetic electron beam, (i.e., cathode rays) strike the heavier metal, then X-rays are produced. Spectrum of X-rays are classified into two categories: (i) continuous spectrum, and (ii) characteristic spectrum. The wavelength of continuous spectrum depends only on the potential difference across the electrode. But wavelength of characteristic spectrum depends on the atomic number (Z).

38. The production of characteristic X-ray is due to the

- (1) continuous acceleration of incident electrons toward the nucleus
- (2) continuous retardation of incident electrons toward the nucleus
- (3) electron transitions between inner shells of the target atom
- (4) electron transitions between outer shells of the target atom

39. The production of continuous X-ray is due to the

- (1) acceleration of incident electrons by the nucleus of the target atom
- (2) electron transitions between inner shells of the target atom
- (3) electron transitions between outer shells of the target atom
- (4) annihilation of the mass of incident electrons

For Problems 40–42

Light from a discharge tube containing hydrogen atoms falls on the surface of a plate of sodium. The kinetic energy of the fastest photoelectrons emitted from sodium is 0.73 eV. The work function for sodium is 1.82 eV.

40. The energy of the photons causing the photoelectric emission is
 (1) 2.55 eV (2) 0.73 eV
 (3) 1.82 eV (4) Information insufficient
41. The quantum numbers of two levels involved in the emission of these photons is
 (1) $4 \rightarrow 2$ (2) $3 \rightarrow 1$
 (3) $3 \rightarrow 2$ (4) $4 \rightarrow 3$
42. The change in the angular momentum of the electron in the hydrogen atom in above transition is
 (1) $\frac{2h}{\pi}$ (2) $\frac{h}{2\pi}$
 (3) $\frac{h}{\pi}$ (4) $\frac{h}{4\pi}$

For Problems 43–45

The electron in a Li^{++} ion is in the n th shell, n being very large. One of the K -electrons in another metallic atom has been knocked out. The second metal has four orbits. Now, we take two samples—one of Li^{++} ions and the other of the second metallic ions. Suppose the probability of electronic transition from higher to lower energy levels is directly proportional to the energy difference between the two shells. Take $hc = 1224 \text{ eV nm}$, where h is Planck's constant and c the velocity of light in vacuum. It is found that major electromagnetic waves emitted from the two samples are identical. Now, answer the following questions:

43. What is the X-ray having least intensity emitted by the second sample?
 (1) K_α (2) L_α
 (3) M_α (4) data insufficient
44. What is the major X-ray emitted by the sample?
 (1) K_α (2) K_β
 (3) K_γ (4) K_δ
45. The wavelength of this major X-ray is
 (1) 0.90 Å (2) 1.0 Å
 (3) 1.1 Å (4) none of these

For Problems 46–47

Two hydrogen-like atoms A and B are of different and each atom has ratio of neutron to proton equal to unity. The difference in the energies between the first Lyman lines emitted by A and B is 81.6 eV. When the atoms A and B moving with the same velocity strike separately a heavy target, they rebound with half of the speed before collision. However, in this process atom B imparts the target a momentum which is three times the momentum imparted to target by atom A .

46. Atom A is
 (1) ${}^1_1\text{H}$ (2) ${}^2_1\text{H}$
 (3) ${}^6_3\text{Li}$ (4) ${}^4_2\text{Li}$

47. The difference in the energies between the first Balmer lines emitted by A and B is

- (1) 12.1 eV (2) 13.6 eV
 (3) 14.3 eV (4) 15.1 eV

For Problems 48–49

1.8 g of hydrogen is excited by irradiation. The study of spectra indicated that 27% of the atoms are in the first excited state, 15% of the atoms in the second excited state, and the rest in the ground state. The ground state ionization potential energy of hydrogen atom is 21.4×10^{-12} ergs.

48. The number of atoms present in the second excited state is
 (1) 1.61×10^{23} (2) 0.805×10^{23}
 (3) 2.92×10^{23} (4) 1.46×10^{23}
49. The total amount of energy that would be evolved when all the atoms return to the ground state is
 (1) 782 kJ (2) 978.7 kJ
 (3) 19.63×10^{11} erg (4) 97.87×10^{11} erg

For Problems 50–52

In a hydrogen like atom in which every atom is in a particular excited state. Now a stream of photons of energy $\frac{64}{225}E_0$ bombarded into it and is absorbed by the hydrogen like atom and subsequently its emission spectrum shows 10 different lines in which some lines have energy less than $\frac{64}{225}E_0$ and some lines have energy more than $\frac{64}{225}E_0$ and some lines have energy equal to $\frac{64}{225}E_0$, (where E_0 is ionization energy of hydrogen atom in ground state).

50. The final quantum number of electron in the atom is
 (1) 2 (2) 3
 (3) 4 (4) 5
51. The initial quantum number of electron in the atom is
 (1) 2 (2) 3
 (3) 4 (4) 5
52. Atomic number of the atom is
 (1) 2 (2) 3
 (3) 4 (4) 5

For Problems 53–55

In a mixture of H-He^+ gas (He^+ is singly ionized He atom), H atoms and He^+ ions are excited to their respective first excited states. Subsequently, H atoms transfer their total excitation energy to He^+ ions (by collisions). Assume that the Bohr model of atom is exactly valid.

	H atom		He^+ atom
$n = 4$	_____ -0.85 eV	_____	-3.4 eV
$n = 3$	_____ -1.51 eV	_____	-6.04 eV
$n = 2$	● _____ -3.4 eV	● _____	-13.6 eV
$n = 1$	_____ -13.6 eV	_____	-54.4 eV

53. The quantum number n of the state finally populated in He^+ ions is
 (1) 2 (2) 3
 (3) 4 (4) 5
54. The wavelength of light emitted in the visible region by He^+ ions after collisions with H atoms is
 (1) $6.5 \times 10^{-7} \text{ m}$ (2) $5.6 \times 10^{-7} \text{ m}$
 (3) $4.8 \times 10^{-7} \text{ m}$ (4) $4.0 \times 10^{-7} \text{ m}$
55. The ratio of the kinetic energy of the $n = 2$ electron for the H atom to that of He^+ ion is
 (1) $\frac{1}{4}$ (2) $\frac{1}{2}$
 (3) 1 (4) 2

Matrix Match Type

1. In each situation of Column I, a physical quantity related to orbiting electron in hydrogen-like atom is given. The terms “ Z ” and “ n ” given in Column II have usual meaning in Bohr’s theory. Match the quantities in Column I with the terms they depend on it Column II

Column I	Column II
i. Frequency of orbiting electron	a. is directly proportional to Z^2
ii. Angular momentum of orbiting electron	b. is directly proportional to n
iii. Magnetic moment of orbiting electron	c. is inversely proportional to n^3
iv. The average current due to orbiting of electron	d. is independent of Z

2. Take the usual meanings of the symbols to match the following:

Column I	Column II
i. Average kinetic energy of photoelectrons	a. zero
ii. Minimum kinetic energy of photoelectrons	b. hc/λ
iii. Maximum wavelength of continuous X-rays	c. hc/eV
iv. Minimum wavelength of continuous X-rays	d. not predictable

3. Match the following:

Column I	Column II
i. The voltage applied to X-ray tube is increased	a. Average KE of the electrons decreases
ii. In photoelectric effect, work function of the target is increased	b. Average KE of the electrons increases
iii. Stopping potential decreases	c. Cut-off wavelength decreased
iv. Wavelength of K_α X-ray increased	d. Atomic number of target material decreases

4. Match the following:

Column I	Column II
i. Characteristic X-ray	a. Inverse process of photoelectric effect
ii. X-ray production	b. High potential difference
iii. Cut-off wavelength	c. Moseley’s law
iv. Continuous X-ray	d. Emission of radiations

- 5.

Column I	Column II
i. K_α photon of aluminium	a. will be most energetic among the four
ii. K_β photon of aluminium	b. will be least energetic among the four
iii. K_α photon from sodium	c. will be more energetic than the K_α photon of lithium
iv. K_β photon of beryllium	d. constant speed

6. Match the entries of Column I with the entries of Column II:

Column I	Column II
i. Emission spectra	a. Discrete
ii. Absorption spectra	b. Continuous
iii. X-ray spectra	c. Electronic transition
iv. Thermal radiation spectra	d. Quantum theory of electromagnetic radiation

- 7.

Column I	Column II
i. Radius of orbit depends on principal quantum number as	a. increase
ii. Due to orbital motion of electron, magnetic field arises at the center of nucleus is proportional to principal quantum number as	b. decrease
iii. If electron is going from lower energy level to higher energy level, then velocity of electron will	c. proportional to $\frac{1}{n^2}$
iv. If electron is going from lower energy level to energy level, then total energy of electron will	d. proportional to n^2
	e. proportional to $\frac{1}{n^5}$

8.

Column I	Column II
i. Radius of orbit is related with atomic number (Z)	a. is proportional to Z
ii. Current associated due to orbital motion of electron with atomic number (Z)	b. is inversely proportional to Z
iii. Magnetic field at the center due to orbital motion of electron related with Z	c. is proportional to Z^2
iv. Velocity of an electron related with atomic number (Z)	d. is proportional to Z^3

9. The spectral lines of hydrogen-like atom fall within the wavelength range from $950 \text{ \AA}$ to $1350 \text{ \AA}$. Then, match the following.

Column I	Column II
i. If it is atomic hydrogen atom and energy $E = -0.85 \text{ eV}$	a. $\lambda = 1212 \text{ \AA}$ and it corresponds to transition from 2 to 1
ii. If it is atomic hydrogen atom and energy $E = -3.4 \text{ eV}$	b. $\lambda = 134 \text{ \AA}$ and it corresponds to transition from 2 to 1
iii. If it is doubly ionized lithium atom, then	c. $\lambda = 303 \text{ \AA}$ and it corresponds to transition from 2 to 1
iv. If it is singly ionized helium atom, then	d. $\lambda = 970 \text{ \AA}$ and it corresponds to transition from 4 to 1

Numerical Value Type

- Find recoil speed (approximately in m s^{-1}) when a hydrogen atom emits a photon during the transition from $n = 5$ to $n = 1$.
- An atom of atomic number $Z = 11$ emits K_α wavelength which is λ . Find the atomic number for an atom that emits K_α radiation with wavelength 4λ (an integer).
- An electron in n th excited state in a hydrogen atom comes down to first excited state by emitting ten different wavelengths. Find value of n (an integer).
- The shortest wavelength of the Brackett series of a hydrogen-like atom (atomic number Z) is the same as the shortest wavelength of the Balmer series of hydrogen atom. Find the value of Z .
- Heat at the rate of 200 W is produced in an X-ray tube operating at 20 kV . Find the current in the circuit. Assume that only a small fraction of the kinetic energy of electrons is converted into X-rays. (in $\times 10^{-2} \text{ A}$)
- In the spectrum of singly ionized helium, the wavelength of a line observed is almost the same as the first line of Balmer

series of hydrogen. It is due to transition of electron from $n_1 = 6$ to $n_2 = \text{"*"}'$. What is the value of $\text{"*"}'$?

- A Bohr hydrogen atom undergoes a transition $n = 5$ to $n = 4$ and emits a photon of frequency f . Frequency of circular motion of electron in $n = 4$ orbit is f_4 . The ratio f/f_4 is found to be $18/5m$. State the value of m .
- The average lifetime for the $n = 3$ excited state of a hydrogen-like atom is $4.8 \times 10^{-8} \text{ s}$ and that for the $n = 2$ state is $12.8 \times 10^{-8} \text{ s}$. The ratio of the average number of revolutions made in the $n = 2$ state to the average number of revolutions made in the $n = 3$ state before any transitions can take place from these states is.
- An electron in hydrogen atom jumps from n_1 state to n_2 state, where n_1 and n_2 represent the quantum number of two states. The time period of revolution of electron in initial state is 8 times that in final state. Then the ratio of n_1 and n_2 is
- Find the quantum number ' n ' corresponding to exciting state of He^+ ion if on transition to the ground state that ion emits two photons in succession with wavelength $1026.7 \text{ \AA}$ and $304 \text{ \AA}$. ($R = 1.096 \times 10^7/\text{m}$)
- The largest wavelength in the ultraviolet region of the hydrogen spectrum is 122 nm . Find the smallest wavelength in the infrared region of the hydrogen spectrum (to the nearest integer in nm).
- The peak emission from a black body at a certain temperature occurs at a wavelength of $9000 \text{ \AA}$. On increasing its temperature, the total radiation emitted is increased 81 times. At the initial temperature when the peak radiation from the black body is incident on a metal surface, it does not cause any photoemission from the surface. After the increase of temperature, the peak radiation from the black body caused photoemission. To bring these photoelectrons to rest, a potential equivalent to the excitation energy between $n = 2$ and $n = 3$ Bohr levels of hydrogen atoms is required. Find the work function of the metal (in eV).
- K_α wavelength emitted by an atom, of atomic number $Z = 11$ is λ . Find the atomic number for an atom that emits K_α radiation with wavelength 4λ .
- In a hydrogen-like atom, an electron is orbiting in an orbit having quantum number n . Its frequency of revolution is found to be $13.2 \times 10^{15} \text{ Hz}$. Energy required to move this electron from the atom to the above orbit is 54.4 eV . In a time of 7 nanosecond the electron jumps back to orbit having quantum number $n/2$. If τ be the average torque acted on the electron during the above process, then find $\tau \times 10^{27}$ in Nm . (Given: $h/\lambda = 2.1 \times 10^{-34} \text{ J-s}$, frequency of revolution of electron in the ground state of H atom $\nu_0 = 6.6 \times 10^{15} \text{ Hz}$ and ionization energy of H atom $E_0 = 13.6 \text{ eV}$)
- A hydrogen-like gas is kept in a chamber having a slit of width $d = 0.01 \text{ mm}$. The atoms of the gas are continuously excited to a certain energy state. The excited electrons make transitions to lower levels; from the initial excited state to the second excited state and then from the second excited state to the ground state. In the process of de-excitation, photons are emitted and come out of the container through a slit. The intensity of the photons is observed on a screen placed

parallel to the plane of the slit. The ratio of the angular width of the central maximum corresponding to the two transitions is $25/2$. The angular width of the central maximum due to first transition is 6.4×10^{-2} radian. Find the atomic number of the gas.

16. A gas containing hydrogen-like ions, with atomic number Z , emits photons in transition $n + 2 \rightarrow n$, where $n = Z$. These photons fall on a metallic plate and eject electrons having minimum de Broglie wavelength λ of $5 \text{ \AA}$. Find the value of Z , if the work function of metal is 4.2 eV
17. A hydrogen atom (mass $= 1.66 \times 10^{-27} \text{ kg}$, ionization potential $= 13.6 \text{ eV}$), moving with a velocity $6.24 \times 10^4 \text{ m s}^{-1}$ makes a completely inelastic head-on collision with another stationary hydrogen atom. Both atoms are in the ground state before collision. Up to what state ($n = ?$) either one atom may be excited?

18. Radiation from hydrogen gas excited to first excited state is used for illuminating certain photoelectric plate. When the radiation from some unknown hydrogen-like gas excited to the same level is used to expose the same plate, it is found that the de Broglie wavelength of the fastest photoelectron has decreased 2.3 times. It is given that the energy corresponding to the longest wavelength of the Lyman series of the unknown gas is 3 times the ionization energy of the hydrogen gas (13.6 eV). Find the work function of photoelectric plate in eV. [Take $(2.3)^2 = 5.25$.]
19. When the voltage applied to an X-ray tube is increased from 10 kV to 20 kV , the wavelength interval between the K_α line and the short wave cut off of the continuous X-ray spectrum increases by a factor 3. Find the atomic number of the element of which the tube anti cathode is made. (Rydberg's constant $= 10^7 \text{ m}^{-1}$).

Archives

JEE ADVANCED

Single Correct Answer Type

1. The wavelength of the first spectral line in the Balmer series of hydrogen atom is $6561 \text{ \AA}$. The wavelength of the second spectral line in the Balmer series of singly ionized helium atom is

- (1) $1215 \text{ \AA}$ (2) $1640 \text{ \AA}$
(3) $2430 \text{ \AA}$ (4) $4687 \text{ \AA}$ (IIT-JEE 2011)

Multiple Correct Answers Type

1. Highly excited states for hydrogen-like atoms (also called Rydberg states) with nuclear charge Ze are defined by their principal quantum number n , where $n \gg 1$. Which of the following statement(s) is (are) true?

- (1) Relative change in the radii of two consecutive orbitals does not depend on Z
(2) Relative change in the radii of two consecutive orbitals varies as $1/n$
(3) Relative change in the energy of two consecutive orbitals varies as $1/n^3$
(4) Relative change in the angular momenta of two consecutive orbitals varies as $1/n$

(JEE Advanced 2016)

2. A free hydrogen atom after absorbing a photon of wavelength λ_a gets excited from the state $n = 1$ to the state $n = 4$. Immediately after that the electron jumps to $n = m$ state by emitting a photon of wavelength λ_e . Let the change in momentum of atom due to the absorption and the

emission are ΔP_a and ΔP_e , respectively. If $\frac{\lambda_a}{\lambda_e} = \frac{1}{5}$.

Which of the option(s) is/are correct ?

[Use $hc = 1242 \text{ eV nm}$; $1 \text{ nm} = 10^{-9} \text{ m}$, h and c are Planck's constant and speed of light, respectively]

- (1) $\lambda_e = 418 \text{ nm}$
(2) The ratio of kinetic energy of the electron in the state $n = m$ to the state $n = 1$ is $\frac{1}{4}$

(3) $m = 2$

(4) $\frac{\Delta p_a}{\Delta p_e} = \frac{1}{2}$

(JEE Advanced 2019)

3. A particle of mass m moves in circular orbits with potential energy $V(r) = Fr$, where F is a positive constant and r is its distance from the origin. Its energies are calculated using the Bohr model. If the radius of the particle's orbit is denoted by R and its speed and energy are denoted by v and E , respectively, then for the n^{th} orbit (here h is the Planck's constant)

(1) $R \propto n^{1/3}$ and $v \propto n^{2/3}$ (2) $R \propto n^{2/3}$ and $v \propto n^{1/3}$

(3) $E = \frac{3}{2} \left(\frac{n^2 h^2 F^2}{4\pi^2 m} \right)^{1/3}$ (4) $E = 2 \left(\frac{n^2 h^2 F^2}{4\pi^2 m} \right)^{1/3}$

(JEE Advanced 2020)

4. In an X-ray tube, electrons emitted from a filament (cathode) carrying current I hit a target (anode) at a distance d from the cathode. The target is kept at a potential V higher than the cathode resulting in emission of continuous and characteristic X-rays. If the filament current, I is decreased to $\frac{I}{2}$, the potential difference V is increased to $2V$, and the separation distance d is reduced to $\frac{d}{2}$, then

- (1) the cut-off wavelength will reduce to half, and the wavelengths of the characteristic X-rays will remain the same
(2) the cut-off wavelength as well as the wavelengths of the characteristic X-rays will remain the same
(3) the cut-off wavelength will reduce to half, and the intensities of all the X-rays will decrease
(4) the cut-off wavelength will become two times larger, and the intensity of all the X-rays will decrease

(JEE Advanced 2020)

5. Which of the following statement(s) is(are) correct about the spectrum of hydrogen atom ?

- (1) The ratio of the longest wavelength to the shortest wavelength in Balmer series is 9/5
- (2) There is an overlap between the wavelength ranges of Balmer and Paschen series.
- (3) The wavelengths of Lyman series are given by $\left(1 + \frac{1}{m^2}\right)\lambda_0$, where λ_0 is the shortest wavelength of Lyman series and m is an integer
- (4) The wavelength ranges of Lyman and Balmer series do not overlap.

(JEE Advanced 2021)

Linked Comprehension Type

For Problems 1–3

The key feature of Bohr's theory of spectrum of hydrogen atom is the quantization of angular momentum when an electron is revolving around a proton. We will extend this to a general rotational motion to find quantized rotational energy of a diatomic molecule assuming it to be rigid. The rule to be applied is Bohr's quantization condition.

(IIT-JEE 2010)

1. A diatomic molecule has moment of inertia I . By Bohr's quantization condition, its rotational energy in the n th level ($n = 0$ is not allowed) is

- | | |
|---|---|
| (1) $\frac{1}{n^2} \left(\frac{h^2}{8\pi^2 I} \right)$ | (2) $\frac{1}{n} \left(\frac{h^2}{8\pi^2 I} \right)$ |
| (3) $n \left(\frac{h^2}{8\pi^2 I} \right)$ | (4) $n^2 \left(\frac{h^2}{8\pi^2 I} \right)$ |

2. It is found that the excitation frequency from ground to the first excited state of rotation for the CO molecule is close to $4/\pi \times 10^{11}$ Hz. Then the moment of inertia of CO molecule about its center of mass is close to (Take $h = 2\pi \times 10^{-34}$ J s)

- | | |
|--|--|
| (1) 2.76×10^{-46} kg m ² | (2) 1.87×10^{-46} kg m ² |
| (3) 4.67×10^{-47} kg m ² | (4) 1.17×10^{-47} kg m ² |

3. In a CO molecule, the distance between C (mass = 12 a.m.u) and O (mass = 16 a.m.u), where 1 a.m.u. = $5/3 \times 10^{-27}$ kg, is close to

- | | |
|-----------------------------|-----------------------------|
| (1) 2.4×10^{-10} m | (2) 1.9×10^{-10} m |
| (3) 1.3×10^{-10} m | (4) 4.4×10^{-11} m |

Numerical Value Type

1. A hydrogen atom in its ground state is irradiated by light of wavelength 970 Å. Taking $hc/e = 1.237 \times 10^{-6}$ eV m and the ground state energy of hydrogen atom as -13.6 eV, the number of lines present in the emission spectrum is

(JEE Advanced 2016)

2. An electron in a hydrogen atom undergoes a transition from an orbit with quantum number n_i to another with quantum number n_f . V_i and V_f are respectively the initial and final potential energies of the electron. If $\frac{V_i}{V_f} = 6.25$, then the smallest possible n_f is _____.

(JEE Advanced 2017)

3. Consider a hydrogen-like ionized atom with atomic number Z with a single electron. In the emission spectrum of this atom, the photon emitted in the $n = 2$ to $n = 1$ transition has energy 74.8 eV higher than the photon emitted in the $n = 3$ to $n = 2$ transition. The ionization energy of the hydrogen atom is 13.6 eV. The value of Z is _____.

(JEE Advanced 2018)

Answers Key

EXERCISES

Single Correct Answer Type

1. (4)	2. (3)	3. (3)	4. (4)	5. (1)
6. (1)	7. (2)	8. (4)	9. (3)	10. (4)
11. (2)	12. (1)	13. (2)	14. (3)	15. (2)
16. (3)	17. (1)	18. (2)	19. (4)	20. (4)
21. (1)	22. (3)	23. (2)	24. (3)	25. (2)
26. (1)	27. (3)	28. (4)	29. (4)	30. (4)
31. (1)	32. (2)	33. (4)	34. (3)	35. (2)
36. (2)	37. (3)	38. (3)	39. (2)	40. (3)
41. (2)	42. (2)	43. (2)	44. (4)	45. (1)
46. (2)	47. (3)	48. (2)	49. (4)	50. (1)
51. (3)	52. (4)	53. (1)	54. (3)	55. (2)
56. (2)	57. (4)	58. (1)	59. (1)	60. (4)
61. (3)	62. (4)	63. (4)	64. (2)	65. (3)
66. (2)	67. (2)	68. (2)	69. (3)	70. (2)
71. (4)	72. (3)	73. (2)	74. (4)	75. (3)
76. (2)	77. (1)	78. (2)	79. (4)	80. (4)
81. (4)	82. (4)	83. (2)	84. (2)	85. (3)
86. (4)	87. (3)	88. (1)	89. (4)	90. (1)
91. (4)	92. (4)	93. (4)	94. (1)	

Multiple Correct Answers Type

1. (1),(2),(4)	2. (2),(3)	3. (1),(3),(4)
4. (2),(3),(4)	5. (1),(2),(3)	6. (1),(2)
7. (1),(2)	8. (2),(4)	9. (2),(3),(4)
10. (1),(3),(4)	11. (2),(3)	12. (1),(2)
13. (1),(2),(3)	14. (1),(2),(3)	15. (1),(3),(4)
16. (2),(3)	17. (1),(3)	18. (2),(4)
19. (1),(3)	20. (2),(3),(4)	21. (3),(4)
22. (1),(3)	23. (1),(2)	24. (1),(3)
25. (1),(3),(4)	26. (1),(3)	27. (1),(2),(3)
28. (1),(2)	29. (1),(2),(4)	30. (2),(3),(4)
31. (3),(4)	32. (1),(4)	

Linked Comprehension Type

1. (2)	2. (2)	3. (2)	4. (1)	5. (3)
6. (4)	7. (3)	8. (2)	9. (3)	10. (2)
11. (3)	12. (1)	13. (3)	14. (2)	15. (2)
16. (4)	17. (2)	18. (1)	19. (3)	20. (2)

21. (3)	22. (1)	23. (4)	24. (3)	25. (4)
26. (3)	27. (2)	28. (2)	29. (1)	30. (2)
31. (4)	32. (4)	33. (4)	34. (4)	35. (1)
36. (2)	37. (1)	38. (3)	39. (1)	40. (1)
41. (1)	42. (3)	43. (1)	44. (3)	45. (4)
46. (2)	47. (4)	48. (3)	49. (1)	50. (4)
51. (2)	52. (1)	53. (3)	54. (3)	55. (1)

Matrix Match Type

- i. $\rightarrow$ a., c.; ii. $\rightarrow$ b., d.; iii. $\rightarrow$ b., d.; iv. $\rightarrow$ b., c.
- i. $\rightarrow$ d.; ii. $\rightarrow$ a.; iii. $\rightarrow$ d.; iv. $\rightarrow$ c.
- i. $\rightarrow$ b., c.; ii. $\rightarrow$ a., c.; iii. $\rightarrow$ a., c.; iv. $\rightarrow$ d.
- i. $\rightarrow$ c.; ii. $\rightarrow$ a., b., c., d.; iii. $\rightarrow$ b.; iv. $\rightarrow$ d.
- i. $\rightarrow$ c., d.; ii. $\rightarrow$ b., c., d.; iii. $\rightarrow$ c., d.; iv. $\rightarrow$ b., c., d.
- i. $\rightarrow$ b., c., d.; ii. $\rightarrow$ a., b., c., d.; iii. $\rightarrow$ b., c., d.; iv. $\rightarrow$ b., c., d.
- i. $\rightarrow$ d.; ii. $\rightarrow$ e.; iii. $\rightarrow$ b.; iv. $\rightarrow$ a.
- i. $\rightarrow$ b.; ii. $\rightarrow$ c.; iii. $\rightarrow$ d.; iv. $\rightarrow$ a.
- i. $\rightarrow$ d.; ii. $\rightarrow$ a.; iii. $\rightarrow$ b.; iv. $\rightarrow$ c.

Numerical Value Type

1. (4)	2. (6)	3. (6)	4. (2)	5. (1)
6. (4)	7. (5)	8. (9)	9. (2)	10. (6)
11. (823)	12. (2.25)	13. (6)	14. (15)	15. (2)
16. (2)	17. (2)	18. (3)	19. (30)	

ARCHIVES

JEE Advanced

Single Correct Answer Type

- (1)

Multiple Correct Answers Type

- (1),(2),(4)
- (2),(3)
- (2),(3)
- (1),(3)
- (1),(4)

Linked Comprehension Type

- (4)
- (2)
- (3)

Numerical Value Type

- (6)
- (5)
- (3)

Chapter 6

Concept Application Exercises

Exercise 6.1

- (b) Because atom is hollow and whole mass of atom is concentrated in a small centre called nucleus.
- (d) α -particles cannot be attracted by the nucleus.
- (a) According to scattering formula

$$N \propto \frac{1}{\sin^4(\theta/2)} \Rightarrow \frac{N_2}{N_1} = \left[\frac{\sin(\theta_1/2)}{\sin(\theta_2/2)} \right]^4$$

$$\Rightarrow \frac{N_2}{N_1} = \left[\frac{\sin \frac{90^\circ}{2}}{\sin \frac{60^\circ}{2}} \right]^4 = \left[\frac{\sin 45^\circ}{\sin 30^\circ} \right]^4$$

$$\Rightarrow N_2 = (\sqrt{2})^4 \times N_1 = 4 \times 56 = 224$$

- (c) At closest distance of approach,

Kinetic energy = Potential energy

$$\Rightarrow 5 \times 10^6 \times 1.6 \times 10^{-19} = \frac{1}{4\pi\epsilon_0} \times \frac{(Ze)(2e)}{r}$$

For uranium $Z = 92$, so $r = 5.3 \times 10^{-12}$ cm

- At the distance r_0 of closest approach,

KE of a proton = PE of proton and the gold nucleus

$$K = \frac{1}{2}mv^2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{Ze \cdot e}{r_0} \quad [q_1 = Ze, q_2 = e]$$

$$\text{Or } r_0 = \frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2}{K}$$

But $K = 5.0 \text{ MeV} = 5.0 \times 1.6 \times 10^{-13} \text{ J}$

For gold, $Z = 79$

$$\therefore r_0 = \frac{9 \times 10^9 \times 79 \times (1.6 \times 10^{-19})^2}{5.0 \times 1.6 \times 10^{-13}} \\ = 2.28 \times 10^{-14} \text{ m} \simeq 2.3 \times 10^{-14} \text{ m}$$

- Here $r_0 = 41.3 \text{ fm} = 41.3 \times 10^{-15} \text{ m}$, $Z = 79$

$$K = \frac{1}{4\pi\epsilon_0} \cdot \frac{2Ze^2}{r_0} \\ = \frac{9 \times 10^9 \times 2 \times 79 \times (1.6 \times 10^{-19})^2}{41.3 \times 10^{-15}} \text{ J} \\ = 8.814 \times 10^{-13} \text{ J} = \frac{8.814 \times 10^{-13}}{1.6 \times 10^{-13}} \text{ MeV} = 5.51 \text{ MeV}$$

Exercise 6.2

- The general expression for the radius of the electron orbit is given by

$$r = \frac{\epsilon_0 n^2 h^2}{\pi m Z e^2}$$

Since $\frac{\epsilon_0 h^2}{\pi m e^2}$ is a constant, for the same radius, $\frac{n^2}{Z}$ is constant.

$$\therefore \frac{n_1^2}{Z_1} = \frac{n_2^2}{Z_2} \Rightarrow n_2^2 = n_1^2 \times \frac{Z_2}{Z_1}$$

For hydrogen, $n_1 = 1$ (ground state), $Z_1 = 1$ and for beryllium, $Z_2 = 4$.

$$\text{Thus, } n_2^2 = (1)^2 \times 4/1 \Rightarrow n_2 = 2$$

Further, the general expression for energy is $E_n = -13.6 \frac{Z^2}{n^2} \text{ (eV)}$

$$\text{Thus, } \frac{E_2}{E_1} = \frac{Z_2^2 / n_2^2}{Z_1^2 / n_1^2}$$

$$\text{or } \frac{E_2}{E_1} = \left(\frac{Z_2}{Z_1} \right)^2 \left(\frac{n_1}{n_2} \right)^2 = \left(\frac{4}{1} \right)^2 \left(\frac{1}{2} \right)^2 = 4$$

- As $E_n = -13.6 \frac{Z^2}{n^2}$, for the same energy $\frac{Z^2}{n^2} = \text{constant}$,

$$\text{i.e., } \frac{Z_1^2}{n_1^2} = \frac{Z_2^2}{n_2^2}$$

For hydrogen (in the ground state), $Z_1 = 1$, $n_1 = 1$.

For Li^{++} , $Z_2 = 3$

$$\text{Thus, } \frac{1^2}{1^2} = \frac{(3)^2}{n_2^2}$$

$$\text{or } n_2 = 3$$

$$\text{Further, as } r \propto \frac{n^2}{Z}, \frac{r_1}{r_2} = \frac{n_1^2 / Z_1}{n_2^2 / Z_2}$$

$$\text{Or } \frac{r_1}{r_2} = \frac{(1)^2 / 1}{(3)^2 / 3} = \frac{1}{3} \text{ whence, } \frac{r_2}{r_1} = 3$$

where the subscripts 1 and 2 stand for hydrogen and Li^{++} respectively.

- Energy of photon, $hf = E_2 - E_1$

$$\text{Frequency, } f = \frac{E_2 - E_1}{h} = \frac{[-3.4 - (-13.6)] \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}} \\ = \frac{10.2 \times 1.6 \times 10^{-19}}{6.6} = 2.47 \times 10^{15} \text{ Hz}$$

- Given $E_1 = -13.6 \text{ eV}$

For third excited state, $n = 4$

$$\therefore E_4 = \frac{E_1}{4^2} = -\frac{13.6}{16} \text{ eV} = -0.85 \text{ eV}$$

$$\therefore \text{KE} = -E_4 = 0.85 \text{ eV}$$

$$\text{and PE} = 2E_4 = -1.7 \text{ eV}$$

$$\Delta E = E_4 - E_1 = [-0.85 - (-13.6)] \text{ eV} = 12.75 \text{ eV}$$

$$\therefore f = \frac{\Delta E}{h} = \frac{12.75 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}} = 3 \times 10^{15} \text{ Hz}$$

- $E_n - E_1 = hf$

$$\text{or } E_n = hf + E_1 = 12.09 - 13.6 = -1.51 \text{ eV}$$

$$\text{As } E_n = \frac{E_1}{n^2} \therefore n^2 = \frac{E_1}{E_n} = \frac{-13.6}{-1.51} \simeq 9$$

$$\text{or } n = 3$$

- Excitation potential = 2.1 V

$$\therefore \text{Excitation energy, } \Delta E = 2.1 \text{ eV} = 2.1 \times 1.6 \times 10^{-19} \text{ J}$$

$$\text{As } hf = \frac{hc}{\lambda} = \Delta E$$

$$\therefore \lambda = \frac{hc}{\Delta E} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{2.1 \times 1.6 \times 10^{-19}} = 5893 \times 10^{-10} \text{ m} = 5893 \text{ \AA}$$

7. As $T_1 = \frac{2\pi r_1}{v_1}$, $v_1 = \frac{1}{T_1} = \frac{v_1}{2\pi r_1}$

Further, $v_1 = \left(\frac{c}{1}\right)\alpha = (3 \times 10^8 \text{ m/s}) \times \frac{1}{137} = 2.2 \times 10^6 \text{ m/s}$

$$r_1 = 0.53 \text{ \AA} = 0.53 \times 10^{-10} \text{ m}$$

Thus, $v_1 = \frac{(2.2 \times 10^6 \text{ m/s})}{2(3.14)(0.53 \times 10^{-10} \text{ m})} = 6.6 \times 10^{15} \text{ Hz}$

8. Energy of an electron in the n^{th} orbit of H-atom, $E_n = -\frac{13.6}{n^2} \text{ eV}$

$$\therefore E_1 = -13.6 \text{ eV}, E_2 = -3.4 \text{ eV}, E_3 = -1.51 \text{ eV}, \dots$$

Clearly, $E_2 - E_1 = -3.4 - (-13.6) = 10.2 \text{ eV}$

Thus the energy state $n = 2$ has an excitation energy of 10.2 eV with respect to the ground state. Hence the electron is making a transition from state $n = n_1$ to the state $n = 2$, where $n_1 > 2$.

Now, $E_{n_1} - E_2 = \frac{hc}{\lambda} = \frac{12400}{4875} \text{ eV} = 2.55 \text{ eV}$

$$\therefore E_{n_1} = E_2 + 2.55 \text{ eV} = -3.4 + 2.55 = -0.85 \text{ eV}$$

Also, $E_{n_1} = -\frac{13.6}{n_1^2} \text{ eV} \therefore -\frac{13.6}{n_1^2} = -0.85$

or $n_1 = \sqrt{\frac{13.6}{0.85}} = \sqrt{16} = 4$

9. (a) The energy E of a photon of wavelength 275 nm is given by

$$E = \frac{hc}{\lambda} = \frac{12400}{275} = 4.5 \text{ eV}$$

This energy corresponds to the transition B for which the energy change = $0 - (-4.5) = 4.5 \text{ eV}$.

(b) Energy of emitted photon, $E = \frac{hc}{\lambda} \propto \frac{1}{\lambda}$

$$\therefore \lambda_{\text{max}} \propto \frac{1}{E_{\text{min}}} \text{ and } \lambda_{\text{min}} \propto \frac{1}{E_{\text{max}}}$$

(i) Transition A , for which the energy emission is minimum, corresponds to the emission of radiation of maximum wavelength,

(ii) Transition D , for which the energy emission is maximum, corresponds to the emission of radiation of minimum wavelength.

10. (a) $Z = 3$ for Li^{2+} .

Further, we know that $r_n = \frac{n^2}{Z} a_0$

Substituting $n = 3$, $Z = 3$, and $a_0 = 0.529 \text{ \AA}$,

We have r_3 for $\text{Li}^{2+} = \frac{(3)^2}{(3)} (0.529) \text{ \AA} = 1.587 \text{ \AA}$

(b) $Z = 2$ for He^+ . Also, we know that $v_n = \frac{Z}{n} v_1$

Substituting $n = 4$, $Z = 2$, and $v_1 = 2.19 \times 10^6 \text{ m s}^{-1}$, we get

$$v_4 \text{ for } \text{He}^+ = \left(\frac{2}{4}\right) (2.19 \times 10^6) \text{ ms}^{-1} = 1.095 \times 10^6 \text{ ms}^{-1}$$

11. $E_1 = -13.60 \text{ eV} \Rightarrow K_1 = -E_1 = 13.60 \text{ eV}$

$$V_1 = -2K_1 = -27.2 \text{ eV}$$

$$E_2 = \frac{E_1}{(2)^2} = -3.40 \text{ eV}$$

$$\Rightarrow K_2 = 3.40 \text{ eV} \text{ and } U_2 = -6.80 \text{ eV}$$

Now, $U_1 = 0$, i.e., potential energy of each shell has been increased by 27.20 eV while kinetic energy will remain unchanged. Changed values in tabular form are as under.

Orbit	K (eV)	U (eV)	E (eV)
First	13.6	0	13.60
Second	3.40	20.40	23.80

12. The force at a distance r is $f = -dU/dr = -2ur$.

Suppose r be the radius of n^{th} orbit. Then, the necessary centripetal force is provided by the above force. Thus,

$$\frac{mv^2}{r} = 2ur \quad \dots(i)$$

Further, the quantization of angular momentum gives

$$mvr = \frac{nh}{2\pi} \quad \dots(ii)$$

Solving Eqs. (i) and (ii) for r , we get

$$r = \left(\frac{n^2 h^2}{8um\pi^2} \right)^{1/4} \quad \dots(iii)$$

Exercise 6.3

1. $\frac{1}{\lambda_1} = R_{\infty} Z^2 \left[\frac{1}{2^2} - \frac{1}{3^2} \right] \quad \dots(i)$

$$\frac{1}{\lambda_2} = R_{\infty} Z^2 \left[\frac{1}{1^2} - \frac{1}{2^2} \right] \quad \dots(ii)$$

$$\frac{1}{\lambda_3} = R_{\infty} Z^2 \left[\frac{1}{1^2} - \frac{1}{3^2} \right] \quad \dots(iii)$$

On adding (i) and (ii), we get $\frac{1}{\lambda_1} + \frac{1}{\lambda_2} = R_{\infty} Z^2 \left[\frac{1}{1^2} - \frac{1}{3^2} \right]$

Thus, $\frac{1}{\lambda_1} + \frac{1}{\lambda_2} = \frac{1}{\lambda_3}$

$$\lambda_3 = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

2. (a) Let n_1 be the initial state of electron. Then, $E_1 = -\frac{13.6}{n_1^2} \text{ eV}$

Here, $E_1 = -0.85 \text{ eV}$

$$\therefore -0.85 = -\frac{13.6}{n_1^2}$$

or $n_1 = 4$

(b) Let n_2 be the final excitation state of the electron. Since excitation energy is always measured with respect to the ground state, therefore

$$\Delta E = 13.6 \left[1 - \frac{1}{n_2^2} \right]$$

Here, $\Delta E = 10.2 \text{ eV}$

$$\therefore 10.2 = 13.6 \left[1 - \frac{1}{n_2^2} \right]$$

or $n_2 = 2$

Thus, the electron jumps from $n_1 = 4$ to $n_2 = 2$.

(c) The wavelength of the photon emitted for a transition between $n_1 = 4$ to $n_2 = 2$, is given by

$$\frac{1}{\lambda} = R_{\infty} \left[\frac{1}{n_2^2} - \frac{1}{n_1^2} \right]$$

$$\frac{1}{\lambda} = 1.09 \times 10^7 \left[\frac{1}{2^2} - \frac{1}{4^2} \right]$$

$$\lambda = 4860 \text{ \AA}$$

3. (a) H_β line of Balmer series corresponds to the transition from $n = 4$ to $n = 2$ level. The corresponding wavelength for H_β line is

$$\frac{1}{\lambda} = (1.097 \times 10^7) \left(\frac{1}{2^2} - \frac{1}{4^2} \right) = 0.2056 \times 10^7 \text{ m}^{-1}$$

$$\lambda = 4.9 \times 10^{-7} \text{ m}$$

(b) $f = \frac{c}{\lambda} = \frac{3.0 \times 10^8}{4.9 \times 10^{-7}} = 6.12 \times 10^{14} \text{ Hz}$

4. The transition equation for Lyman series is given by

$$\frac{1}{\lambda} = R \left(\frac{1}{1^2} - \frac{1}{n^2} \right), n = 2, 3, \dots$$

The largest wavelength is corresponding to $n = 2$

$$\therefore \frac{1}{\lambda_{\max}} = 1.097 \times 10^7 \left(\frac{1}{1} - \frac{1}{4} \right) = 0.823 \times 10^7$$

$$\Rightarrow \lambda_{\max} = 1.2154 \times 10^{-7} \text{ m} = 1215 \text{ \AA}$$

The shortest wavelength corresponds to $n = \infty$

$$\therefore \frac{1}{\lambda_{\min}} = 1.097 \times 10^7 \left(\frac{1}{1} - \frac{1}{\infty} \right)$$

$$\text{or } \lambda_{\min} = 0.911 \times 10^{-7} \text{ m} = 911 \text{ \AA}$$

Both of these wavelengths lie in the ultraviolet (UV) region of electromagnetic spectrum.

5. (a) We have

$$\frac{m_p v^2}{r_n} = \frac{1}{4\pi\epsilon_0} \frac{4e^2}{r_n^2} \quad \dots(i)$$

The quantization of angular momentum gives

$$m_p v r_n = \frac{nh}{2\pi} \quad \dots(ii)$$

Solving Eqs. (i) and (ii), we get $r = \frac{n^2 h^2 \epsilon_0}{4\pi m_p e^2}$

Substituting $m_p = 100 m$, where $m =$ mass of electron, we get

$$r_n = \frac{n^2 h^2 \epsilon_0}{400\pi m e^2}$$

- (b) As we know, $E_1 = -13.60 \text{ eV}$ (For H-atom)

$$\text{and } E_n \propto \left(\frac{z^2}{n^2} \right) m$$

$$\text{For the given particle, } E_4 = \frac{(-13.60)(4)^2}{(4)^2} \times 100 = -1360 \text{ eV}$$

$$\text{and } E_2 = \frac{(-13.60)(4)^2}{(2)^2} \times 100 = -5440 \text{ eV}$$

$$\Delta E = E_4 - E_2 = 4080 \text{ eV}$$

$$\lambda (\text{in \AA}) = \frac{12375}{\Delta E (\text{in eV})} = \frac{12375}{4080} = 3.03 \text{ \AA}$$

6. Given that electrostatic potential energy = -1.7 eV .

$$\therefore \text{KE} = \frac{1.7}{2} = 0.85 \text{ eV}$$

So, total energy = $-1.7 + 0.85 = -0.85 \text{ eV}$

$$\text{Now, } E_n = -\frac{13.6}{n^2} = -0.85$$

$$\therefore n^2 = \frac{13.6}{0.85} \quad \text{or } n = 4$$

Hence, the atom is excited to state $n = 4$.

$$\Delta E = -0.85 - (-13.6) = 12.75 \text{ eV}$$

Using Einstein's equation, we have $h\nu = W + (\text{KE})_{\max}$

$$\text{or } (\text{KE})_{\max} = h\nu - W = 12.75 - 2.3 = 10.45 \text{ eV}$$

The minimum de-Broglie wavelength is given by

$$\lambda_{\min} = \frac{h}{(p)_{\max}} = \frac{h}{\sqrt{2m(\text{KE})_{\max}}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times (9.1 \times 10^{-31})(10.45 \times 1.6 \times 10^{-19})}}$$

$$= 3.8 \times 10^{-10} \text{ m} = 3.8 \text{ \AA}$$

7. If mass of nucleus is considered (not infinite), then the reduced mass of nucleus-electron system can be taken as

$$\mu = \frac{mM}{m+M}$$

Here, m is the mass of electron and M is that of the nucleus. The binding energy in ground state of hydrogen atom can now be given as

$$E = \frac{2\pi^2 K^2 e^4 \mu}{h^2} = 13.6 \times \frac{\mu}{m} \text{ eV} = \frac{13.6M}{m+M} \text{ eV}$$

We have hydrogen atom Rydberg constant given as

$$R = \frac{2\pi^2 K^2 e^4 m}{ch^3}$$

If effect of mass of nucleus is considered, the new value of Rydberg constant can be given as

$$R_\mu = \frac{2\pi^2 K^2 e^4 \mu}{ch^3} \quad \text{or} \quad R_\mu = \frac{RM}{m+M}$$

Percentage difference in the values of R and R_μ is given as

$$\frac{\Delta R}{R} \times 100 = \frac{(R_\mu - R) \times 100}{R_\mu} = \frac{m}{M} \times 100 = 0.055\%$$

8. The energy lost by the electron in exciting the hydrogen atom equals the energy corresponding to $\lambda = 1.216 \times 10^7 \text{ m}$

$$h\nu = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3.0 \times 10^8}{1.216 \times 10^{-7}} = 16.36 \times 10^{-19} \text{ J}$$

Now, the initial energy of electron = $20 \text{ eV} = 32 \times 10^{-19} \text{ J}$.

Hence, the kinetic energy of the scattered electron,

$$E = 32 \times 10^{-19} \text{ J} - 16.36 \times 10^{-19} \text{ J} = 15.64 \times 10^{-19} \text{ J}$$

The velocity v of the scattered electron is given by $\frac{1}{2} mv^2 = E$

$$\text{or } v = \left(\frac{2E}{m} \right)^{1/2} = \left(\frac{2 \times 15.64 \times 10^{-19}}{9.11 \times 10^{-31}} \right)^{1/2} = 1.86 \times 10^6 \text{ ms}^{-1}$$

9. The orbital frequency of n^{th} orbit is given by

$$v_n = \frac{\text{electron speed}}{\text{orbital circumference}}$$

$$= \frac{e^2}{2nh\epsilon_0} \times \frac{1}{2\pi} \times \frac{\pi m e^2}{n^2 \epsilon_0 h^2} = \frac{m e^4}{4 \epsilon_0^2 n^3 h^3} \quad \left[\text{As } K = \frac{1}{4\pi\epsilon_0} \right]$$

From the laws of electromagnetic theory, the frequency of the radiation emitted by this electron will also be ν_n .

According to Bohr's theory, the frequency of the radiation for the adjoining orbits is given by

$$\nu_n = \frac{E_n - E_{n-1}}{h} = \frac{m e^4}{8 \epsilon_0^2 h^2} \left[\frac{1}{(n-1)^2} - \frac{1}{n^2} \right]$$

$$= \frac{m e^4}{8 \epsilon_0^2 h^2} \frac{(2n-1)}{(n-1)^2 \times n^2}$$

A comparison of this expression with the classical expression above show that the difference in their predictions will be large for small n , i.e., for $n = 2$, $\frac{1}{n^3} = \frac{1}{8}$ and $\frac{(2n-1)}{2n^2(n-1)^2} = \frac{3}{8}$, so that the frequency given by quantum theory is 3 times that calculated from classical theory. However, for very large n , the quantum expression reduces to that obtained from classical theory because for $n \rightarrow \infty$,

$$\frac{2n-1}{2n^2(n-1)^2} \cong \frac{2n}{2n^2 n^2} = \frac{1}{n^3}$$

Consequently, the predictions of Bohr's theory agrees with that of the classical theory in the limit of very large quantum numbers. This correspondence is called as Bohr's correspondence principle.

10. Here, $\lambda = 275 \text{ nm} = 275 \times 10^{-9} \text{ m}$

Therefore, energy of the emitting photon,

$$E = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{275 \times 10^{-9}} = 7.22 \times 10^{-19} \text{ J} = \frac{7.22 \times 10^{-19}}{1.6 \times 10^{-19}} = 4.5 \text{ eV}$$

The photon of energy 4.5 eV (or of wavelength 275 nm) will be emitted, corresponding to the transition B.

11. (a) The second excited state corresponds to $n = 3$,

$$\text{Now, } E_3 = -\frac{13.6}{n^2} = -\frac{13.6}{3^2} = -\frac{13.6}{9} = 1.51 \text{ eV}$$

The kinetic energy of an electron in the second excited state $= -(-1.51) = 1.51 \text{ eV}$

- (b) Energy emitted, when the electron jumps from the second excited state to the ground state,

$$E = -1.51 - (-13.6) = 12.09 \text{ eV} = 12.09 \times 1.6 \times 10^{-19} \text{ J}$$

The wavelength of the spectral line emitted,

$$\lambda = \frac{hc}{E} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{12.09 \times 1.6 \times 10^{-19}} = 1.027 \times 10^{-7} \text{ m} = 1.027 \text{ Å}$$

12. (a) Since $E_n \propto \frac{Z^2}{n^2}$ in a hydrogen-like atom, the energy levels will be $29^2 = 841$ times the corresponding energies for hydrogen.

$$\therefore E_n = 841 \times \left(\frac{-13.6}{n^2} \right)$$

$$\text{or } E_1 = -11.44 \text{ keV}; E_2 = -2.86 \text{ keV}; E_3 = -1.27 \text{ keV}$$

- (b) Making use of $\Delta E = E_n - E_1$

$$\Rightarrow \lambda_1 = 1.44 \text{ Å}, \lambda_2 = 1.22 \text{ Å}, \text{ and } \lambda_3 = 1.15 \text{ Å}$$

Ionization potential is the potential corresponding to energy $+|E_1|$.

$$V_\infty = \frac{|E_1|}{e} = 11.44 \text{ kV}$$

Exercise 6.4

1. (a) True. According to the theory of electromagnetism, electromagnetic waves are generated by the acceleration or retardation of charges.
- (b) True. Line spectrum of electromagnetic waves are produced due to the transition of electron from the higher level to the lower level.
- (c) False. The cut-off wavelength depends only on the kinetic energy of the striking electrons.
- (d) True. The K_α transition is more probable than the K_β transition.

- (e) False. Frequency of K_β is more than that of K_α because $\lambda_\beta < \lambda_\alpha$ as shown:

$$\frac{1}{\lambda_\beta} = R_\infty (Z-1)^2 \left[\frac{1}{1^2} - \frac{1}{3^2} \right] = \frac{8}{9} R_\infty (Z-1)^2$$

$$\text{and } \frac{1}{\lambda_\alpha} = R_\infty (Z-1)^2 \left[\frac{1}{1^2} - \frac{1}{2^2} \right] = \frac{3}{4} R_\infty (Z-1)^2$$

$$2. \text{ We know that } \lambda_{\min} = \frac{hc}{eV} = \frac{12400}{V} = \frac{12400}{40 \times 10^3} = 0.31 \text{ Å}$$

$$3. \text{ We know that for } K_\alpha\text{-line } \frac{1}{\lambda_\alpha} = R_\infty (Z-1)^2 \left[1 - \frac{1}{2^2} \right] = \frac{3}{4} R_\infty (Z-1)^2$$

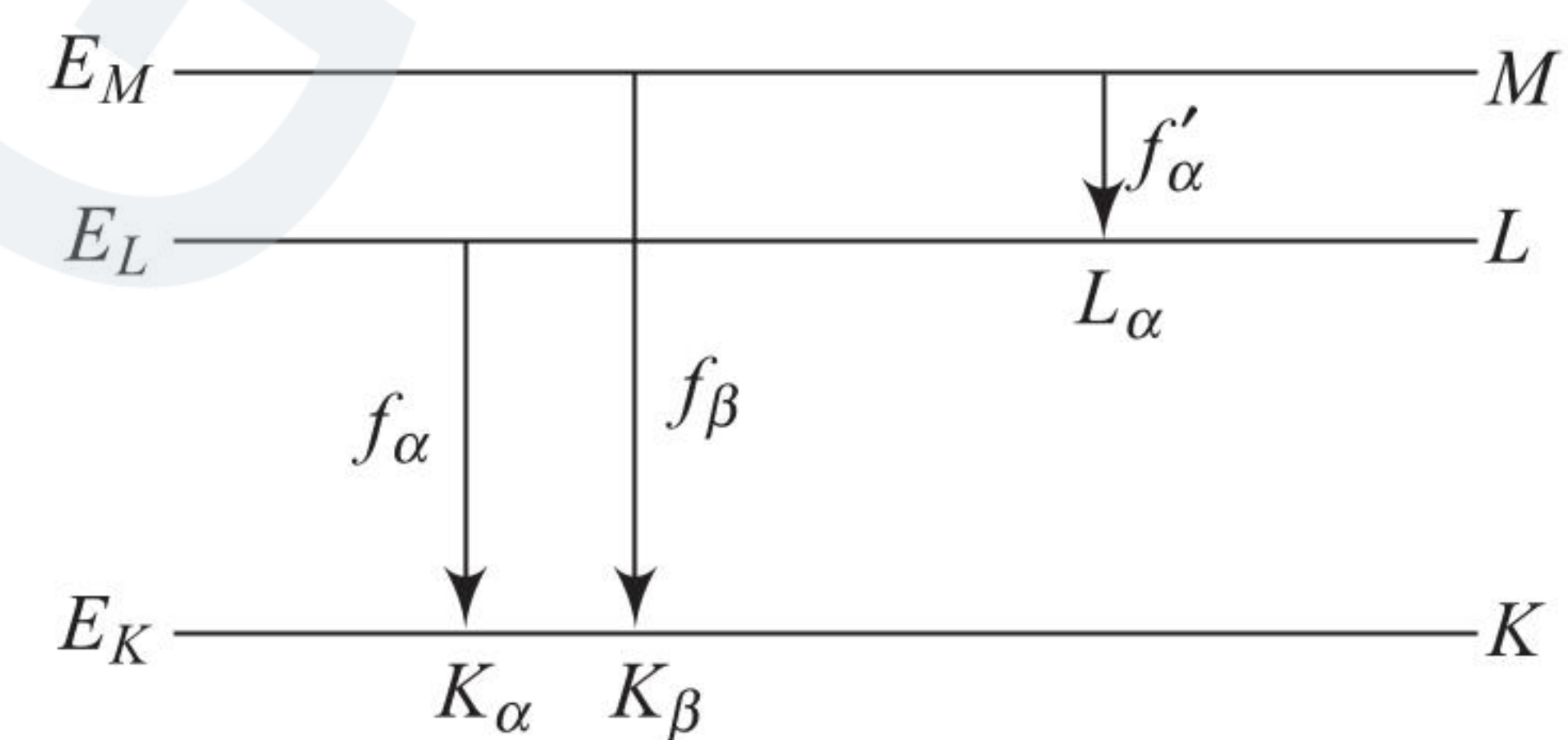
$$\text{or } \lambda_\alpha = \frac{4}{3R_\infty (Z-1)^2}$$

Here, $R_\infty = 1.096 \times 10^7 \text{ m}^{-1}$ and $Z = 74$, therefore

$$\lambda_\alpha = \frac{4}{3(1.096 \times 10^7)(74-1)^2} = 2.28 \times 10^{-11} \text{ m}$$

$$\text{or } \lambda_\alpha = 0.228 \text{ Å}$$

4. From the energy diagram as shown in the following figure, we know that



$$\text{Frequency of } K_\alpha\text{-line is } f_\alpha = \frac{E_L - E_K}{h}$$

$$\text{Frequency of } K_\beta\text{-line is } f_\beta = \frac{E_M - E_K}{h}$$

$$\text{Frequency of } L_\alpha\text{-line is } f'_\alpha = \frac{E_M - E_L}{h}$$

$$\text{Since } E_M - E_K = (E_M - E_L) + (E_L - E_K)$$

$$\therefore f_\beta = f_\alpha + f'_\alpha$$

5. When an electron of charge e is accelerated through a potential difference V , it acquires energy eV . If m be the mass of the electron and $v_{\max}$ the maximum speed of the electron, then

$$\frac{1}{2} m v_{\max}^2 = eV \quad \text{or} \quad v_{\max} = \sqrt{\left(\frac{2eV}{m} \right)}$$

Substituting the given values, we get

$$v_{\max} = \sqrt{\left(\frac{2 \times (1.6 \times 10^{-19}) \times 20,000}{9 \times 10^{-31}} \right)} = 8.4 \times 10^7 \text{ ms}^{-1}$$

6. (a) $\lambda_{\min} = 0.45 \text{ Å}$

$$E_{\max} = h v_{\max} = \frac{hc}{\lambda_{\min}} = \frac{12431}{0.45} = 27624.44 \text{ eV} = 27.624 \text{ keV}$$

- (b) The minimum accelerating voltage for electrons is

$$\frac{27.6 \text{ keV}}{e} = 27.6 \text{ kV, i.e., of the order of 30 kV.}$$

7. According to Moseley's law, the frequency for K series is proportional to $V(Z-1)^2$

$$\text{or } \frac{c}{\lambda} \propto (Z-1)^2$$

$$\text{or } \frac{1}{\lambda} = k(Z-1)^2 \quad \dots(i)$$

where k is a constant. Let λ' be the wavelength of K_α line emitted from molybdenum, then

$$\frac{1}{\lambda'} = k(Z-1)^2 \quad \dots(ii)$$

Dividing (i) and (ii), we get

$$\lambda' = \left(\frac{Z-1}{Z'-1} \right)^2 \lambda = \left(\frac{30-1}{42-1} \right)^2 \times 1.415 \text{ \AA} = 0.708 \text{ \AA}$$

8. The short series limit of the Balmer series is corresponding to transition $n = \infty$ to $n = 2$ which is given by

$$\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{\infty^2} \right) = \frac{R}{4}$$

$$\text{or } R = \frac{4}{\lambda} = \frac{4}{3644} (\text{\AA})^{-1}$$

The shortest wavelength corresponds to transition from $n = \infty$ to $n = 1$ which is given as

$$\frac{1}{\lambda_c} = R(Z-1)^2 \left[\frac{1}{1^2} - \frac{1}{\infty^2} \right]$$

$$\text{or } (Z-1)^2 = \frac{1}{\lambda_c R} = \frac{1}{1 \text{ \AA} \times \frac{4}{3644} (\text{\AA})^{-1}} = \frac{3644}{4} = 911$$

$$\text{or } Z-1 = 30.2$$

$$\text{or } Z = 31.2 \approx 31$$

Thus, the atomic number of the element is 31 which is gallium.

9. The binding energy for K shell in eV is

$$E_k = \frac{hc}{\lambda_k} = \frac{12431}{0.2} \text{ eV} = 62.155 \text{ keV}$$

The energy of the incident photon in eV is

$$E = \frac{hc}{\lambda} = \frac{12431}{0.15} = 82.873 \text{ keV}$$

Therefore, the maximum energy of the photoelectrons emitted from the K shell is

$$E_{\max} = E - E_k = (82.873 - 62.155) \text{ keV} = 20.718 \text{ keV}$$

10. Tungsten is a multielectron atom. Due to the shielding of the nuclear charge by the negative charge of the inner core electrons, each electron is subject to an effective nuclear charge Z_{eff} , which is different for different shells.

For an electron in the K shell, $\sigma = 1$.

Thus, $Z_{\text{eff}} = Z - \sigma = Z - 1$

Here, as electron drops from M shell ($n = 3$) to K shell ($n = 1$), we call the radiated emission K_β X-ray and from Moseley's law the wavelength emitted of K_β X-ray is given as

$$\frac{1}{\lambda_{k_\beta}} = R(Z-1)^2 \left[\frac{1}{1^2} - \frac{1}{3^2} \right]$$

$$\text{or } \frac{1}{\lambda_{k_\beta}} = 10967800 \times (74-1)^2 \left[\frac{8}{9} \right]$$

$$\text{or } \lambda_{k_\beta} = 0.192 \text{ \AA}$$

11. The X-rays produced under an accelerating potential V will have varying wavelengths with the minimum due to the entire energy of the accelerated electron being lost in a single collision with the target atoms. Here, the shortest wavelength generated of X-rays can be given by

$$\lambda_c = \frac{12431}{V} \text{ \AA} = \frac{12431}{20000} = 0.6215 \text{ \AA}$$

12. K_α X-rays are produced when an electron makes a transition from $n = 2$ to $n = 1$ to fill a vacancy in K shell. The wavelength of X-ray lines is given by

$$\frac{1}{\lambda_{K_\alpha}} = R(Z-1)^2 \left(\frac{1}{1^2} - \frac{1}{2^2} \right) = \frac{3}{4} R(Z-1)^2$$

$$\text{or } (Z-1)^2 = \frac{4}{3R\lambda_{K_\alpha}} = \frac{4}{3 \times (1.097 \times 10^7) \times (0.76 \times 10^{-10})} = 1599.25$$

$$\text{or } (Z-1)^2 \approx 1600 \quad \text{or } Z-1 = 40$$

$$\text{or } Z = 41$$

Exercises

Single Correct Answer Type

$$1. (4) \quad \frac{1}{\lambda} = R \left[\frac{1}{2^2} - \frac{1}{4^2} \right]$$

$$\text{or } \frac{f}{c} = R \left[\frac{1}{4} - \frac{1}{16} \right]$$

$$\text{or } f = cR \left[\frac{1}{4} - \frac{1}{16} \right] = 3 \times 10^8 \times 10^7 \times \frac{3}{16} = \frac{9}{16} \times 10^{15} \text{ Hz}$$

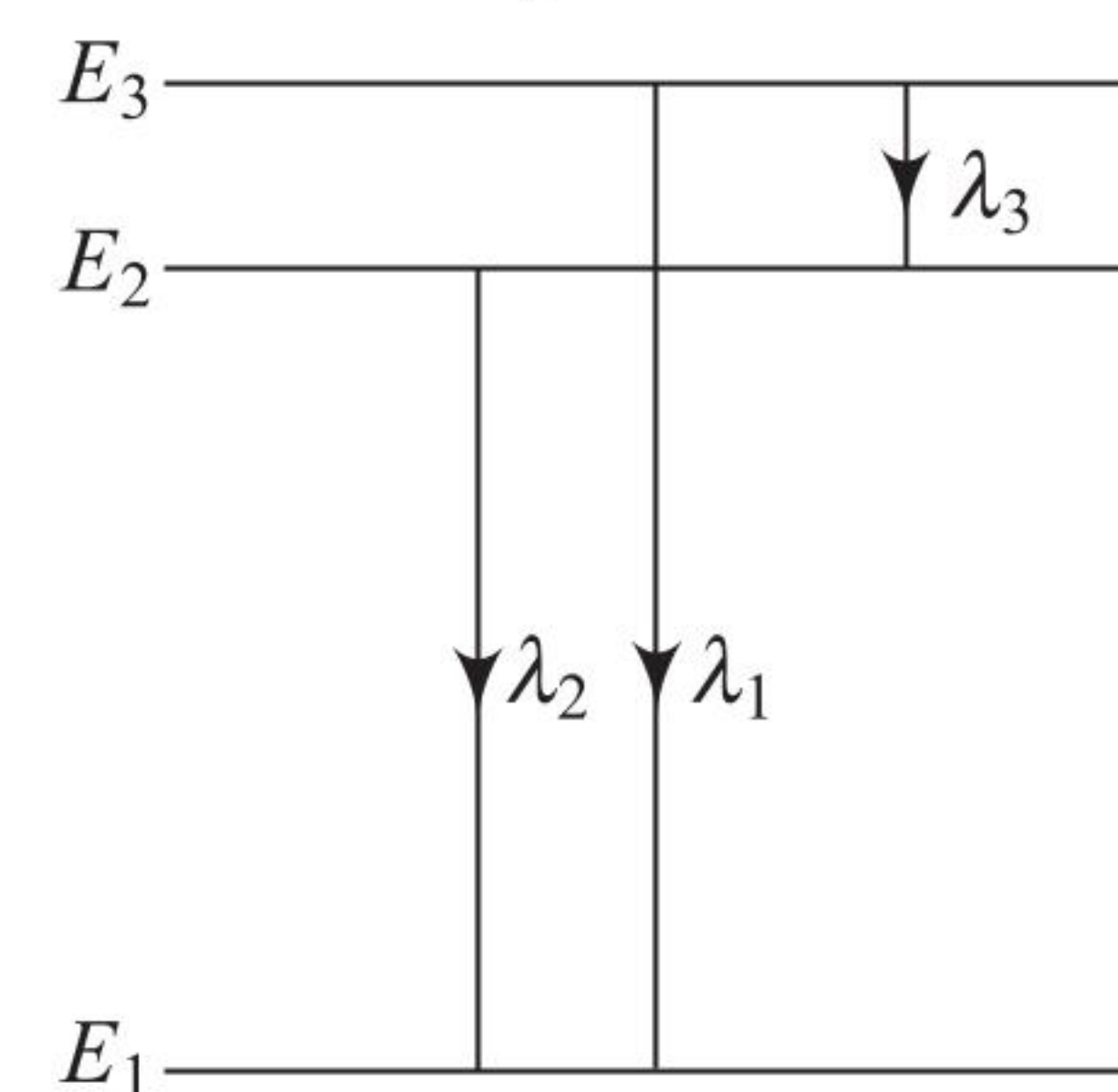
$$2. (3) \quad \text{Volume occupied by one mole of gold} = \frac{197 \text{ g}}{19.7 \text{ g cm}^{-3}} = 10 \text{ cm}^3$$

$$\text{Volume of one atom} = \frac{10}{6 \times 10^{23}} = \frac{5}{3} \times 10^{-23} \text{ cm}^3$$

$$\text{Let } r \text{ be the radius of the atom. Therefore, } \frac{4}{3} \pi r^3 = \frac{5}{3} \times 10^{-23}$$

$$\text{or } r \approx 1.5 \times 10^{-10} \text{ m}$$

3. (3) Let the three energy levels be E_1 , E_2 , and E_3 . The wavelengths λ_1 , λ_2 , and λ_3 of the spectral lines corresponding to the three energy transitions are depicted as shown in figure.



$$E = hf = \frac{h}{\lambda} \quad \text{or } E \propto \frac{1}{\lambda}$$

(given $\lambda_1 < \lambda_2 < \lambda_3$)

Thus, for the three wavelengths, we have

$$E_3 - E_2 = \frac{h}{\lambda_3} \quad \dots(i)$$

$$E_2 - E_1 = \frac{h}{\lambda_2} \quad \dots(ii)$$

$$E_3 - E_1 = \frac{h}{\lambda_1} \quad \dots(iii)$$

$$\text{Now, } E_3 - E_1 = (E_3 - E_2) + (E_2 - E_1)$$

$$\Rightarrow \frac{h}{\lambda_1} = \frac{h}{\lambda_3} + \frac{h}{\lambda_2} \Rightarrow \frac{1}{\lambda_1} = \frac{1}{\lambda_2} + \frac{1}{\lambda_3}$$

$$4. (4) \quad v_n = k \frac{2\pi e^2}{nh}$$

We know that in cgs system $k = 1$

$$\therefore v_n = \frac{2\pi e^2}{nh} \Rightarrow v_1 = \frac{2\pi e^2}{h}$$

$$\text{So } \frac{v_1}{c} = \frac{2\pi e^2}{ch}$$

$$5. (1) \quad \text{Barrier height} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r_e} \text{ J} = \frac{1}{4\pi\epsilon_0} \frac{e}{r_e} \text{ eV}$$

$$= \frac{9 \times 10^9 \times 1.6 \times 10^{-19}}{10^{-14}} \text{ eV} = 1.44 \times 10^5 \text{ eV}$$

6. (1) The recoil momentum of atom is same as that of photon but in opposite direction.

Hence, recoil momentum,

$$P = \frac{E}{c} = \frac{12.09 \times 1.6 \times 10^{-19}}{3 \times 10^8} \text{ Ns} = 6.45 \times 10^{-27} \text{ Ns}$$

Note that almost whole of the energy will be carried away by the photon because it is very light in comparison to H atom.

$$7. (2) \quad f = cZ^2 R \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

$$\Rightarrow 2.7 \times 10^{15} = cz^2 R \left[\frac{1}{1^2} - \frac{1}{2^2} \right]$$

$$f' = cZ^2 R \left[\frac{1}{1^2} - \frac{1}{3^2} \right]$$

Divide and solve to get: $f = 3.2 \times 10^{15} \text{ Hz}$

$$8. (4) \quad E_p = -\frac{ke^2}{r}, E = -\frac{ke^2}{2r}$$

So, $E_p = 2E = 2(-13.6) \text{ eV} = -27.2 \text{ eV}$.

Potential energy of electron in the ground state of Li^{2+} ion
 $= -3^2 \times 27.2 \text{ eV}$ or -244.8 eV .

$$9. (3) \quad \text{Required energy} = \left[\left(\frac{-13.6}{9} \right) - \left(\frac{-13.6}{1} \right) \right] \times 9$$

$$= \left[13.6 - \frac{13.6}{9} \right] 9 = 8 \times 13.6 \text{ eV}$$

$$\text{Wavelength} = \frac{12375}{8 \times 13.6} = 113.7 \text{ \AA}$$

$$10. (4) \quad F = \frac{mv^2}{r}$$

But $v \propto \frac{1}{n}$ and $r \propto n^2$

$$\Rightarrow F \propto \frac{1}{n^4}$$

$$11. (2) \quad U = -\frac{ke^2}{2R^3}, F = -\frac{dU}{dR} = -\frac{3ke^2}{2R^4}$$

$$\text{But, } F = \frac{mv^2}{R} \Rightarrow \frac{mv^2}{R} = \frac{3ke^2}{2R^4}$$

$$\text{Also, } mvR = \frac{nh}{2\pi}$$

$$\text{Solve to get } R = \frac{6\pi^2 ke^2 m}{n^2 h^2}$$

$$12. (1) \quad \text{Frequency of electron revolution, } f = \frac{mZ^2 e^4}{4\epsilon_0^2 n^3 h^3},$$

Put the various values to get $f = 6.62 \times 10^{15} \frac{Z^2}{n^3}$

Now, put $Z = 1$ and $n = 1$ to get $f = 6.62 \times 10^{15} \text{ Hz}$

$$13. (2) \quad r_n \propto n^2$$

$$\frac{n'^2}{n^2} = \frac{21.2 \times 10^{-11}}{5.3 \times 10^{-11}} \quad \text{or} \quad \frac{n'^2}{n^2} = 4$$

$$\text{or } \frac{n'^2}{1^2} = 4 \quad \text{or } n' = 2$$

$$14. (3) \quad \frac{1}{\lambda} = R \left[\frac{1}{2^2} - \frac{1}{3^2} \right]$$

$$\text{or } R = \frac{36}{5\lambda} = \frac{36}{5 \times 6563 \times 10^{-10}} \text{ m}^{-1}$$

$$= \frac{36000}{5 \times 6563} \times 10^7 \text{ m}^{-1} = 1.097 \times 10^7 \text{ m}^{-1}$$

15. (2) l will be same for both because it does not depend upon Z . But for energy

$$(E_n)_{\text{Li}} = -\frac{Z^2 \times 13.6}{n^2} \quad \text{and} \quad (E_n)_{\text{H}} = -\frac{13.6}{n^2}$$

Clearly, $|E_{\text{H}}| < |E_{\text{Li}}|$

$$16. (3) \quad \frac{1}{\lambda} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

For longest wavelength, $n_1 = 2, n_2 = 3$

$$\therefore \frac{1}{\lambda} = R \left[\frac{1}{2^2} - \frac{1}{3^2} \right] \quad \text{or} \quad \frac{1}{\lambda} = R \left[\frac{1}{4} - \frac{1}{9} \right]$$

$$\Rightarrow \lambda = \frac{36}{5R} \text{ for electron,}$$

But $\lambda \propto \frac{1}{m}$

$$\text{So } \lambda' = \frac{1}{2} \times \frac{36}{5R} = \frac{18}{5R}$$

$$17. (1) \quad |F| = \left| \frac{-dU}{dr} \right| = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{U_0}{m}}, \text{ which is a constant.}$$

$$\text{So, } mv_n r_n = \frac{nh}{2\pi} \Rightarrow r_n \propto n$$

$$18. (2) \quad \lambda = \frac{912 \text{ \AA}}{Z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]}$$

For singly ionized helium atom $Z = 2$.

For the wavelength to be longest, $n_1 = 1, n_2 = 2$

$$\therefore \lambda = \frac{912 \text{ \AA}}{(2^2) \left[1 - \frac{1}{4} \right]} = \frac{912}{3} = 304 \text{ \AA}$$

19. (4) We know that frequency of orbital motion:

$$f \propto \frac{1}{n^3} \quad \text{and} \quad \text{given } f_1 = \frac{1}{27} f_2$$

$$\Rightarrow \left(\frac{n_2}{n_1} \right)^3 = \frac{f_1}{f_2} \Rightarrow \frac{n_2}{n_1} = \left(\frac{1}{27} \right)^{1/3} = \frac{1}{3}$$

Among given options, option (4) is correct.

20. (4) Total energy received by the atom will be 25.2 eV. 13.6 eV energy is needed to remove the electron from the attraction of the nucleus. Rest of the energy will be almost available in

the form of KE of electron. One electron of 11.6 eV will be detected.

21. (1) As magnetic moment, $\mu_B = \frac{1}{2} e v r$

$$L = m v r \Rightarrow v r = \frac{L}{m}$$

$$\therefore \mu_B = \frac{1}{2} \frac{e L}{m} \Rightarrow \frac{\mu_B}{L} = \frac{e}{2m}$$

22. (3) $\frac{1}{\lambda} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$

$$\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{3^2} \right)$$

$$\Rightarrow \lambda = \frac{36}{5R} = \frac{36 \times 10^{-7}}{5 \times 1.097} = 6.566 \times 10^{-7}$$

$$\lambda = 6566 \text{ \AA}$$

$$E = \frac{hc}{\lambda} = \frac{6.64 \times 10^{-34} \times 3 \times 10^8}{6.566 \times 10^{-7}} = 3.03 \times 10^{-19} \text{ J}$$

$$\therefore E = 1.89 \text{ eV}$$

23. (2) For shortest wavelength in Balmer series, $n_1 = 2$; $n_2 = \infty$

$$\therefore \frac{1}{\lambda} = R \left[\frac{1}{4} - \frac{1}{\infty} \right] \text{ or } \lambda = \frac{4}{R}$$

For shortest wavelength in Brackett series, $n_1 = 4$; $n_2 = \infty$

$$\therefore \frac{1}{\lambda'} = R \left[\frac{1}{4^2} - \frac{1}{\infty^2} \right]$$

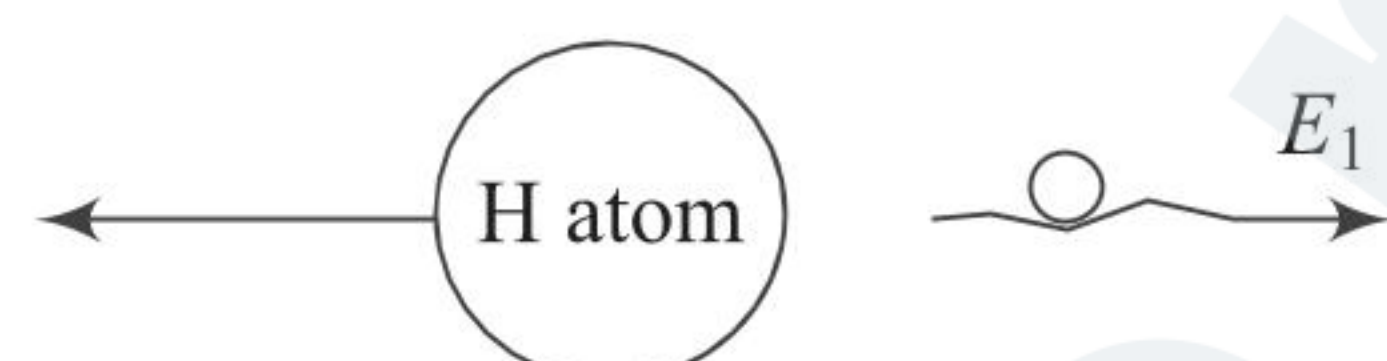
$$\text{or } \lambda' = \frac{16}{R} = 4 \times \frac{4}{R} = 4\lambda$$

24. (3) First excitation energy is $Rhc \left(\frac{1}{1^2} - \frac{1}{2^2} \right) = Rhc \frac{3}{4}$

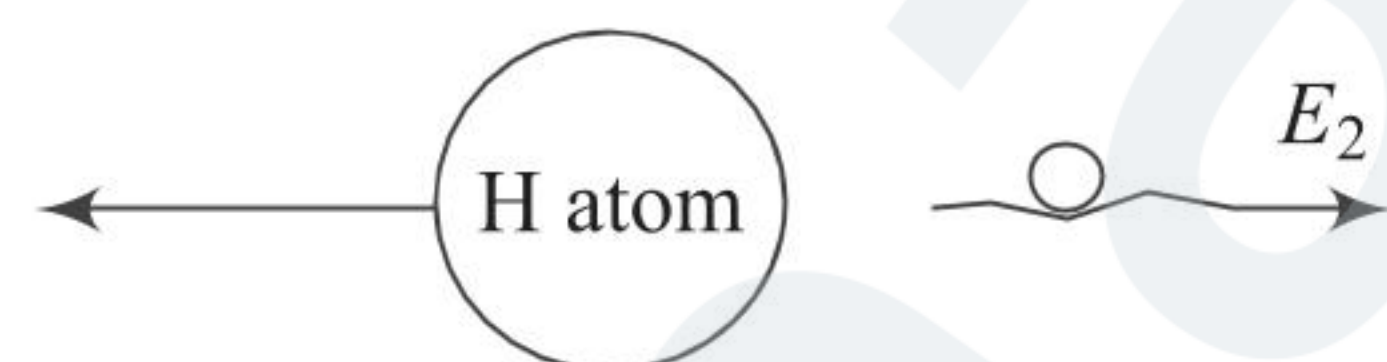
$$\frac{3}{4} Rhc = V \text{ eV}$$

$$\therefore Rhc = \frac{4V}{3} \text{ eV}$$

25. (2)



Case I



Case II

In the first case, KE of H atom increases due to recoil whereas in the second case KE decreases due to recoil.

$$\therefore E_2 > E_1$$

26. (1) Linear momentum, $mv \propto \frac{1}{n}$

Angular momentum, $mvr \propto n$

Therefore, product of linear momentum and angular momentum $\propto n^0$

27. (3) Energy of photon is given by mc^2 . Now, the maximum energy of photon is equal to the maximum energy of electrons = eV

Hence, $mc^2 = eV$

$$\Rightarrow m = \frac{eV}{c^2} = \frac{1.6 \times 10^{-19} \times 18 \times 10^3}{(3 \times 10^8)^2} = 3.2 \times 10^{-32} \text{ kg}$$

28. (4) Using $\frac{1}{\lambda} = R(Z-1)^2 \left[\frac{1}{n_2^2} - \frac{1}{n_1^2} \right]$

For K_α line; $n_1 = 2$, $n_2 = 1$

$$\text{For metal A; } \frac{1875R}{4} = R(Z_1 - 1)^2 \left(\frac{3}{4} \right)$$

$$\Rightarrow Z_1 = 26$$

$$\text{For metal B; } 675R = R(Z - 1)^2 \left(\frac{3}{4} \right)$$

$$\Rightarrow Z_2 = 31$$

Therefore, 4 elements lie between A and B.

29. (4) For second line of Balmer series in hydrogen spectrum, corresponds to $4 \rightarrow 2$ transition of hydrogen atom.

It is equivalent to $4 \times 3 \rightarrow 2 \times 3$ i.e., $12 \rightarrow 6$ transition of Li^{2+}

30. (4) Energy of n^{th} state in hydrogen is same as energy of $3n^{\text{th}}$ state in Li^{++} .

$\therefore 3 \rightarrow 1$ transition in H would give same energy as the $3 \times 3 \rightarrow 1 \times 3$ i.e., $9 \rightarrow 3$ transition in Li^{++} .

31. (1) $\frac{1}{\lambda_\alpha} = \frac{3R}{4} (Z-1)^2$

$$(Z-1) = \sqrt{\frac{4}{3R\lambda_\alpha}} = \sqrt{\frac{4}{3 \times 1.1 \times 10^7 \times 1.8 \times 10^{-10}}}$$

$$= \frac{200}{3} \sqrt{\frac{5}{33}} = \frac{78}{3} = 26 \Rightarrow Z = 27$$

32. (2) λ_m will increase to $3\lambda_m$ due to decrease in the energy of bombarding electrons. Hence, no characteristic X-rays will be visible, only continuous X-ray will be produced.

33. (4) $B = \frac{\mu_0 I}{2r}$ and $I = \frac{e}{T}$

$$B = \frac{\mu_0 e}{2rT}$$

$$[r \propto n^2, T \propto n^3]$$

$$B \propto \frac{1}{n^5}$$

34. (3) $i = \frac{q}{T}$

$$\text{Now } T^2 \propto r^3 \propto n^6 \Rightarrow i \propto \frac{1}{n^3}$$

35. (2) The cut-off wavelength when $V = V_1 = 10 \text{ kV}$ is

$$\lambda_1 = \frac{hc}{eV_1} = 1243.125 \times 10^{-13} \text{ m}$$

The cut-off wavelength when $V = V_2 = 20 \text{ kV}$ is,

$$\lambda_2 = \frac{hc}{eV_2} = 621.56 \times 10^{-13} \text{ m}$$

The wavelength corresponding to K_α line is

$$\frac{1}{\lambda} = \frac{3R}{4} (Z-1)^2$$

From given information, $(\lambda - \lambda_2) = 3(\lambda - \lambda_1)$

Solving above equation, we get $Z = 29$

36. (2) In the emission spectrum 10 lines are observed, so the energy level (n) to which the sample has been excited after absorbing the radiation is given by

$$\frac{n(n-1)}{2} = 10$$

which gives $n = 5$

$$\text{So, } \frac{hc}{\lambda} = 13.6 \left(1 - \frac{1}{5^2} \right) \text{ eV}$$

$$\frac{1242}{\lambda} \text{ eV-nm} = 13.6 \times \frac{24}{25} \text{ eV}$$

$$\therefore \lambda = 95 \text{ nm}$$

37. (3) Let n be the number of electrons per second striking the target.
 $P = VI$

$$16 \text{ W} = (5 \times 10^3 \text{ V}) \times ne$$

$$\therefore n = \frac{16}{5 \times 10^3 \times 1.6 \times 10^{-19}} = 2 \times 10^{16}$$

38. (3) In first case, transition is from n^{th} state to 2^{nd} (1^{st} excited) state.

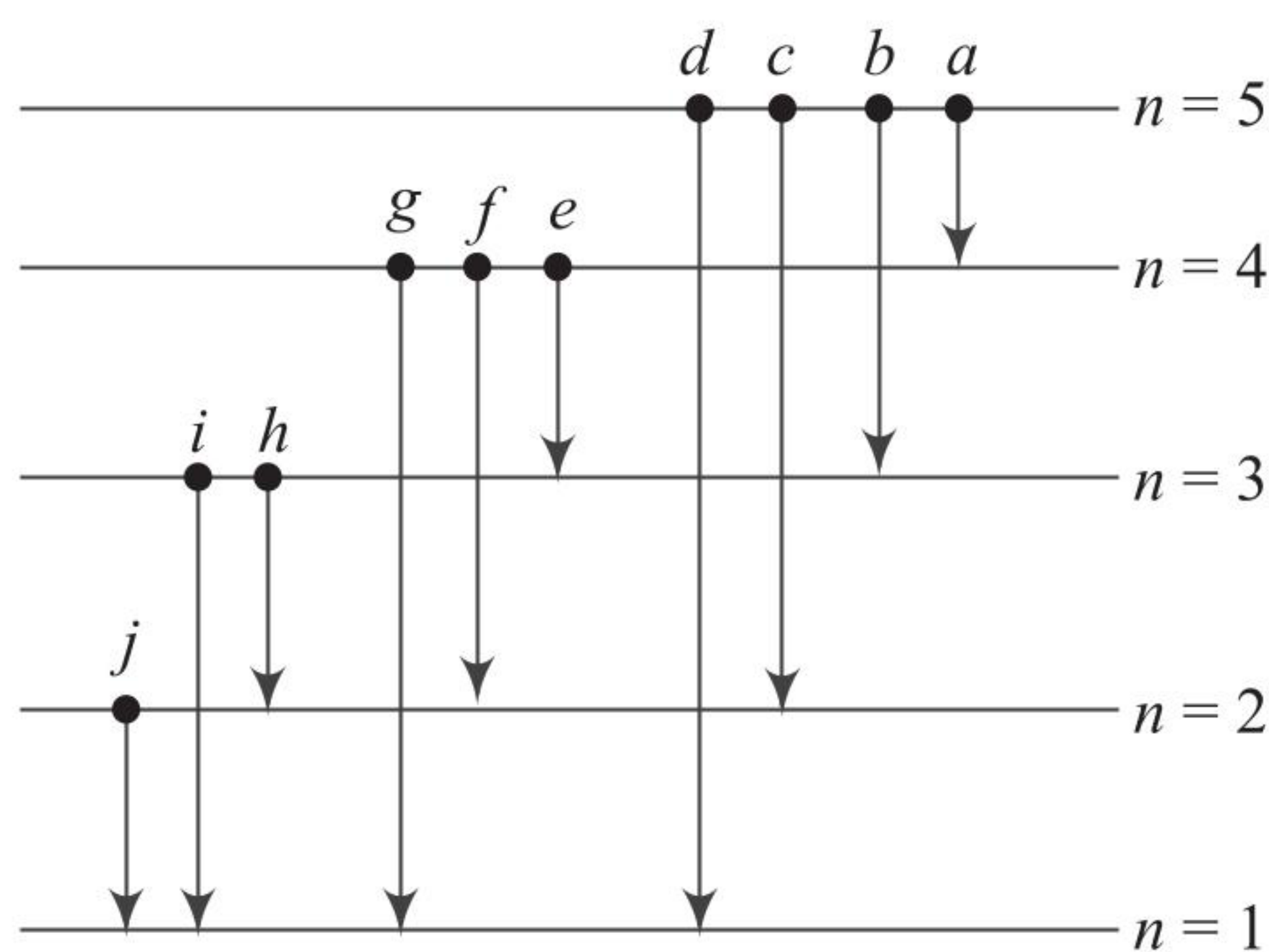
$$\therefore (10.2 + 17.0) \text{ eV} = 13.6 \times Z^2 \left[\frac{1}{2^2} - \frac{1}{n^2} \right]$$

In 2^{nd} case, transition is from n^{th} state to 3^{rd} state

$$\therefore (4.25 + 5.95) \text{ eV} = 13.6 Z^2 \left[\frac{1}{3^2} - \frac{1}{n^2} \right]$$

Solving above equations, we get $n = 6$ and $Z = 3$.

39. (2) The wavelengths present in emission spectra are shown in figure.



Transitions a , e , h and j can be performed by a single atom also. This is also true about transitions b and i , other transitions require one atom each.

40. (3) Through filter, only those photons will pass through whose energy is less than $E = \frac{hc}{800 \text{ nm}} = 1.55 \text{ eV}$.

From hydrogen's energy level diagram, we can easily identify that when electron jumps from 3^{rd} excited state (4^{th} energy level) to 2^{nd} excited state (3^{rd} energy level), then photons of 1.55 eV energy have been emitted. So, the required initial energy level is the 3^{rd} excited state.

41. (2) Option (1) explains the production of X-rays on the basis of electromagnetic theory of light, which is not able to explain the characteristic X-rays and cut-off wavelength, so option (1) is wrong.

Option (2) correctly explains the production of characteristic X-rays.

Option (3) is wrong as X-ray spectra is a continuous spectra having some peaks representing characteristic X-rays.

42. (2) The energy taken by hydrogen atom corresponds to its transition from $n = 1$ to $n = 3$ state.

$$\Delta E = 13.6 \left(1 - \frac{1}{9} \right) = 13.6 \times \frac{8}{9} = 12.1 \text{ eV}$$

43. (2) ΔE_1 (for 4^{th} to 3^{rd} excited states) $= 13.6 \times 3^2 \left[\frac{1}{4^2} - \frac{1}{5^2} \right] = 2.75 \text{ eV}$

$$\Delta E_2$$
 (for 3^{rd} to 2^{nd} excited states) $= 13.6 \times 3^2 \left[\frac{1}{3^2} - \frac{1}{4^2} \right] = 5.95 \text{ eV}$

For shorter wavelength, i.e., for ΔE_2 , $V_{01} = 3.95 \text{ volt}$

$$\text{From } eV_{01} = hc - \phi, 3.95 = 5.95 - \phi$$

$$\therefore \phi = 2 \text{ eV}$$

$$\text{For longer wavelength, } eV_{02} = 2.75 - 2 = 0.75 \text{ eV}$$

$$\text{So, } V_{02} = 0.75 \text{ V}$$

44. (4) In transition of electron from higher energy level to lower energy level, the wavelength is given by $\lambda = hc/\Delta E$, where ΔE is the energy difference between two levels.

For minimum λ , ΔE should be maximum, so (4) is the correct option.

45. (1) For a collision of neutron with hydrogen atom in ground state to be inelastic (partial or complete), the minimum KE of striking neutron must be 20.4 eV . [This condition is derived in theory.]

As the energy of the given incident neutron is less than 2.4 eV , the collision must be elastic.

46. (2) Time period for n^{th} energy level electron is,

$$T = \frac{2\pi r_n}{v_n} = \frac{4\pi^2 m}{h} \times \frac{r_n^2}{n}$$

$$r_n = n^2 a_0$$

$$T = \frac{4\pi^2 m}{h} \times n^3 a_0^2$$

$$\text{Required number of revolutions, } N = \frac{10^{-8}}{T}$$

After substituting $n = 2$, $m = 9.1 \times 10^{-31} \text{ kg}$,

and $h = 6.63 \times 10^{-34} \text{ J-s}$, we get $N = 8 \times 10^6$.

47. (3) As the collision is inelastic, it means a part of kinetic energy is transformed into some other form due to collision. In this case, the kinetic energy of incident electron can be absorbed by H atom and it can absorb only 10.2 eV out of 11.2 eV , so that it can reach to 1^{st} excited state and the electron leaves with remaining energy, i.e., 1.0 eV .

48. (2) $E = R_{\infty} ch \times \left[1 - \frac{1}{2^2} \right] = \frac{3}{4} R_{\infty} \times hc$

$$\text{Momentum of photon emitted is, } p = \frac{E}{c} = \frac{3R_{\infty} h}{4}$$

Recoiling speed of hydrogen atom is given by $v = p/m$, where m is the mass of hydrogen atom.

$$v = \frac{3R_{\infty} h}{4m} = \frac{3 \times 1.1 \times 10^7 \times 6.63 \times 10^{-34}}{4 \times 1.67 \times 10^{-27}} = 3.3 \text{ m s}^{-1}$$

49. (4) As the electron beam is having energy of 13 eV , it can excite the atom to the states whose energy is less than or equal to 0.6 eV ($13.6 - 13$). $E_5 = 0.544 \text{ eV}$ and $E_4 = 0.85 \text{ eV}$. So, the electron beam can excite the hydrogen gas maximum to 4^{th} energy state, hence the transit electron can come back to ground state from either of three excited states, thus emitting Lyman, Balmer and Paschen series.

50. (1) $\frac{hc}{\lambda} = 13.6 \left[\frac{1}{1^2} - \frac{1}{10^2} \right] = 13.6 \times 0.99$

$$\lambda = \frac{1242}{13.6 \times 0.99} \text{ nm} = 92.25 \text{ nm}$$

The line belongs to UV part of electromagnetic spectrum.

51. (3) As angular momentum of electron is $4h/2\pi$, it means electron is in the 4^{th} orbit.

TE of atom in 4^{th} orbit is -0.85 eV

KE of electron $= |\text{TE}| = 0.85 \text{ eV}$

52. (4) We know, $r_n \propto n^2$

$$\text{So, } (n+1)^2 - n^2 = (n-1)^2 \Rightarrow n = 4$$

53. (1) From Moseley's law, $\sqrt{\nu} = a(Z - 1)$ for K_α X-ray

$$\text{i.e., } \frac{1}{\sqrt{\lambda}} = a(Z - 1)$$

$$\text{Given, } \frac{1}{\sqrt{\lambda}} = a(11 - 1) \quad \text{and} \quad \frac{1}{\sqrt{4\lambda}} = a(Z - 1)$$

Dividing, we get $Z = 6$.

54. (3) The protons move toward each other till their relative velocity becomes equal to zero. At the closest distance of approach, both the protons will be moving with the same velocity.

As coulombian repulsive force is internal for the system of protons, we can apply the law of conservation of momentum.

$$\therefore mv_0 = 2mv$$

$$\text{Change in KE} = \frac{1}{2}mv_0^2 - 2 \times \frac{1}{2}m\left(\frac{v_0}{2}\right)^2$$

This change in energy is equal to the electrical potential energy

$$\frac{mv_0^2}{2} - m\left(\frac{v_0}{2}\right)^2 = \frac{e^2}{4\pi\epsilon_0 r}$$

$$\therefore r = \frac{e^2}{\pi\epsilon_0 mv_0^2}$$

55. (2) Kinetic energy gained by a charge q after being accelerated through a potential difference V volt, is given by

$$qV = \frac{1}{2}mv^2$$

$$mV = \sqrt{2MqV}$$

$$\text{As } \lambda_b = \frac{h}{mV} = \frac{h}{\sqrt{2qmV}}$$

For cut-off wavelength of X-rays, we have $qV = \frac{hc}{\lambda_m}$

$$\text{or } \lambda_m = \frac{hc}{qV}$$

$$\text{Now, } \frac{\lambda_b}{\lambda_m} = \frac{\sqrt{qV}}{\sqrt{2m}}$$

$$\text{As } \frac{q}{m} = 1.8 \times 10^{11} \text{ C kg}^{-1} \text{ for electron,}$$

$$\text{we have } \frac{\lambda_b}{\lambda_m} = \frac{\sqrt{1.8 \times 10^{11} \times 10 \times 10^3 / 2}}{3 \times 10^8} = 0.1$$

Therefore, the answer is (2).

56. (2) $P = VI$

Therefore, total power drawn by Coolidge tube $P_T = VI = 200 \text{ W}$.

As 0.5% of the energy is carried by electron,

$$\text{Power carried by X-rays is } 0.5\% \text{ of } P_T = \frac{0.5}{100} \times 200 = 1 \text{ W}$$

The answer is (2).

57. (4) As $\lambda_0 = \frac{hc}{E} = 1.55 \times 10^{-11} \text{ m}$

$$\therefore \lambda_0 = 0.155 \text{ \AA}$$

This is the minimum wavelength of continuous X-rays which carry energy equivalent to energy of incident electrons.

Now, as the energy of incident radiation is more than that of K-shell electrons, the characteristic X-rays appear as peaks on the continuous spectrum.

Therefore, (4) is the answer.

58. (1) From conservation of momentum, two identical photons travel in opposite directions with equal magnitude of momentum and energy hc/λ .

From conservation of energy, we have $\frac{hc}{\lambda} + \frac{hc}{\lambda} = m_0c^2 + m_0c^2$

$$\lambda = \frac{h}{m_0c}$$

Therefore, the answer is (1).

59. (1) For the incident electron, $\frac{1}{2}mv^2 = Ve$

$$p^2 = 2meV$$

de-Broglie wavelength of incident electron, $\lambda_1 = \frac{h}{p} = \frac{h}{\sqrt{2mVe}}$

Shortest X-ray wavelength, $\lambda_2 = \frac{hc}{Ve}$

$$\therefore \frac{\lambda_1}{\lambda_2} = \frac{Ve}{c\sqrt{2mVe}} = \frac{1}{c} \sqrt{\left(\frac{V}{2}\right) \frac{e}{m}} = \frac{\sqrt{\frac{10^4}{2} \times 1.8 \times 10^{11}}}{3 \times 10^8} = 0.1 = \frac{1}{10}$$

$$\text{or } \lambda_1 : \lambda_2 = 1 : 10$$

60. (4) The K, L, and M lines have different intercepts. The intercept of K is more than that of L, which in turn is more than that of M.

61. (3) $L_1 = (1) \frac{h}{2\pi} \dots(i)$

(Using Bohr's quantization rule)

In the first excited state of Li,

$$L_2 = (2) \frac{h}{2\pi} \dots(ii)$$

$$\therefore \frac{L_2}{L_1} = 2$$

62. (4) $\frac{0.001}{2} \times 10^{-3} = (0.5 \times 10^{-10})n^2$

$$\left\{ \begin{array}{l} \text{Because radius of } n^{\text{th}} \text{ orbit is equal to} \\ r_n = n^2 r_0, \quad \text{where } r_0 = 0.529 \text{ \AA} \end{array} \right\}$$

$$\therefore n^2 = 1000$$

$$\text{or } n = 31$$

63. (4) For each principal quantum number n , number of electrons permitted equals the number of elements corresponding to the quantum number.

$$\text{Total number of electrons} = \sum 2n^2 = \frac{n(n+1)(2n+1)}{3}$$

64. (2) $\nu_n = \alpha \left(\frac{cZ}{n} \right)$, where $\alpha = \frac{e^2}{2h\epsilon_0 c}$ is the fine structure constant

$$\left(\alpha = \frac{1}{137} \right)$$

$$\nu_{\text{He}^+} = \alpha \frac{c(2)}{2} = \alpha c \text{ and } \nu_{\text{H}} = \alpha \frac{c(1)}{1} = \alpha c = \nu_{\text{He}^+}$$

65. (3) $E = R_\infty hc \left(1 - \frac{1}{25} \right)$

$$\text{Momentum of photon emitted is } p = \frac{E}{c} = R_\infty h \left(\frac{24}{25} \right)$$

Recoil momentum of H atom will also be p .

$$mv = p$$

$$\nu = \frac{p}{m} = \frac{(1.097 \times 10^7)(6.626 \times 10^{-34})24}{(25)(1.67 \times 10^{-27})} = 4.178 \text{ m s}^{-1}$$

$$66. (2) \text{ Since } E = -\frac{13.6}{n^2} \text{ eV}$$

$$E_1 = -13.6 \text{ eV}$$

$$E_2 = -3.4 \text{ eV}$$

$$E_3 = -1.50 \text{ eV}$$

$$E_4 = -0.85 \text{ eV}$$

From above, we can see that $E_3 - E_1 = 12.1 \text{ eV}$

i.e., the electron must be making a transition from $n = 3$ to $n = 1$ level.

$$\Delta L = (3 - 1) \frac{h}{2\pi} = \frac{h}{\pi} = 2.11 \times 10^{-34} \text{ Js}$$

67. (2) For an elastic collision to take place, there must be no loss in the energy of electron. The hydrogen atom will absorb energy from the colliding electron only if it can go from ground state to first excited state, i.e., from $n = 1$ to $n = 2$ state. For this, hydrogen atom must absorb energy $E_2 - E_1 = -3.4 - (-13.6) = 10.2 \text{ eV}$.

So, if the electron possesses energy less than 10.2 eV , it would never lose it and hence collision would be elastic.

$$68. (2) \frac{1}{\lambda} = Z^2 R_\infty \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

For K_α line, $n_1 = 1$ and $n_2 = 2$

$$\frac{1}{\lambda} = Z^2 R_\infty \left(\frac{3}{4} \right)$$

$$Z = \sqrt{\frac{4}{3\lambda R_\infty}} = 39.9 \approx 40$$

69. (3) Making potential energy zero increases the value of total energy by $13.6 - (-13.6) = 27.2 \text{ eV}$

Now, actual energy in second orbit = -3.4 eV

Hence, new value is $(-3.4 + 27.2) \text{ eV} = 23.8 \text{ eV}$

$$70. (2) \text{ KE}_{\lambda_1} = \frac{hc}{\lambda_1} - \psi = e\Delta V$$

$$\text{KE}_{\lambda_2} = \frac{hc}{\lambda_2} - \psi = 2e\Delta V$$

$$\Rightarrow 3 \left(\frac{hc}{\lambda_1} - \psi \right) = \frac{hc}{\lambda_2} - \psi$$

$$\Rightarrow \psi = hc \left(\frac{3}{2\lambda_1} - \frac{1}{2\lambda_2} \right)$$

$$\Rightarrow \text{KE}_{\lambda_3} = \frac{hc}{\lambda_3} - hc \left[\frac{3}{2\lambda_1} - \frac{1}{2\lambda_2} \right] = hc \left[\frac{1}{\lambda_3} + \frac{1}{2\lambda_2} - \frac{3}{2\lambda_1} \right]$$

$$e\Delta V = hc \left[\frac{1}{\lambda_3} + \frac{1}{2\lambda_2} - \frac{3}{2\lambda_1} \right]$$

$$\Delta V = \frac{hc}{e} \left[\frac{1}{\lambda_3} + \frac{1}{2\lambda_2} - \frac{3}{2\lambda_1} \right]$$

$$71. (4) E = \frac{Z^2}{n^2} E_0 = 13.6 \times (11^2) \text{ eV}$$

$$72. (3) \lambda_{\min} = \frac{hc}{eV_{\max}}$$

73. (2) Use Moseley's law

$$74. (4) \lambda_{\min} = \frac{hc}{eV_{\max}}$$

75. (3) Assuming that ionization occurs as a result of a completely inelastic collision, we can write

$$mv - 0 = (m + m_H) u$$

where m is the mass of incident particle, m_H the mass of hydrogen atom, v_0 the initial velocity of incident particle, and u the final common velocity of the particle after collision. Prior to collision, the KE of the incident particle was

$$E_0 = \frac{mv_0^2}{2}$$

The total kinetic energy after collision

$$E = \frac{(m + m_H)u^2}{2} = \frac{m^2 v_0^2}{2(m + m_H)}$$

The decrease in kinetic energy must be equal to ionization energy. Therefore,

$$E_1 = E_0 - E = \left(\frac{m_H}{m + m_H} \right) E_0$$

$$\text{i.e., } \frac{E_1}{E_0} = \frac{1}{1 + \frac{m}{m_H}}$$

i.e., the greater the mass m , the smaller the fraction of initial kinetic energy that be used for ionization.

$$76. (2) a = \frac{v^2}{r}$$

$$\therefore a \propto \frac{(z)^2}{(1/z)} \quad (\text{for } n = 1)$$

$$\text{or } a \propto z^3$$

$$\therefore \frac{a_1}{a_2} = \left(\frac{2}{1} \right)^3 = 8$$

77. (1) Shortest wavelength of Brackett series corresponds to the transition of electron between $n_1 = 4$ and $n_2 = \infty$ and the shortest wavelength of Balmer series corresponds to the transition of electron between $n_1 = 2$ and $n_2 = \infty$. So,

$$(Z^2) \left(\frac{13.6}{16} \right) = \left(\frac{13.6}{4} \right)$$

$$\therefore Z^2 = 4$$

$$\text{or } Z = 2$$

78. (2) Let v = speed of neutron before collision,

v_1 = speed of neutron after collision,

v_2 = speed of proton or hydrogen atom after collision,

and ΔE = energy of excitation

From conservation of linear momentum,

$$mv = mv_1 + mv_2 \quad \dots(i)$$

From conservation of energy,

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \Delta E \quad \dots(ii)$$

$$\text{From Eq. (i), } v^2 = v_1^2 + v_2^2 + 2v_1v_2$$

$$\text{From Eq. (ii), } v^2 = v_1^2 + v_2^2 + \frac{2\Delta E}{m}$$

$$\therefore 2v_1v_2 = \frac{2\Delta E}{m}$$

$$\therefore (v_1 - v_2)^2 = (v_1 + v_2)^2 - 4v_1v_2$$

$$\Rightarrow (v_1 - v_2)^2 = v^2 - 4 \frac{\Delta E}{m}$$

As $v_1 - v_2$ must be real, therefore $v^2 - 4 \frac{\Delta E}{m} \geq 0$

$$\text{or } \frac{1}{2}mv^2 \geq 2\Delta E$$

The minimum energy that can be absorbed by hydrogen atom in ground state to go into excited state is 10.2 eV. Therefore,

$$\frac{1}{2}mv_{\min}^2 = 2 \times 10.2 \text{ eV} = 20.4 \text{ eV}$$

$$79. (4) \quad r = \frac{\epsilon_0 n^2 h^2}{e^2 \pi m} = \frac{\epsilon_0 (2\pi L)^2}{e^2 \pi m} \quad \left(L = n \frac{h}{2\pi} \text{ or } nh = 2\pi L \right)$$

$$\therefore Lr^{\frac{1}{2}} = \text{constant}$$

$$80. (4) \quad \frac{E_{4n} - E_{2n}}{E_{2n} - E_n} = \frac{\frac{E_1}{16n^2} - \frac{E_1}{4n^2}}{\frac{E_1}{4n^2} - \frac{E_1}{n^2}} = \frac{1}{4} = \text{constant}$$

$$81. (4) \quad E_n \propto \frac{1}{n^2} \text{ and } r_n \propto n^2$$

Therefore, $E_n r_n$ is independent of n .

$$\text{Hence, } E_1 r_1 = (13.6 \text{ eV}) (0.53 \text{ \AA}) = 7.2 \text{ eV-\AA} = \text{constant}$$

$$82. (4) \quad \Delta\lambda = \lambda_{K_\alpha} - \lambda_{\min}$$

When V is halved, $\lambda_{\min}$ becomes two times but λ_{K_α} remains the same.

$$\therefore \Delta\lambda' = \lambda_{K_\alpha} - 2\lambda_{\min} = 2(\Delta\lambda) - \lambda_{K_\alpha}$$

$$\therefore \Delta\lambda' < 2(\Delta\lambda)$$

$$83. (2) \quad \text{Potential energy} = -2 \times \text{Kinetic energy} = -2E$$

$$\text{Total energy} = -2E + E = -3.4 \text{ eV} = -E$$

$$\text{or } E = 3.4 \text{ eV}$$

$$p = \text{momentum, } m = \text{mass of electron}$$

$$E = \frac{p^2}{2m}$$

$$\text{or } p = \sqrt{2mE}$$

$$= \sqrt{2 \times 9.1 \times 10^{-31} \times 3.4 \times 1.6 \times 10^{-19}} \approx 10^{-24}$$

$$\text{de Broglie wavelength } \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = 6.6 \times 10^{-10} \text{ m}$$

$$84. (2) \quad \text{de Broglie wavelength of electron in hydrogen atom}$$

$$= \frac{h}{mv} = \frac{2\pi r_n}{n}$$

$$\text{For second Bohr orbit, } \lambda = \frac{600 \times 10^{-9}}{2} = 3000 \times 10^{-9} \text{ m}$$

$$\lambda = \sqrt{\frac{150}{V}} \text{ \AA} = 300 \text{ \AA}$$

$$\therefore V = \frac{150}{(3000)^2} = \frac{5}{3} \times 10^{-5} \text{ V}$$

$$85. (3) \quad \text{Using Moseley's law, } v^{1/2} = a(Z - b)$$

$$\left(\frac{c}{\lambda_1} \right)^{1/2} = a(Z_1 - b) \text{ and } \left(\frac{c}{\lambda_2} \right)^{1/2} = a(Z_2 - b)$$

$$\left(\frac{\lambda_2}{\lambda_1} \right)^{\frac{1}{2}} = \frac{a(Z_1 - b)}{a(Z_2 - b)} \Rightarrow \left(\frac{7.12}{15.42} \right)^{\frac{1}{2}} = \frac{(29 - b)}{(42 - b)}$$

$$(42 - b) = 1.47(29 - b) \Rightarrow b = 1.44$$

$$\left(\frac{\lambda_1}{\lambda} \right)^{\frac{1}{2}} = \frac{(Z - 1.44)}{(Z_1 - 1.44)}$$

$$\left(\frac{15.42}{22.85} \right)^{\frac{1}{2}} (27.56) = Z - 1.44 \Rightarrow Z = 24$$

$$86. (4) \quad \text{Angular momentum, } L = 4.2176 \times 10^{-34} = \frac{n_2 h}{2\pi} \Rightarrow n_2 = 4$$

For the transition from $n_2 = 4$ to $n_1 = 3$, the wavelength of spectral line = λ

$$\frac{1}{\lambda} = \frac{13.6}{hc} \left(\frac{1}{3^2} - \frac{1}{4^2} \right) = \frac{13.6 \text{ eV}}{1240 \text{ eV nm}} \left(\frac{7}{9 \times 16} \right)$$

$$\lambda = \frac{1240 \times 144}{13.6 \times 7} = 1876 \text{ nm} = 18760 \text{ \AA} = 1.876 \times 10^4 \text{ \AA}$$

$$87. (3) \quad \text{The angular momentum is } mvr = \frac{nh}{2\pi} \Rightarrow n = 1$$

$$\text{Centripetal force, } \frac{mv^2}{r} = \frac{Ze^2}{4\pi\epsilon_0 r^2}$$

$$r = \frac{\epsilon_0 n^2 h^2}{\pi m_e e^2 Z} = \left(\frac{\epsilon_0 h^2}{\pi m_e e^2} \right) \left(\frac{m_e}{m_\pi} \right) \frac{1}{Z} = \frac{0.53 \times 10^{-10}}{264 Z} = \frac{200 \times 10^{-15}}{Z} \quad \left[\because \frac{m_\pi}{m_e} = 264 \right]$$

Since r cannot be less than nuclear radius, $r > 1.6 Z^{\frac{1}{3}} \times 10^{-15} \text{ m}$

$$\text{or } \frac{200 \times 10^{-15}}{Z} > 1.6 \times 10^{-15} Z^{\frac{1}{3}}$$

$$\Rightarrow Z < \left(\frac{200}{1.6} \right)^{\frac{3}{4}} < 37$$

$$88. (1) \quad \text{For the first line of Balmer series of hydrogen}$$

$$\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{3^2} \right) = \frac{5R}{36} \Rightarrow \lambda = \frac{36}{5R}$$

$$\text{For singly ionized helium } (Z = 2), \frac{1}{\lambda'} = 4R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$\text{Given } \lambda' = \lambda$$

$$\text{For } n_1 = 6 \text{ to } n_2 = 4$$

$$\frac{1}{\lambda'} = 4R \left(\frac{1}{4^2} - \frac{1}{6^2} \right) = \frac{20R}{144} = \frac{5R}{36}$$

It corresponds to transition from $n_1 = 6$ to $n_2 = 4$.

$$89. (4) \quad \text{The energy of the } K_\alpha \text{ X-ray photon}$$

$$E_{K_\alpha} = E_i - E_f = (Z - 1)^2 (-3.4 \text{ eV} + 13.6 \text{ eV}) = (Z - 1)^2 (10.2 \text{ eV})$$

$$\text{Given } E_{K_\alpha} = 7.46 \text{ keV}$$

$$\therefore 7.46 \times 10^3 \text{ eV} = (Z - 1)^2 (10.2 \text{ eV})$$

$$\text{or } (Z - 1)^2 = \frac{7.46 \times 10^3}{10.2} = 731.4$$

$$\text{or } (Z - 1) = 27 \Rightarrow Z = 28$$

$$90. (1) \quad \text{The maximum number of electrons in an orbit is } 2n^2. \text{ Since } n > 4 \text{ is not possible, therefore the maximum number of electrons that can be in the first four orbits are}$$

$$2(1)^2 + 2(2)^2 + 2(3)^2 + 2(4)^2 = 2 + 8 + 18 + 32 = 60$$

Therefore, possible elements are 60.

$$91. (4) \quad \text{We know that } \frac{1}{\lambda} = RZ^2 \left[\frac{1}{n_2^2} - \frac{1}{n_1^2} \right] \Rightarrow \frac{1}{\lambda} \propto Z^2$$

λ is shortest when $1/\lambda$ is largest, i.e., when Z is big. Z is highest for lithium.

92. (4) For hydrogen and hydrogen-like atoms, $E_n = -13.6 \frac{(Z)^2}{(n)^2} \text{ eV}$

Therefore, ground state energy of doubly ionized lithium atom ($Z = 3, n = 1$) will be

$$E_1 = (-13.6) \frac{(3)^2}{(1)^2} = -122.4 \text{ eV}$$

Therefore, ionization energy of an electron in ground state of doubly ionized lithium atom will be 122.4 eV.

93. (4) For an atom following Bohr's model, the radius is given by

$$r_m = \frac{r_0 m^2}{Z}$$

where r_0 = Bohr's radius and m = orbit number.

For Fm, $m = 5$ (i.e. fifth orbit in which the outermost electron is present)

$$\therefore r_m = \frac{r_0 5^2}{100} = r_0 n \text{ (given)} \Rightarrow n = \frac{1}{4}$$

94. (1) The cut-off wavelength is given by

$$\lambda_0 = \frac{hc}{eV} \quad \dots(i)$$

According to de Broglie equation,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}} \Rightarrow \lambda^2 = \frac{h^2}{2meV} \quad \dots(ii)$$

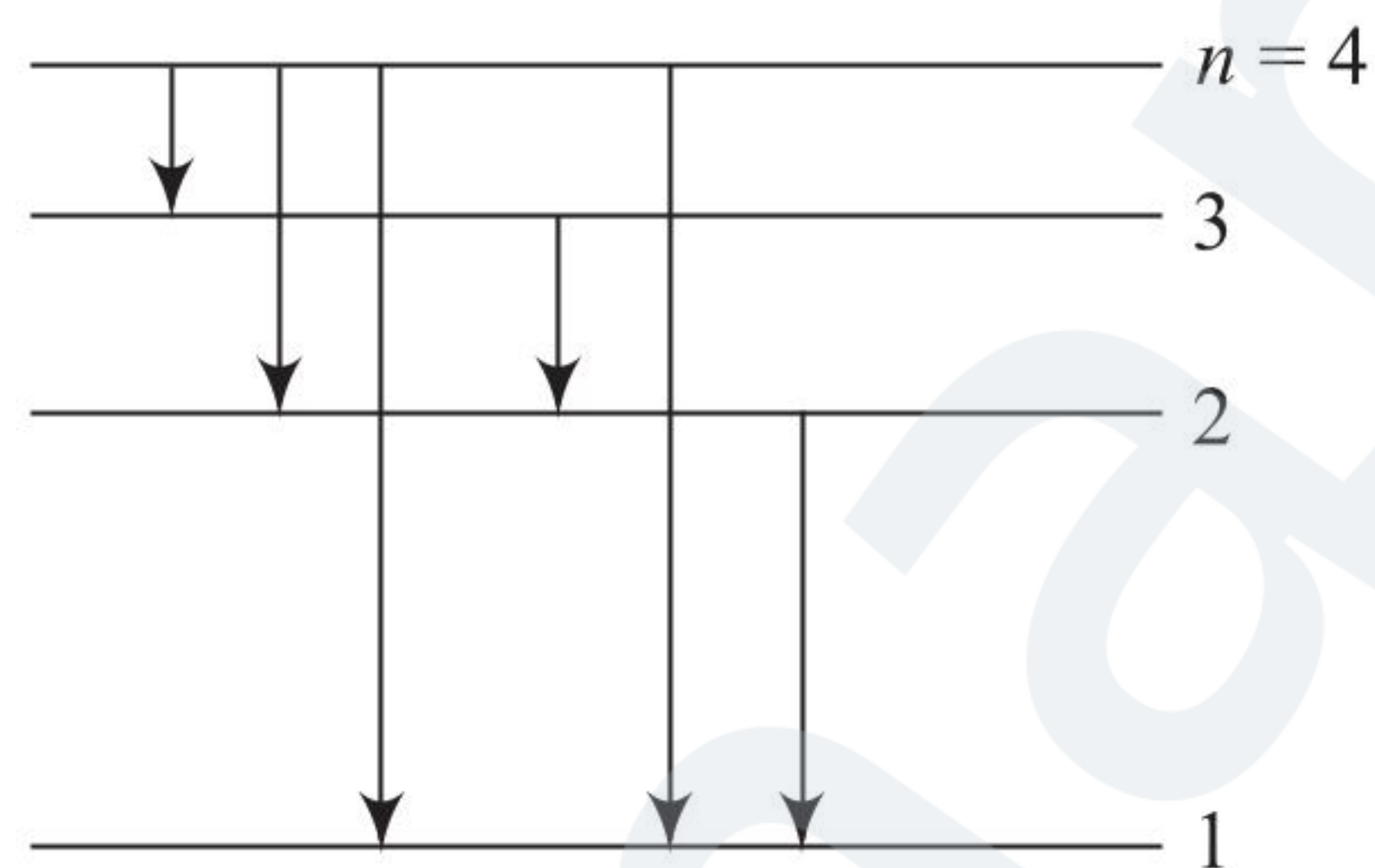
$$\Rightarrow V = \frac{h^2}{2me\lambda^2}$$

$$\text{From (i) and (ii), } \lambda_0 = \frac{hc \times 2me\lambda^2}{eh^2} = \frac{2mc\lambda^2}{h}$$

Multiple Correct Answers Type

1. (1), (2), (4)

$$\frac{n(n-1)}{2} = 6 \Rightarrow n = 4$$



If the initial state were $n = 3$, in the emission spectrum, no wavelengths shorter than λ_0 would have occurred.

This is possible if initial state were $n = 2$.

2. (2), (3)

$$E_0 z^2 \left(1 - \frac{1}{9}\right) - E_0 z^2 \left(\frac{1}{4} - \frac{1}{9}\right) = 3E_0$$

$$\Rightarrow \frac{27}{36} E_0 z^2 = 3E_0$$

$$\Rightarrow Z = 2$$

$$\frac{\lambda_1}{\lambda_2} = 3$$

$$\text{KE}_1 = E_0 \left(1 - \frac{1}{9}\right) - \phi; \text{KE}_2 = E_0 z^2 \left(1 - \frac{1}{4}\right) - \phi$$

$$\text{KE} \propto \frac{1}{\lambda^2} = 8.5 \text{ eV}$$

3. (1), (3), (4)

(1) As $v \propto 1/n$, so momentum $= mv \propto 1/n$

(2) is not true as radius $r \propto n^2$

$$(3) \text{ KE} = \frac{1}{2} mv^2,$$

$$\text{As } v \propto 1/n, \text{ so KE} \propto \frac{1}{n^2}$$

(4) is true.

4. (2), (3), (4)

Ground state $n = 1$

First excited state $n = 2$

$$\text{KE} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{2r} (Z = 1)$$

$$\text{KE} = \frac{14.4 \times 10^{-10}}{2r} \text{ eV}$$

Now $r = 0.53 n^2 \text{ \AA} (Z = 1)$

$$(\text{KE})_1 = \frac{14.4 \times 10^{-10}}{2 \times 0.53 \times 10^{-10}} \text{ eV} = 13.58 \text{ eV}$$

$$(\text{KE})_2 = \frac{14.4 \times 10^{-10}}{2 \times 0.53 \times 10^{-10} \times 4} \text{ eV} = 3.39 \text{ eV}$$

KE decreases by 10.2 eV

$$\text{Now, PE} = \frac{-1}{4\pi\epsilon_0} \frac{e^2}{r} = \frac{-14.4 \times 10^{-10}}{r} \text{ eV}$$

$$(\text{PE})_1 = \frac{-14.4 \times 10^{-10}}{0.53 \times 10^{-10}} \text{ eV} = -27.1 \text{ eV}$$

$$(\text{PE})_2 = \frac{-14.4 \times 10^{-10}}{0.53 \times 10^{-10} \times 4} = -6.79 \text{ eV}$$

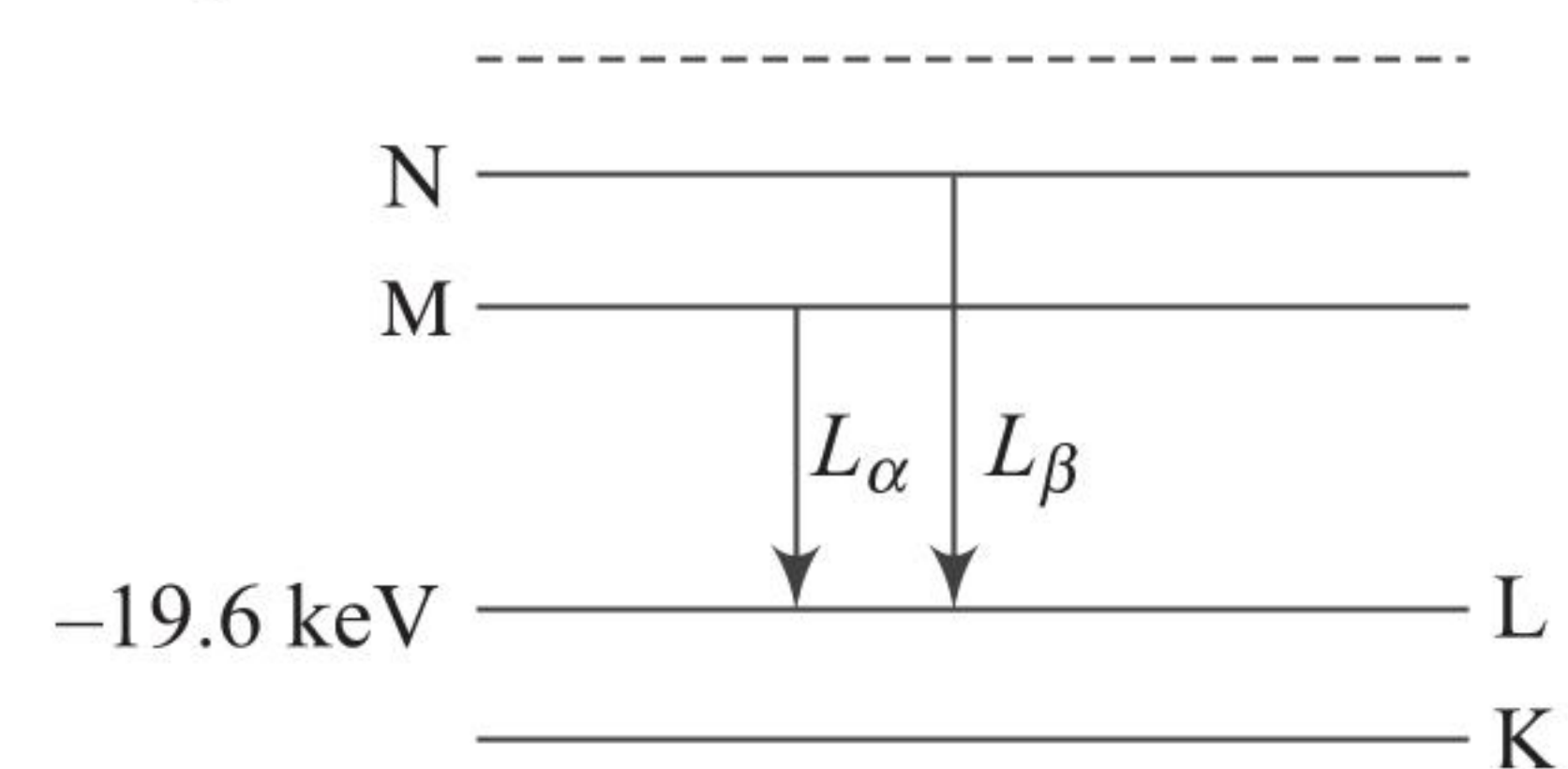
PE increases by 20.4 eV

$$\text{Now, angular momentum, } L = mvr = \frac{nh}{2\pi}$$

$$L_2 - L_1 = \frac{h}{2\pi} = \frac{6.6 \times 10^{-34}}{6.28} = 1.05 \times 10^{-34} \text{ J s}$$

5. (1), (2), (3)

$$\lambda_{\min} = \frac{12400}{V_0} \text{ \AA} = \frac{12400}{20,000} = 62 \text{ \AA}$$



6. (1), (2)

$$|F| = \frac{dU}{dt} = \frac{Ke^2}{r^4} \quad \dots(i)$$

$$\frac{Ke^2}{r^4} = \frac{mv^2}{r} \quad \dots(ii)$$

$$\text{and } mvr = \frac{nh}{2\pi} \quad \dots(iii)$$

By (ii) and (iii),

$$r = \frac{Ke^2 4\pi^2}{h^2} \frac{m}{n^2} = K_1 \frac{m}{n^2} \quad \dots(iv)$$

where $K_1 = \frac{Ke^2 4\pi^2}{h^2}$

$$\begin{aligned} \text{Total energy} &= \frac{1}{2} (\text{potential energy}) \\ &= \frac{Ke^2}{6r^3} = \frac{-Ke^2}{6\left(\frac{K_1 m}{n^2}\right)^3} = \frac{-Ke^2 n^6}{6K_1^3 m^3} \end{aligned}$$

Total energy $\propto n^6$

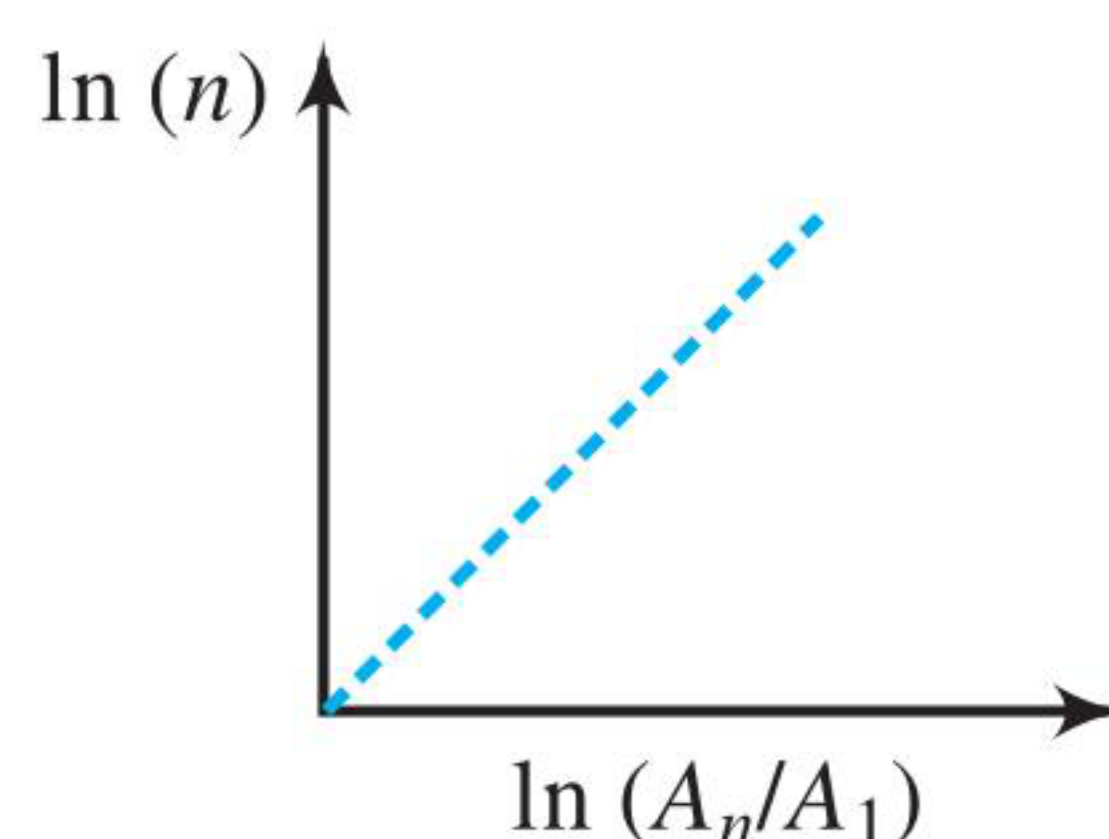
Total energy $\propto m^{-3}$

Therefore, (1) and (2) are correct.

7. (1), (2)

$$r_n = n^2 r_1$$

$$\ln \left(\frac{A_n}{A_1} \right) = \ln \left(\frac{\pi n_n^2}{\pi r_1^2} \right) = \ln n^4 = 4 \ln n$$



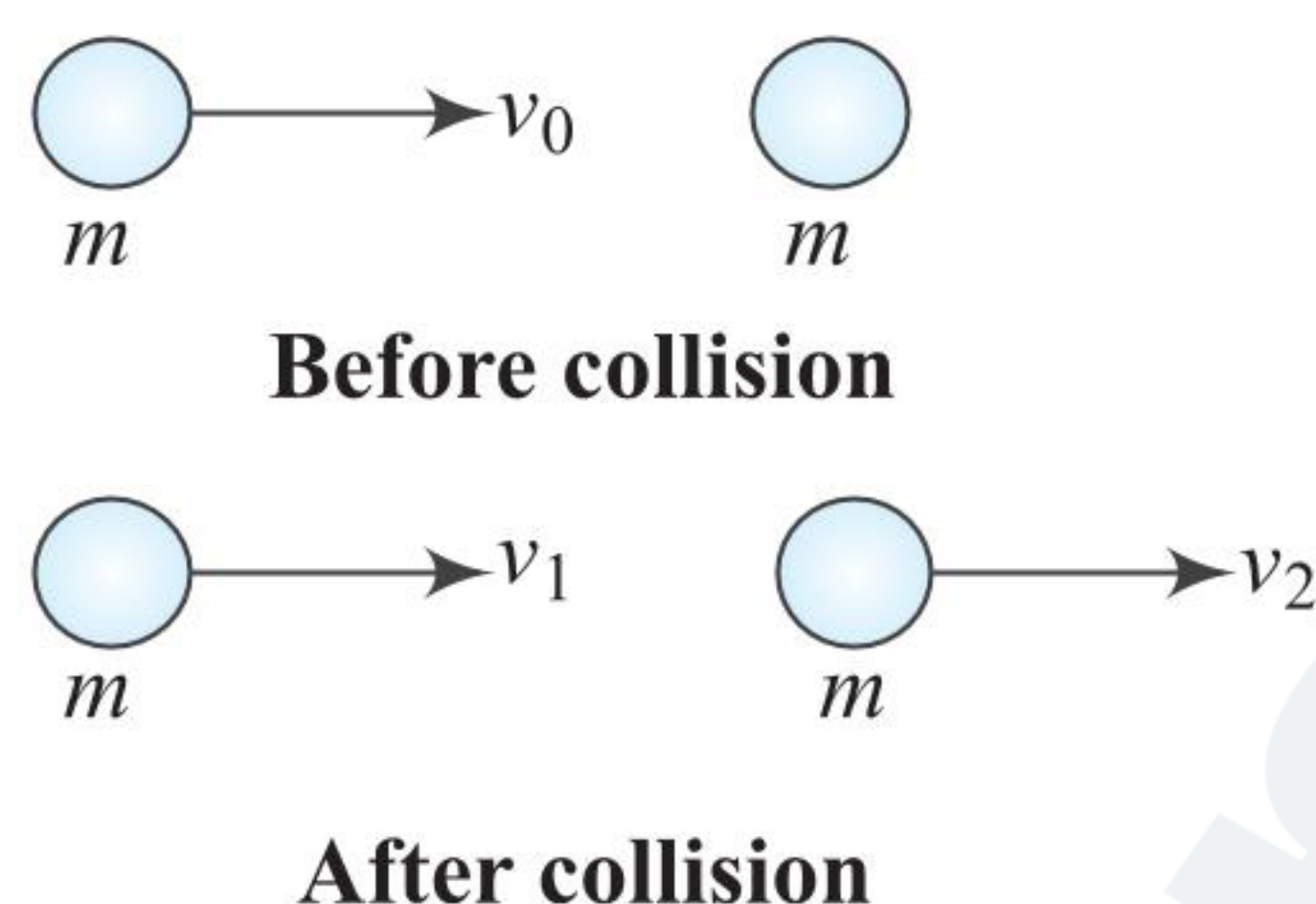
8. (2), (4)

Line emission spectra can be obtained for atoms and molecules both. An atom or molecule in an excited state emits photons by making a transition from excited state to ground state thus constituting line emission spectra.

The wavelengths emitted by the molecular energy levels which are generally grouped into several bunches are also grouped and each group is well separated from the other. The spectrum in this case looks like a band spectrum.

9. (2), (3), (4)

Let collision between two atoms be an inelastic one. From momentum conservation, $mv_0 = mv_1 + mv_2$



From energy conservation, $\frac{mv_1^2}{2} + \frac{mv_2^2}{2} - \frac{mv_0^2}{2} = -\Delta E$

where ΔE is the energy absorbed by the initially stationary atom to change its state.

Solving above equations, we get $(v_1 - v_2)^2 = v_0^2 - \frac{4\Delta E}{m}$

For collision to be inelastic, $(v_1 - v_2)^2 \geq 0$: a real quantity [equal to sign for perfect inelastic collision].

The minimum value of ΔE is 10.2 eV, so for collision to be inelastic, $E \geq 20.4$ eV.

For perfectly inelastic collision, $v_1 = v_2$ and hence $E = 20.4$ eV.

For $E = 18$ eV, the collision is elastic one and as masses are the same, velocities would be interchanged during collision.

10. (1), (3), (4)

(1) $r_n = \frac{n^2 h^2}{4\pi^2 m k e^2}$, i.e., $r_n \propto n^2$

(3) Bohr's 2nd postulate, $mvr = \frac{nh}{2\pi}$

(4) $K_n = \frac{kZe^2}{2r_n}$, $U_n = \frac{kZe^2}{r_n}$

11. (2), (3)

Lyman series lies in the ultraviolet region, Balmer series in visible region, and Paschen series in infrared region.

$$\lambda_R > \lambda_Y > \lambda_B > \lambda_V$$

12. (1), (2)

Continuous spectrum is obtained from the bulk state of matter, it has no relation with the atomic or molecular state. It is produced by thermal vibrations of atoms in the macroscopic matter (not by the transition between the energy states of atoms). Every vibrating atom emits light of frequency of its vibrations. In a white hot matter, atoms vibrating with all frequencies within a definite range are present, so this matter emits frequencies of continuous range.

13. (1), (2), (3)

$E_n^H = E_1^H + \Delta E = -13.6 \text{ eV} + 12.75 \text{ eV} = -0.85 \text{ eV}$ i.e., hydrogen atoms are excited to $n = 4$ level, i.e., transitions $4 \rightarrow 1$, $3 \rightarrow 1$, $2 \rightarrow 1$ are possible which correspond to Lyman series, then transitions $4 \rightarrow 2$ and $3 \rightarrow 2$ are possible which correspond to Balmer series, and then transition $4 \rightarrow 3$ is also possible which correspond to Paschen series.

14. (1), (2), (3)

(1) $U_1 = E$, then total energy in the orbit $= \frac{U_1}{2} = \frac{E}{2}$

(2) $IE = (TE)_{n=\infty} - (TE)_{n=1} = 0 - (TE)_{n=1} = -\left(\frac{E}{2}\right)$

(3) $(KE)_{n=1} = -(TE)_{n=1} = -\left(\frac{E}{2}\right)$

15. (1), (3), (4)

$$p_n = mv_n, p_n \propto v_n \propto \frac{1}{n}, r_n \propto n^2 \text{ and } K_n = \frac{Ke^2}{2r_n}$$

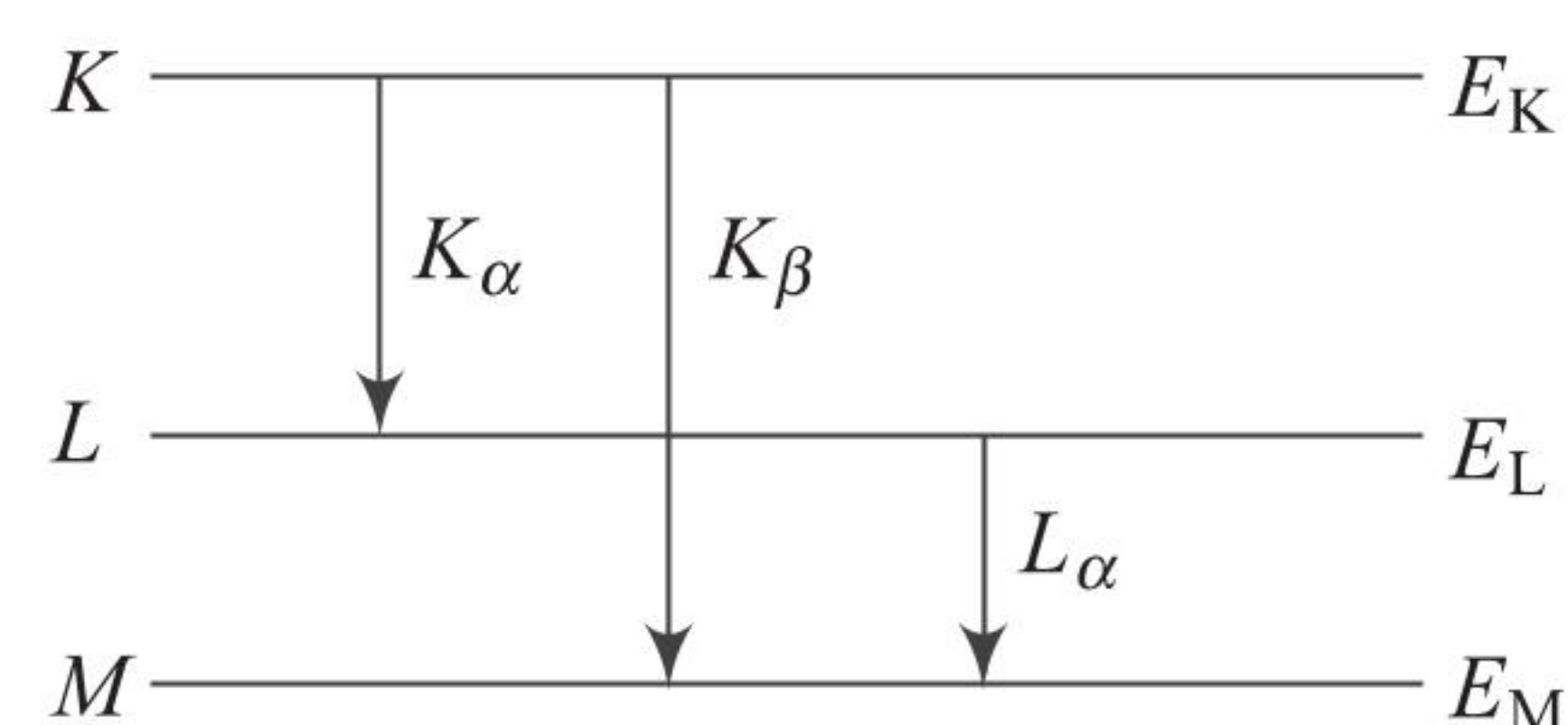
i.e., $K_n \propto \frac{1}{n^2}$ and $L \propto n$

16. (2), (3)

Any transition causing a photon to be emitted in the Balmer series must end at $n = 2$. This must be followed by the transition from $n = 2$ to $n = 1$, emitting a photon of energy 10.2 eV, which corresponds to a wavelength of about 122 nm. This belongs to the Lyman series.

17. (1), (3)

$$E_K - E_L = \frac{hc}{\lambda_\alpha} \quad \dots(i)$$



$$E_K - E_M = \frac{hc}{\lambda_\beta} \quad \dots(ii)$$

$$E_L - E_M = \frac{hc}{\lambda'_\alpha} \quad \dots(iii)$$

$$(ii) - (i) \Rightarrow E_L - E_M = \frac{hc}{\lambda'_\alpha} = \frac{hc}{\lambda_\beta} - \frac{hc}{\lambda_\alpha}$$

$$\frac{1}{\lambda_\beta} = \frac{1}{\lambda_\alpha} + \frac{1}{\lambda'_\alpha}$$

$$\text{Also, } (E_K - E_M) > (E_K - E_L) > (E_L - E_M)$$

$$\frac{hc}{\lambda_\beta} > \frac{hc}{\lambda_\alpha} > \frac{hc}{\lambda'_\alpha}$$

18. (2), (4)

When potential difference between filament and target is increased, then KE of striking electrons gets increased. Since most energetic electrons now strike the target, therefore more energetic electrons are emitted. It means, frequency of X-ray photons increases. It means, penetration power of X-rays gets increased.

To increase the photon flux, rate of collision of electrons with the target must be increased. This can be achieved only when rate of emission of electrons from the filament is increased. To achieve this, filament current must be increased. Therefore, options (2) and (4) are correct.

19. (1), (3)

First line of Lyman series is obtained during transition of hydrogen atom from $n = 2$ to $n = 1$. Hence, its energy is equal to $E_2 - E_1 = (13.6 \text{ eV} - (-3.4 \text{ eV})) = -1.2 \text{ eV}$.

$\therefore$ Wavelength of the first line of Lyman series is equal to

$$\frac{12375}{10.2} \text{ \AA} = 1215 \text{ \AA}$$

Therefore, it lies in ultraviolet region.

Since energy of all the other lines of Lyman series is greater than that of first line, therefore all the lines of Lyman series lie in ultraviolet region. Hence, option (1) is correct.

First line of Balmer series is obtained during transition of hydrogen atom from $n = 3$ to $n = 2$. Hence, its energy is equal to $E_3 - E_2 = 1.89 \text{ eV}$.

$\therefore$ Wavelength of first line of Balmer series is equal to $12375/1.89 \text{ \AA} = 6563 \text{ \AA}$. It lies in visible region.

Energy of last line of Balmer series is equal to 3.4 eV . Therefore, its wavelength is equal to $12375/3.4 \text{ \AA} = 3640 \text{ \AA}$.

Since it is less than $4000 \text{ \AA}$, therefore it lies in ultraviolet region. Hence, option (2) is wrong. Energy of last line of Paschen series is equal to 1.51 eV . It lies in infrared region. Since energy of all the other lines of Paschen series is less than its energy, therefore all the lines of Paschen series will lie in infrared region. Hence, option (3) is also correct.

20. (2), (3), (4)

Statement (1) is false. The shortest wavelength of the X-rays emitted depends on the energy of the electrons incident on the target. This, in turn, depends on the potential through which they have fallen. In

$$\text{fact, } \lambda_{\min} = \frac{hc}{eV}$$

Statement (2) is true. X-ray spectra of all heavy elements are similar in character.

Statement (3) is also true. The short wavelength of the X-rays (compared to the grating constant of optical grating) makes it difficult to observe X-ray diffraction with ordinary gratings.

Statement (4) is also true. The sharp limit on the short wavelength side is dependent on the voltage applied to the incident electrons

$$\text{and is given by } \lambda_{\min} = \frac{hc}{eV}$$

21. (3), (4)

$$V = 6.6 \text{ kV} = 6600 \text{ V}$$

$$\text{Now, } \nu_{\max} = \frac{eV}{h} = \frac{1.6 \times 10^{-19} \times 6600}{6.6 \times 10^{-34}} = 1.6 \times 10^{18} \text{ Hz}$$

Thus, the frequency of the X-rays cannot exceed $1.6 \times 10^{18} \text{ Hz}$. Hence, the correct choices are (3) and (4).

22. (1), (3)

Statement (1) is correct. The angular momentum of the earth ($= mr^2 \omega$) has to be equal to $nh/2\pi$. This gives, $n = 2\pi mr^2 \omega/h$.

Putting the numerical values of the Earth's mass, radius, and the angular velocity, we get the given value of n .

Statement (2) is incorrect. The maximum number of electrons allowed in an orbit being $2n^2$, the required number is $2(1^2 + 2^2 + 3^2 + 4^2) = 60$.

Statement (3) is correct. The 'reduced mass' of the electron [$\mu = mM/(m + M)$] being dependent on the mass of the nucleus (M), the Rydberg constant also varies with the mass number of the given element.

Statement (4) is incorrect. The ratio is not exactly equal to 4 but slightly different from 4 because of the dependence of the Rydberg constant on the mass number of the element concerned.

23. (1), (2)

The correct choices are (1) and (2). The last two statements are incorrect because they violate the principle of conservation of charge. We always have either an electron-positron 'pair production' or an electron-positron 'pair annihilation'. It is only then that the total charge remains zero both before and after reaction.

24. (1), (3)

$$\text{Moseley's law: } \lambda \propto \frac{1}{(z-1)^2}; \frac{\lambda_z}{\lambda_1} = \frac{(z_1-1)^2}{(z-1)^2} = 4$$

$$z_1 - 1 = (z - 1) 2; z_1 = 2z - 1;$$

$$\text{Similarly, } \frac{\lambda_z}{\lambda_2} = \frac{1}{4} = \left(\frac{z_2 - 1}{z - 1} \right)^2; z_2 = \frac{z + 1}{2}$$

25. (1), (3), (4)

In Bohr model of hydrogen atom,

$$R \propto n^2, V \propto \frac{1}{n}, T \propto n^3 \text{ and } E \propto \frac{1}{n^2}$$

$$\Rightarrow VR \propto n; TE \propto n; \frac{T}{R} \propto n \text{ and } \frac{V}{E} \propto n$$

26. (1), (3)

Power loss increases the temperature.

27. (1), (2), (3)

Energy of K absorption edge

$$E_K = \frac{1242 \text{ eVnm}}{0.0172 \text{ nm}} = 72.21 \times 10^3 \text{ eV} = 72.21 \text{ keV}$$

$$\text{Energy of } K_\alpha \text{ line is } E_{K_\alpha} = \frac{he}{e\lambda_\alpha} = \frac{1242 \text{ eVnm}}{0.021 \text{ nm}} = 59.14 \text{ keV}$$

$$\text{Similarly, } E_{K_\beta} = \frac{1242}{0.0192} = 64.69 \text{ keV}$$

$$E_{K_\gamma} = \frac{1242}{0.0180} = 69 \text{ keV}$$

$$\text{Energy of } K \text{ shell} = (E_{K_\alpha} - E_K)$$

$$= (59.14 - 72.21) \text{ keV} = -13.04 \text{ keV}$$

$$\text{Energy of } L \text{ shell} = E_{K_\beta} - 72.21 \text{ keV}$$

$$= 64.69 \text{ keV} - 72.21 \text{ keV} = -7.52 \text{ keV}$$

$$\text{Energy of } M \text{ shell} = E_{K_\gamma} - E_K = \frac{1242 \text{ eVnm}}{0.018 \text{ nm}} - 72.21 \text{ keV}$$

$$= 69 \text{ keV} - 72.21 \text{ keV} = -3.21 \text{ keV}$$

28. (1), (2)

For the third line of Balmer series, $n_1 = 2, n_2 = 5$

$$\therefore \frac{1}{\lambda} = RZ^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) = RZ^2 \left(\frac{1}{2^2} - \frac{1}{5^2} \right) = \frac{21RZ^2}{100}$$

$$E = -13.6 \text{ eV}$$

$$Z^2 \times \frac{21}{100} = \frac{hc}{\lambda} = \frac{1242 \text{ eVnm}}{108.5 \text{ nm}}$$

$$Z^2 = \frac{1242 \times 100}{108.5 \times 21 \times 13.6} = 4 \Rightarrow Z = 2$$

Binding energy of an electron in the ground state of hydrogen-like ion = $13.6Z^2/n^2 = 54.4 \text{ eV}$ ($n = 1$).

29. (1), (2), (4)

$$28.3 = 13.6 Z^2 \left[\frac{1}{n_1^2} - \frac{1}{n^2} \right] \quad \dots(i)$$

$$302.2 = 13.6 Z^2 \left[1 - \frac{1}{n_1^2} \right] \quad \dots(ii)$$

$$\text{For } K_{\alpha}, \frac{1}{\lambda} = \frac{3R}{4} (Z-1)^2$$

$$\frac{10^9}{7.64} = \frac{3 \times 1.09677 \times 10^7}{4} (Z-1)^2$$

$$(Z-1)^2 = \frac{4 \times 100}{7.6 \times 3 \times 1.09}$$

$$Z = 5 \quad \dots(iii)$$

Solving Eqs. (ii) and (iii)

$$302.2 = 13.6(5)^2 \left[1 - \frac{1}{n_1^2} \right]$$

$$n_1 = 3 \quad \dots(iv)$$

From Eqs. (i), (iii) and (iv)

$$n = 6$$

30. (2), (3), (4)

Since Rydberg constant $R \propto$ mass of electron

$$\frac{1}{\lambda} = 3R \left(\frac{1}{n^2} - \frac{1}{3^2} \right) \text{ where } n = 1, 2$$

So, the possible emitted wavelength are $\frac{3}{8R}$, $\frac{4}{9R}$ and $\frac{12}{5R}$

31. (3), (4)

In the case of hydrogen, atomic number = mass number.

In other atoms, atomic number < mass number.

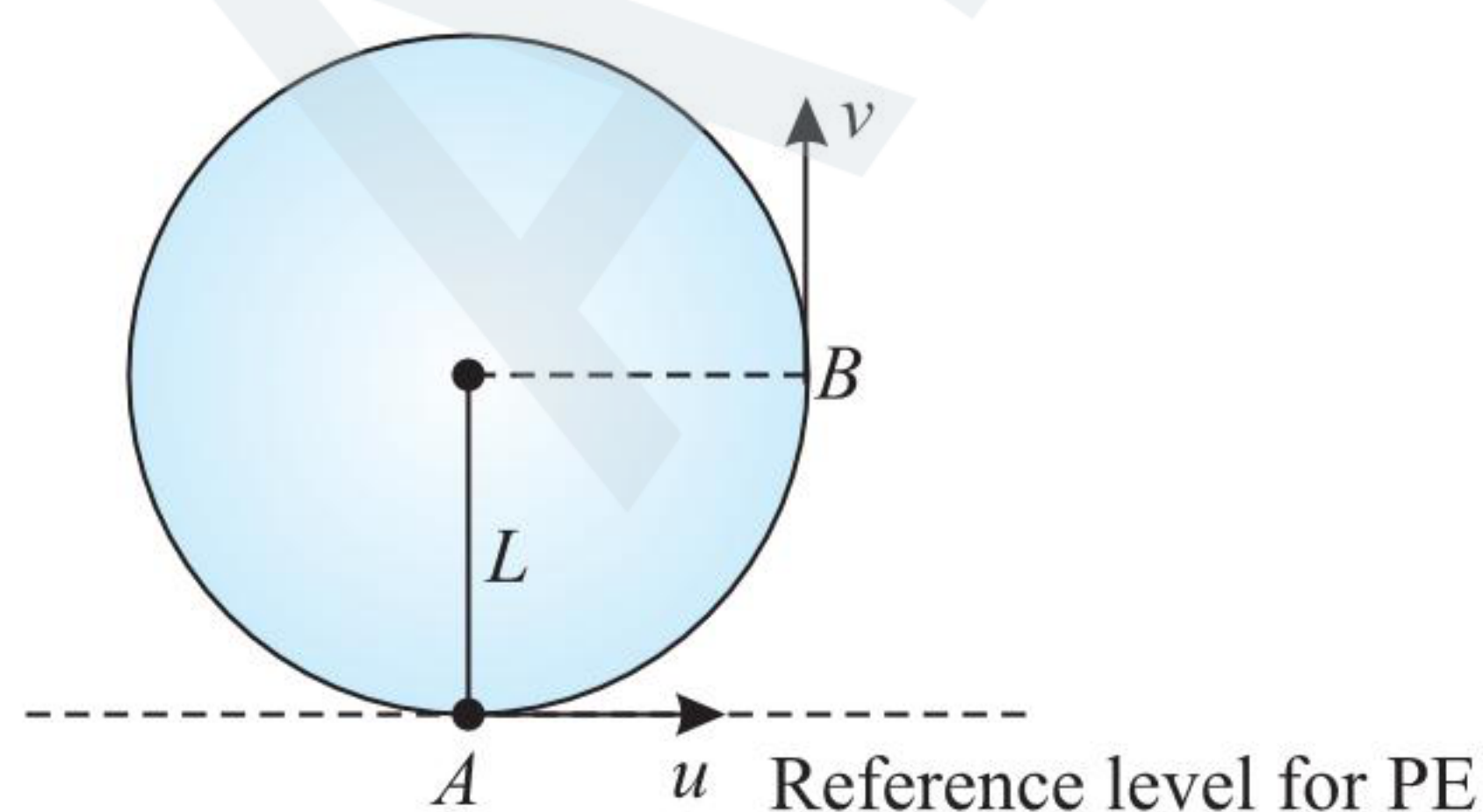
Therefore, (3) and (4) are the correct options.

32. (1), (4)

The time period of the electron in a Bohr orbit is given by $T = 2\pi r/v$

Since for the n^{th} Bohr orbit, $mvr = n(h/2\pi)$, the time period becomes

$$T = \frac{2\pi r}{nh/(2\pi mr)} = \left(\frac{4\pi^2 m}{nh} \right) r^2$$



Since the radius of the orbit r depends on n , we replace r .

The expression of Bohr radius of a hydrogen atom is

$$r = n^2 \left(\frac{h^2 \epsilon_0}{\pi m e^2} \right)$$

$$\text{Hence, } T = \left(\frac{4\pi^2 m}{nh} \right) \left(\frac{n^4 h^4 \epsilon_0^2}{\pi^2 m^2 e^4} \right) = n^3 \left(\frac{4h^3 \epsilon_0^2}{m e^4} \right)$$

$$\text{For two orbits, } \frac{T_1}{T_2} = \left(\frac{n_1}{n_2} \right)^3$$

It is given that $T_1/T_2 = 8$. Hence, $n_1/n_2 = 2$.

Linked Comprehension Type

For Problems 1–3

$$1. (2) \text{ On other planet: } mvr = 2n \frac{h}{2\pi} \Rightarrow v = \frac{nh}{\pi mr}$$

$$\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} \Rightarrow \frac{mn^2 h^2}{\pi^2 m^2 r^3} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$$

$$\text{Putting } n = 1, \text{ we get } r = \frac{4h^2 \epsilon_0}{m\pi e^2}$$

$$2. (2) \text{ On our planet: } v_0 = \frac{e^2}{2\epsilon_0 nh}$$

$$\text{On other planet: } v = \frac{e^2}{2\epsilon_0 (2n)h} = \frac{v_0}{2}$$

$$3. (2) \text{ On our planet: } E_n = -\frac{13.6}{n^2}$$

$$\text{On other planet: } E'_n = -\frac{13.6}{(2n)^2}$$

$$\Rightarrow E'_n = \frac{E_n}{4} = -3.4 \text{ eV}$$

For Problems 4–6

4. (1) Internal forces act between electron and proton, then how can the atom get an acceleration.

5. (3) Change in angular momentum = $I\omega$

$$\Rightarrow \frac{mh}{2\pi} - \frac{nh}{2\pi} = I\omega \Rightarrow \omega = \frac{(m-n)h}{6.8I}$$

6. (4) Since no external force and torque act on the atom, still it gets an acceleration and angular acceleration. So, none is true.

For Problems 7–12

7. (3) Given that $E_1 = -15.6 \text{ eV}$, $E_{\infty} = 0 \text{ eV}$.

Ionization energy of the atom: $E_{\infty} - E_1 = 0 - (-15.6 \text{ eV}) = 15.6 \text{ eV}$

So, ionization potential = 15.6 V

8. (2) For short wavelength limit of the series terminating at $n = 2$, a transition must take place from $n = \infty$ state to $n = 2$ state. For this, $\Delta E = 5.30 \text{ eV}$

$$\lambda = \frac{12400}{\Delta E (\text{eV})} \text{ \AA} = \frac{12400}{5.30} \text{ \AA} = 2339 \text{ \AA}$$

9. (3) The excitation energy for the $n = 3$ state is

$$\Delta E = E_3 - E_1 = 15.6 - 3.08 = 12.52 \text{ eV}$$

Excitation potential = 12.52 V

$$10. (2) \lambda = \frac{12400}{E_3 - E_1} = \frac{12400}{12.52} \text{ \AA} = 990 \text{ \AA}$$

$$\text{Wave number} = \frac{1}{\lambda} = \frac{1}{990 \times 10^{-10} \text{ m}} = 1.009 \times 10^7 \text{ m}^{-1}$$

11. (3) Because excitation energy for the second shell is more than 6 eV, hence electron having initial kinetic energy of 6 eV will

not interact with the atom. Because it cannot transfer its energy to the electron in the atom.

12. (1) As $E_2 - E_1 = 15.6 - 5.3 = 10.3 \text{ eV}$

hence energy of the electron after interaction is

$$11 - 10.3 \text{ eV} = 0.7 \text{ eV}$$

For Problems 13–14

13. (3) We can use the equation:

$$\frac{1}{\lambda} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) = R \left(\frac{1}{1^2} - \frac{1}{2^2} \right) = \frac{3R}{4}$$

$$\lambda = \frac{4}{3R} = \frac{4}{3(1.097 \times 10^7)} = 1.215 \times 10^{-7} \text{ m} = 121.5$$

The frequency of the photon is

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^8}{1.215 \times 10^{-7}} = 2.47 \times 10^{15} \text{ Hz}$$

The energy of the photon is $E = hf$

$$= (4.136 \times 10^{-15})(2.47 \times 10^{15}) = 10.2 \text{ eV}$$

14. (2) From conservation of momentum, as the total momentum before emission is zero, the total momentum after emission must also be zero. The photon and atom therefore move off in opposite directions, with $mv = E_{\text{photon}}/c$, where m and v are the mass and recoil speed of the hydrogen atom, E_{photon} is the actual energy of the photon (less than 10.2 eV), and c is the speed of light. The energy difference between the $n = 2$ and $n = 1$ levels, E is the source of both the photon energy and the recoil kinetic energy of the atom.

From energy conservation, we have $E = E_{\text{photon}} + \frac{1}{2}mv^2$

Atom being massive, we can assume that its recoil speed v and kinetic energy are so small that $E = E_{\text{photon}}$. Substituting $E_{\text{photon}} = 10.2 \text{ eV}$ into the expression for mv yields.

$$mv = 10.2 \text{ eV}/c$$

The recoil kinetic energy of the hydrogen atom can now be calculated.

$$K = \frac{1}{2}mv^2 = \frac{1}{2} \frac{(mv)^2}{m} = (0.5) \frac{(10.2)^2}{mc^2}$$

$$= \frac{(0.5)(10.2)^2}{938.8 \times 10^6} = 5.54 \times 10^{-8} \text{ eV}$$

Thus, the fraction of the energy difference between the $n = 2$ and $n = 1$ levels that goes into atomic recoil energy is very small.

$$\frac{K}{E} = \frac{5.54 \times 10^{-8}}{10.2} = 5.43 \times 10^{-9}$$

That is why by equating the photon's energy to the atomic energy level separation yields accurate answers because little energy is needed to conserve momentum.

For Problems 15–16

15. (2) As we know, energy of a photon is given by $E = \frac{hc}{\lambda}$

From the given condition, $\lambda = \frac{1500p^2}{p^2 - 1}$

$$\text{Hence, } E = \frac{hc}{\lambda} = \frac{hc}{1500} \left(1 - \frac{1}{p^2} \right) \times 10^{10} \text{ J}$$

$$= \frac{hc}{(1500)(1.6 \times 10^{-19})} \left(1 - \frac{1}{p^2} \right) \times 10^{10} \text{ eV}$$

$$= 8.28 \left(1 - \frac{1}{p^2} \right) \text{ eV}$$

Hence, energy of n th state is given by $E_n = -\frac{8.28}{n^2} \text{ eV}$

$$n = 3 \text{ ————— } -0.91 \text{ eV}$$

$$n = 2 \text{ ————— } -2.07 \text{ eV}$$

$$n = 1 \text{ ————— } -8.28 \text{ eV}$$

Maximum energy is released for transition from $p = \infty$ to $p = 1$; hence, wavelength of most energetic photon is 1500 Å.

16. (4) The ionization potential corresponds to energy required to liberate an electron from its ground state, i.e. ionization energy = 8.28 eV.

Hence, ionisation potential = 8.28 V

For Problems 17–20

17. (2), 18. (1), 19. (3), 20. (2)

Consider energy level diagram of hydrogen atom. After absorbing photons of energy 10.2 eV, it would reach the first excited state of -3.4 eV , since energy difference corresponding to $n = 1$ and $n = 2$ is 10.2 eV. When this excited hydrogen atom de-excites, it would release 10.2 eV energy, which is absorbed by He^+ and Li^{2+} .

Energy of the n th state of a hydrogen-like atom with atomic number Z is given by $E_n = \frac{13.6Z^2}{n^2} \text{ eV}$

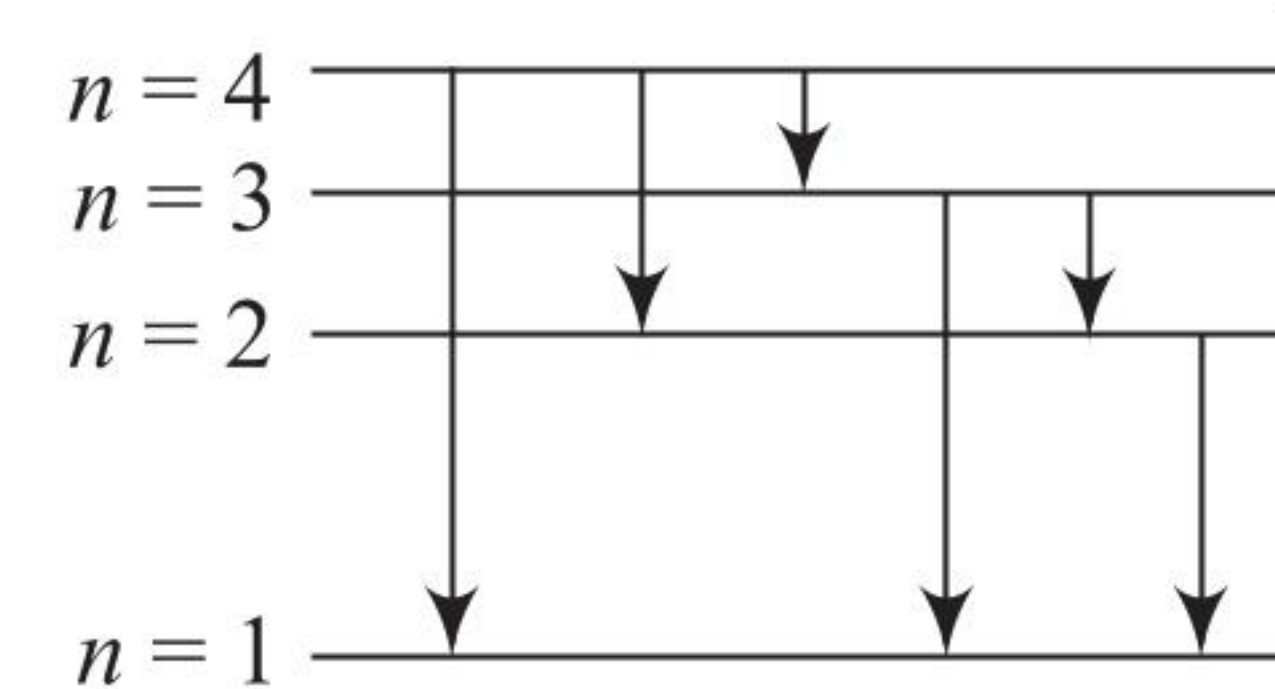
After absorbing 10.2 eV, He^+ electron moves from $n = 2$ to $n = 4$ and Li^{2+} electron moves from $n = 3$ to $n = 6$.

$$\begin{array}{l} n = 4 \text{ ————— } -3.4 \text{ eV} \\ n = 3 \text{ ————— } -6.04 \text{ eV} \\ n = 2 \text{ ————— } -13.6 \text{ eV} \end{array}$$

$$n = 1 \text{ ————— } -54.4 \text{ eV}$$

Energy levels of He^+

In the spectrum of He^+ , there would be ${}^4C_2 = 6$ lines.



Transition in He^+

$$\lambda_{\min} = \frac{hc}{\Delta E} = \frac{1242}{13.6 \left[4 - \frac{4}{16} \right]} = 24.4 \text{ nm}$$

$$\begin{array}{l} n = 6 \text{ ————— } -3.4 \text{ eV} \\ n = 5 \text{ ————— } -4.89 \text{ eV} \\ n = 4 \text{ ————— } -7.65 \text{ eV} \\ n = 3 \text{ ————— } -13.6 \text{ eV} \\ n = 2 \text{ ————— } -30.6 \text{ eV} \\ n = 1 \text{ ————— } -122.4 \text{ eV} \end{array}$$

Similarly, in spectrum of Li^{2+} there will be ${}^6C_2 = 15$ lines.

$$\lambda_{\min} = \frac{1242}{13.6 \left[9 - \frac{9}{36} \right]} = 10.4 \text{ nm}$$

For Problems 21–22

21. (3) For a conservative force field, $-\frac{dU}{dr} = F$

$$\text{Since } U = k \log r, -\frac{dU}{dr} = -\frac{k}{r} = F$$

This force $F = -k/r$ provides the centripetal force for circular motion of electron.

$$\frac{mv^2}{r} = \frac{k}{r} \quad \dots(i)$$

Applying Bohr's quantization rule,

$$mvr = \frac{nh}{2\pi} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get $r = \frac{nh}{2\pi\sqrt{mk}}$

22. (1) From Eq. (i), KE of electron $= \frac{1}{2}mv^2 = \frac{1}{2}k$

Total energy of electron = KE + PE

$$\text{PE of electron} = k \log r = \frac{1}{2}k + k \log r$$

$$E = \frac{k}{2} \left[1 + \log \frac{n^2 h^2}{4\pi^2 mk} \right]$$

For Problems 23–25

23. (4) For Balmer series, $n_1 = 2$; $n_2 = 3, 4, \dots$
(lower) (higher)

Therefore, in transition (VI), proton of Balmer series is absorbed.

24. (3) In transition II, $E_2 = -3.4$ eV, $E_4 = -0.85$ eV

$$\Delta E = 2.55 \text{ eV}$$

$$\Delta E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\Delta E} = 487 \text{ nm}$$

25. (4) Wavelength of radiation $= 103 \text{ nm} = 1030 \text{ \AA}$

$$\Delta E = \frac{12400}{1030 \text{ \AA}} = 12.0 \text{ eV}$$

So, difference of energy should be 12.0 eV (approx)

Hence $n_1 = 1$ and $n_2 = 3$

$$(-13.6) \text{ eV} \quad (-1.51) \text{ eV}$$

Therefore, transition is V.

For Problems 26–28

26. (3), 27. (2), 28. (2)

Let n be the lowest energy level of the given series of lines, then maximum wavelength is given by

$$\frac{hc}{\lambda_{\max}} = 13.6 \times Z^2 \left[\frac{1}{n^2} - \frac{1}{(n+1)^2} \right]$$

$$\frac{hc}{\lambda_{\min}} = 136 \times Z^2 \left[\frac{1}{n^2} - \frac{1}{\infty^2} \right]$$

where $\lambda_{\max} = 41.02 \text{ nm}$ and $\lambda_{\min} = 22.79 \text{ nm}$.

Solving above equations, we get $n = 2$, which corresponds to Balmer series.

$$Z = 4$$

For next to longest wavelength, transition takes place from $n_1 = 4$ to $n_2 = 2$.

$$\text{So, } \frac{hc}{\lambda} = 13.6 \times 4^2 \left[\frac{1}{4} - \frac{1}{16} \right]$$

$$\therefore \lambda = 30.47 \text{ nm}$$

For Problems 29–31

29. (1) The minimum energy required to remove the electron from K-level is $3 \times 10^{-15} \text{ J}$.

Let V be the potential difference required, then $eV = 3 \times 10^{-15} \text{ J}$

$$V = \frac{3 \times 10^{-15}}{1.6 \times 10^{-19}} = 18750 \text{ V}$$

30. (2) The energy released due to transition of electron from L-level to K-level is

$$\Delta E = 3 \times 10^{-15} - 2 \times 10^{-16} = 28 \times 10^{-16} \text{ J}$$

The wavelength of the electromagnetic wave is

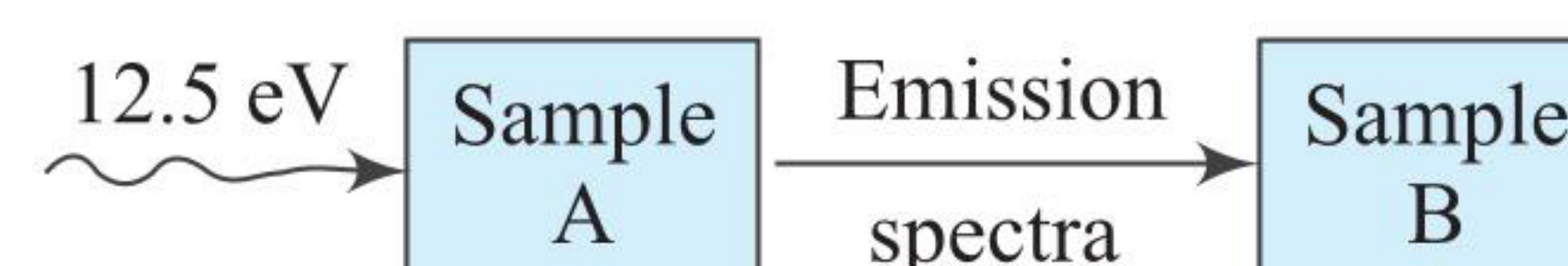
$$\lambda = \frac{hc}{\Delta E} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{28 \times 10^{-16}} \text{ m} = 7.104 \times 10^{-11} \text{ m}$$

31. (4) The minimum energy required to eject the electron from M-level is $3 \times 10^{-17} \text{ J}$. The energy used up in ejecting the electron from M-level is, $28 \times 10^{-16} \text{ J}$. So, the kinetic energy of emitted electrons from M-level is $K = (28 \times 10^{-16} - 3 \times 10^{-17}) \text{ J}$ or $K = 277 \times 10^{-17} \text{ J}$

For Problems 32–34

32. (4), 33. (4), 34. (4)

As photon energy of incident light is not equal to energy difference between two energy states, that is why sample A does not absorb any photon and therefore remains in ground state, and hence, no emission spectra. Thus, sample B remains as it is, but it will de-excite itself to ground state by emitting radiations of energy 10.2 eV.



When photon energy is replaced by electron beam, then atoms of sample A can absorb either 10.2 eV (to reach 1st excited state) or 12.1 eV (to reach 2nd excited state). In emission spectra of A, we will have 3 lines corresponding to $n = 2$ to 1, $n = 3$ to 2 and $n = 3$ to 1, having energies equal to 10.2 eV, 1.9 eV, and 12.1 eV, respectively.

As least energy of this emission spectra is corresponding to transition from $n = 2$ to 1 and sample B is in 1st excited state, so B can be excited to some higher energy level and if B absorbs energy corresponding to $n = 1$ to 2 and $n = 1$ to 3, then it may ionize.

For Problems 35–37

35. (1) $\Delta E = -13.6Z^2 \left[\frac{1}{2^2} - \frac{1}{3^2} \right]$

$$47.2 = 13.6Z^2 \left[\frac{1}{4} - \frac{1}{9} \right] \Rightarrow Z = 5$$

36. (2) $r_n = \frac{0.529}{Z} \times 10^{-10} \text{ m} = \frac{0.529}{5} \times 10^{-10} = 0.106 \times 10^{-10} \text{ m}$

37. (1) $L = mvr = \frac{nh}{2\pi}$

$$\text{For first Bohr orbit, } L = 1 \times \frac{h}{2\pi} = 1.05 \times 10^{-13} \text{ Js}$$

For Problems 38–39

38. (3) Characteristic X-ray arises due to transition of electrons.

39. (1) When the accelerated electron beam strikes the nucleus of target atom, continuous X-rays are produced.

For Problems 40–42
40. (1) In photoelectric effect

$$E = W + K_{\max} = 0.73 + 1.82 = 2.55 \text{ eV}$$

41. (1) $E_n = \frac{-13.6z^2}{n^2} \text{ eV}$

$$\therefore E_1 = -13.6 \text{ eV}; E_2 = -3.4 \text{ eV}; E_3 = -1.51 \text{ eV}; E_4 = -0.85 \text{ eV}$$

$$E_4 - E_2 = -(0.85) - (-3.4) = 2.55 \text{ eV}$$

42. (3) $L_4 - L_2 = \frac{4h}{2\pi} = \frac{2h}{\pi} = \frac{2h}{2\pi} = \frac{h}{\pi} \quad \left[L = \frac{nh}{2\pi} \right]$

For Problems 43–45
43. (1) Least intensity $2 \rightarrow 1 \Rightarrow K_\alpha, X\text{-rays}$
44. (3) $4 \rightarrow 1 \Rightarrow K_\gamma, X\text{-rays}$
45. (4) Calculate from energy of Li^{++} .

For Problems 46–47
46. (2), 47. (4)

$$\Delta E = \left\{ 13.6 Z_B^2 \left(\frac{1}{1^2} - \frac{1}{2^2} \right) - 13.6 Z_A^2 \left(\frac{1}{1^2} - \frac{1}{2^2} \right) \right\} \text{ eV}$$

$$81.6 \text{ eV} = \frac{1.36 \times 3}{4} (Z_B^2 - Z_A^2) \text{ eV}$$

$$Z_B^2 - Z_A^2 = 8 \quad \dots(i)$$

Using conservation of momentum,

For A, $m_A u = MV_1 - m_A \frac{u}{2}$

$$\frac{3}{2} m_A u = MV_1$$

For B, $\frac{3}{2} m_B u = MV_2$

But, $MV_2 = 3MV_1$

$$\therefore m_B = 3m_A$$

Since both A and B carry same number of protons and neutrons, we have

$$Z_B = 3Z_A \quad \dots(ii)$$

But $Z_B^2 - Z_A^2 = 8$

$$9Z_A^2 - Z_A^2 = 8$$

$$Z_A = 1, Z_B = 3$$

 Hence, A is ${}^2_1\text{H}$ and B is ${}^6_4\text{Li}$

Now, the difference in energy between the first Balmer lines emitted by A and B

$$\Delta E = 13.6 \left(\frac{1}{2^2} - \frac{1}{3^2} \right) Z_B^2 - 13.6 \left(\frac{1}{2^2} - \frac{1}{3^2} \right) Z_A^2$$

$$= 13.6 \times \frac{5}{36} \times (Z_B^2 - Z_A^2) = \frac{13.6 \times 5 \times 8}{36} = 15.1 \text{ eV}$$

For Problems 48–49
48. (3) Number of atoms in the second excited state

$$N_2 = \frac{N_{\text{av}}}{M} \times m \times \frac{15}{100} = \frac{6.023 \times 10^{23}}{1.008} \times 1.8 \times \frac{15}{100} = 1.61 \times 10^{23}$$

In the first excited state, $N_1 = \frac{N_{\text{av}}}{M} \times m \times \frac{27}{100} = 2.9 \times 10^{23}$

49. (1) Amount of energy that would be evolved $= U_i - U_f$

$$U_1 = N_1 E_1 + N_2 E_2$$

$$= -1.61 \times 10^{23} \times \frac{21 \times 10^{-12}}{3^2} - \frac{2.9 \times 10^{23} \times 21.7 \times 10^{-12}}{2^2}$$

$$= -3.88 \times 10^{11} - 15.75 \times 10^{11} = -19.63 \times 10^{11} \text{ erg}$$

$$U_f = (N_1 + N_2) E_0$$

$$= -(1.61 \times 10^{23} + 2.9 \times 10^{23}) 21.7 \times 10^{-12} = -97.87 \times 10^{11} \text{ erg}$$

$$\text{Energy evolved} = U_i - U_f = (-19.63 \times 10^{11} + 97.87 \times 10^{11})$$

$$= 78.237 \times 10^{11} = 782 \text{ kJ}$$

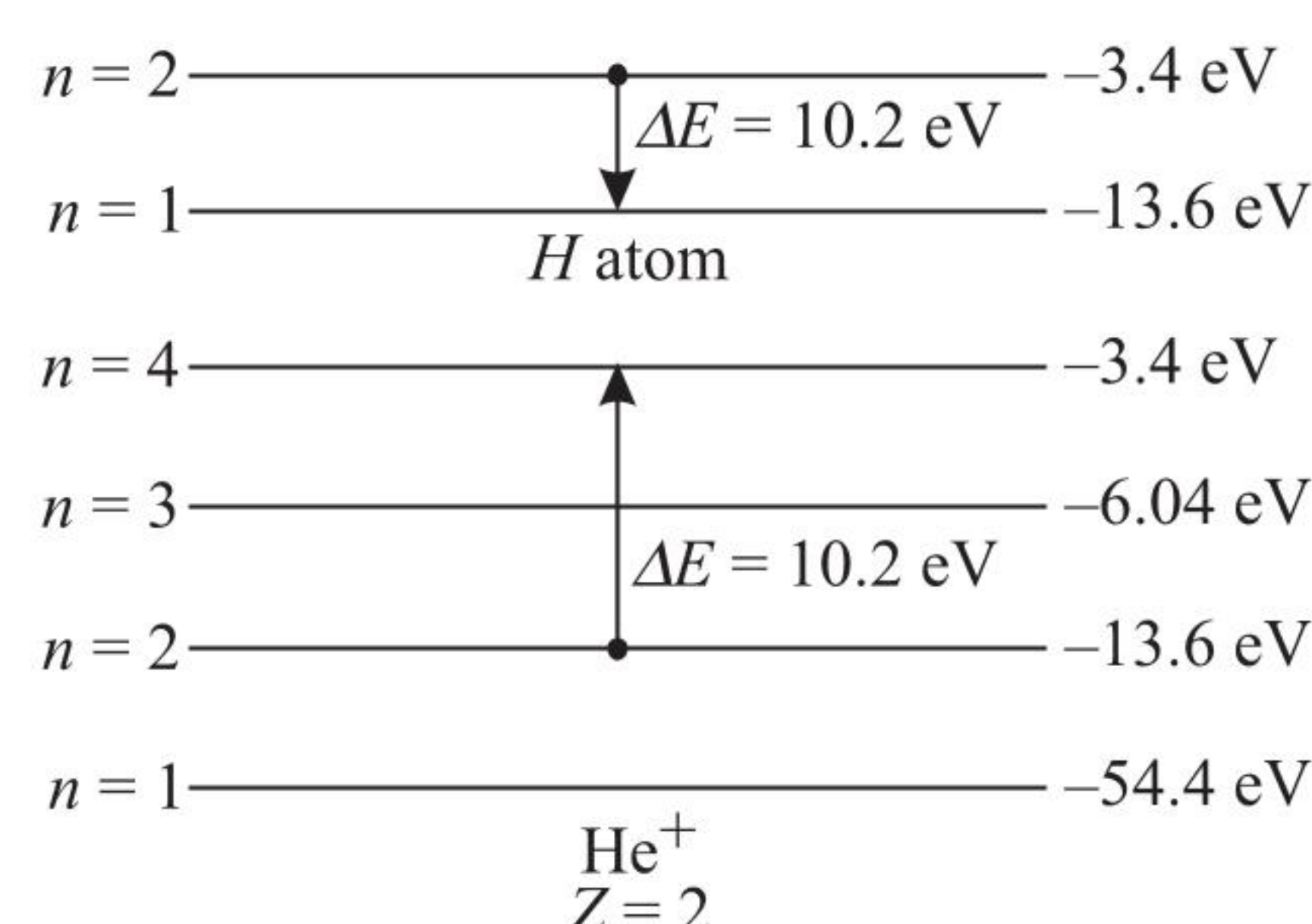
For Problems 50–52
50. (4) As emission spectrum of the atom shows 10 different lines, so the highest excited state of the electron is $n = 5$.

51. (2) As some lines have energy less than $\frac{64}{225} E_0$ and some have more than $\frac{64}{225} E_0$ and some have equal to it. So initial state of electron can be $n = 2$ or $n = 3$.

 Again if initial level is $n = 2$, then

$$Z^2 \left(-\frac{E_0}{25} \right) - Z^2 \left(-\frac{E_0}{4} \right) = \frac{64}{225} E_0,$$

 Z is in fraction (not possible)

52. (1) If $n = 3$, then $Z^2 \left(-\frac{E_0}{25} \right) - Z^2 \left(-\frac{E_0}{9} \right) = \frac{64}{225} E_0, Z = 2$
For Problems 53–55
53. (3)

 Energy given by H atom in transition from $n = 2$ to $n = 1$ is equal to energy taken by He^+ atom in transition from $n = 2$ to $n = 4$.

54. (3) Visible light lies in the range, $\lambda_1 = 4000 \text{ \AA}$ to $\lambda_2 = 7000 \text{ \AA}$. Energy of photons corresponding to these wavelengths (in eV) would be

$$E_1 = \frac{12375}{4000} = 3.09 \text{ eV}, \text{ and } E_2 = \frac{900}{11R} = 1.77 \text{ eV}$$

 From energy level diagram of He^+ atom, we can see that in transition from $n = 4$ to $n = 3$, energy of photon released will lie between E_1 and E_2 .

$$\Delta E_{43} = -3.4 - (-6.04) = 2.64 \text{ eV}$$

Wavelength of photon corresponding to this energy,

$$\lambda = \frac{12375}{264} \text{ \AA} = 4687.5 \text{ \AA} = 4.68 \times 10^{-7} \text{ m}$$

55. (1) Kinetic energy, $K \propto Z^2$

$$\frac{K_{\text{H}}}{K_{\text{He}^+}} = \left(\frac{1}{2} \right)^2 = \frac{1}{4}$$

Matrix Match Type

 1. i. $\rightarrow$ a., c.; ii. $\rightarrow$ b., d.; iii. $\rightarrow$ b., d.; iv. $\rightarrow$ b., c.

$$(i) f = \frac{mZ^2 e^4}{4\epsilon_0^2 h^3 n^3} \Rightarrow f \propto \frac{Z^2}{n^3}$$

$$(ii) L = \frac{nh}{2\pi} \Rightarrow L \propto n$$

$$(iii) \text{ Magnetic moment, } M = IA = \frac{e}{2\pi r} v \pi r^2$$

$$= \frac{e}{2} vr = \frac{e}{2m} (mvr) = \frac{e}{2m} L$$

$$\Rightarrow M = \frac{e}{2m} \left(\frac{nh}{2\pi} \right) \Rightarrow M \propto n$$

$$(iv) I = \frac{ev}{2\pi r} = \frac{e}{2\pi} \left(\frac{\pi m Z e^2}{n^2 h^2 \epsilon_0} \right) \frac{Z e^2}{2 \epsilon_0 n h} \Rightarrow I \propto \frac{Z^2}{n^3}$$

2. i. → d.; ii. → a.; iii. → d.; iv. → c.

(i) Photoelectrons can have KE anywhere from 0 to $KE_{\max} = hc/\lambda - \phi$.
So, average KE is unpredictable.

(ii) Minimum KE can be zero.

(iii), (iv) In continuous X-rays, wavelength can vary from $\lambda_{\min} = hc/eV$ to ∞ .

3. i. → b., c.; ii. → a., c.; iii. → a., c.; iv. → d.

(i) KE of electrons striking the target: $KE = eV$.

So, if V increases, KE increases.

$$\text{Cut-off wavelength, } \lambda_{\min} = \frac{12400}{V} \text{ \AA}$$

So, if V increases, $\lambda_{\min}$ decreases.

(ii) If work function of target is increased in photoelectric effect, the KE of photoelectrons emitted decreases from

$$KE = hf - W_0$$

$$\text{Now, } hf_0 = W_0 \Rightarrow \frac{hc}{\lambda_0} = W_0 \Rightarrow \lambda_0 = \frac{hc}{W_0}$$

If work function (W_0) increases, then cut-off wavelength (λ_0) decreases.

(iii) $eV_0 = KE$, where V_0 is stopping potential. If V_0 decreases, then KE decreases. V_0 is also decreased by increasing W_0 , hence λ_0 decreases as explained above.

(iv) $f_{K_\alpha} = \frac{3}{4} R c (Z - 1)^2$. If f_{K_α} increases, then f_{K_α} decreases and hence Z decreases.

4. i. → c.; ii. → a., b., c., d.; iii. → b., iv. → d.

(i) Moseley's law is about characteristic X-rays.

(ii) In photoelectric effect, from electromagnetic radiation, electrons are ejected. In X-rays, the process is reverse; here, electromagnetic radiation (X-ray) is produced from fast moving electrons.

Also X-rays are produced using high potential difference.

(iii) Cut-off wavelength is related to potential difference applied in continuous X-rays.

(iv) In continuous X-rays, electromagnetic radiations are emitted.

5. i. → c., d.; ii. → b., c., d.; iii. → c., d.; iv. → b., c., d.

$$\text{From Moseley's law: } f = \frac{1}{\lambda} = R(Z - \sigma)^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

For Al, Z is highest and for K_β , $n_1 = 1$ and $n_2 = 3$; and for K_α , $n_1 = 1$, and $n_2 = 2$

Hence, order of frequency:

$$f_{K_\beta}(\text{Al}) > f_{K_\alpha}(\text{Al}) > f_{K_\alpha}(\text{Na}) > f_{K_\beta}(\text{Be}) > f_{K_\alpha}(\text{Li})$$

Speed will be same (iii) for all photons of any frequency.

6. i. → b., c., d.; ii. → a., b., c., d.; iii. → b., c., d.; iv. → b., c., d.

(i) Emission spectra is discrete as only those wavelengths are emitted which correspond to energy difference of two energy levels.

It is due to electronic transition, i.e., due to transition of electron from one energy level (higher) to the another energy level (lower). It is explained by quantum theory of light.

(ii) For energies of incident light less than ionization energy, the absorption spectra is discrete. For energies of incident photon greater than ionization energy, the absorption spectra is continuous. It occurs due to electronic transition which is explained by quantum theory of light.

(iii) Continuous X-rays constitute continuous spectra and characteristic X-rays arise due to electronic transition.

(iv) Thermal radiation spectra is a continuous spectra and is explained by atomic transition and quantum theory.

7. i. → d.; ii. → e.; iii. → b.; iv. → a.

$$r_n = \frac{0.529 n^2}{Z} \text{ \AA}$$

So, (i) → (d)

$$\text{Magnetic field, } B = \frac{12.5 Z^3}{n^5} \text{ T}$$

So, (ii) → (e)

$$B \propto \frac{1}{n^5}$$

$$V_n = \frac{2.2 \times 10^6 Z}{n}$$

$$V_n \propto \frac{1}{n}$$

$$n \uparrow V_n \uparrow$$

So, (iii) → (b)

$$\text{Total energy, } E_n = \frac{-13.6 Z^2}{n^2} \text{ eV}$$

$$n \uparrow E_n \uparrow$$

So, (iv) → (a)

8. i. → b.; ii. → c.; iii. → d.; iv. → a.

$$r_n = \frac{0.529 n^2}{Z} \text{ \AA}$$

So, (i) → (b)

$$V_n = \frac{2.2 \times 10^6 Z}{n} \text{ m s}^{-1}$$

So, (iv) → (a)

$$I = \frac{1.06 Z^2}{n^3} \text{ mA}$$

So, (ii) → (c)

$$\text{Magnetic field, } B = \frac{12.5 Z^3}{n^5} \text{ T}$$

So, (iii) → (d).

9. i. → d.; ii. → a.; iii. → b.; iv. → c.

$$\frac{1}{\lambda} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

Wavelength range from 950 Å to 1350 Å corresponds to ultraviolet region of electromagnetic spectrum, i.e., Lyman series.

$$\text{For transition } 4 \rightarrow 1: \frac{1}{\lambda_1} = 1.1 \times 10^7 \left(\frac{15}{16} \right)$$

$$\therefore \lambda_1 = \frac{16}{15 \times 1.1 \times 10^7} = 970 \text{ \AA}$$

$$\text{For transition } 3 \rightarrow 1: \lambda_2 = \frac{9}{8 \times 1.1 \times 10^7} = 1023 \text{ \AA}$$

$$\text{For transition } 2 \rightarrow 1: \lambda_3 = \frac{4}{3 \times 1.1 \times 10^7} = 1212 \text{ \AA}$$

$$\text{For Li}^2 \text{ atom } (Z=3), \text{ for transition } 2 \rightarrow 1: \frac{1}{\lambda'} = RZ^2 \left(1 - \frac{1}{2^2}\right)$$

$$\lambda' = \frac{1}{1.1 \times 10^7 \times 9 \times 0.75} = 134 \text{ \AA}$$

$$\text{For He}^+ \text{ atom } (Z=2) \text{ for transition } 2 \rightarrow 1: \frac{1}{\lambda'} = RZ^2 \left(1 - \frac{1}{2^2}\right)$$

$$\lambda' = \frac{1}{1.1 \times 10^7 \times 4 \times 0.75} = 303 \text{ \AA}$$

Numerical Value Type

$$1. (4) E_{\text{photon}} = 13.6 \left(1 - \frac{1}{25}\right) \text{ eV} = 13.0 \text{ eV}$$

$$E/c = mv \text{ (momentum conserved)}$$

$$v = \frac{E}{mc} = \frac{(13 \times 1.6 \times 10^{-19})}{(1.67 \times 10^{-27} \times 3 \times 10^8)} = 4 \text{ m/s}$$

$$2. (6) \frac{\lambda_1}{\lambda_2} = \frac{(Z_2 - 1)^2}{(Z_1 - 1)^2} \quad \left(\text{since, } \frac{1}{\lambda} \propto (Z - 1)^2\right)$$

$$\frac{1}{4} = \frac{(Z_2 - 1)^2}{(11 - 1)^2}; \text{ on solving, } Z_2 = 6$$

$$3. (6) \text{ When electron jumps from } n^{\text{th}} \text{ state to ground state, number of possible emission lines} = n(n-1)/2. \text{ But here transition takes place from } n^{\text{th}} \text{ state to state } n_1 = 2.$$

$$\text{Here, number of possible emission lines} = n(n-1)(n-2)/2 = 10 \text{ (given).}$$

$$\text{On solving, } n = 6.$$

$$4. (2) \text{ The shortest wavelength of Brackett series is corresponding to transition of electron between } n_1 = 4 \text{ and } n_2 = \infty.$$

$$\text{Similarly, the shortest wavelength of Balmer series is corresponding to transition of electron between } n_1 = 2 \text{ and } n_2 = \infty.$$

$$(Z)^2 \left(\frac{13.6}{16}\right) = \left(\frac{13.6}{4}\right) \quad \text{or} \quad Z = 2$$

$$5. (1) \text{ Heat produced/sec} = 200 \text{ W}$$

$$\Rightarrow i = \frac{200}{V} = \frac{200}{20 \times 10^3} = 10 \text{ mA}$$

$$6. (4) \text{ For the first line of Balmer series of hydrogen}$$

$$\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R}{36} \Rightarrow \lambda = \frac{36}{5R}$$

$$\text{For singly ionized helium } Z = 2$$

$$\frac{1}{\lambda'} = 4R \left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

$$\text{Given } \lambda' = \lambda \Rightarrow 4R \left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right) = \frac{5R}{36}$$

$$\Rightarrow n_2 = 4$$

$$7. (5) E_n = -\frac{mZ^2e^4}{8\epsilon_0^2n^2h^2}, \text{ so } hf = +\frac{mZ^2e^4}{8\epsilon_0^2h^2} \left[\frac{1}{16} - \frac{1}{25}\right]$$

$$\therefore f = \frac{mZ^2e^4}{8\epsilon_0^2h^3} \left[\frac{9}{16 \times 25}\right] \quad \dots(i)$$

$$\text{and frequency } f_4 = \frac{Z^2e^4m}{4\epsilon_0^2n^3h^3} = \frac{Z^2e^4m}{4\epsilon_0^2(4)^3h^3} \quad \dots(ii)$$

$$\therefore f/f_4 = 18/25, \text{ so } m = 5$$

$$8. (9) \text{ Number of revolutions before transition} = \text{frequency} \times \text{time}$$

$$\text{Also, frequency} \propto \frac{1}{n^3}$$

$$\text{So required ratio} = \frac{(1/2)^3 12.8 \times 10^{-8}}{(1/3)^3 4.8 \times 10^{-8}} = 9$$

$$9. (2) \frac{T_1}{T_2} = \frac{n_1^3}{n_2^3} \Rightarrow \frac{8}{1} = \frac{n_1^3}{n_2^3} \Rightarrow \frac{n_1}{n_2} = \frac{2}{1}$$

$$10. (6) \text{ According to conservation of energy}$$

$$\frac{hc}{\lambda_1} + \frac{hc}{\lambda_2} = RchZ^2 \left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

$$n = 6$$

$$11. (823)$$

The smallest frequency and largest wavelength in ultra-violet region will be for transition of electron from orbit 2 to orbit 1. Therefore,

$$\frac{1}{\lambda} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\Rightarrow \frac{1}{122 \times 10^{-9} \text{ m}} = R \left[\frac{1}{1^2} - \frac{1}{2^2}\right] = R \left[1 - \frac{1}{4}\right] = \frac{3R}{4}$$

$$\Rightarrow R = \frac{4}{3 \times 122 \times 10^{-9}} \text{ m}^{-1}$$

The highest frequency and smallest wavelength for infrared region will be for transition of electron from ∞ to 3rd orbit.

$$\therefore \frac{1}{\lambda} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\Rightarrow \frac{1}{\lambda} = \frac{4}{3 \times 122 \times 10^{-9}} \left(\frac{1}{3^2} - \frac{1}{\infty}\right)$$

$$\therefore \lambda = \frac{3 \times 122 \times 9 \times 10^{-9}}{4} = 823.5 \text{ nm}$$

$$12. (2.25)$$

Let T_1 be the initial temperature and T_2 be the increased temperature of the black body. According to Stefan's law,

$$\left(\frac{T_2}{T_1}\right)^4 = 81 = (3)^4$$

$$T_2 = 3T_1$$

$$\dots(i)$$

$$\text{Also, } \lambda_1 T_1 = \lambda_2 T_2$$

$$\lambda_2 = \frac{\lambda_1 \times T_1}{T_2} = \frac{9000 \times T_1}{3T_1} = 3000 \text{ \AA}$$

$$\text{Now, } \frac{hf}{\lambda_2} - W = eV_0 \text{ or } \frac{hf}{\lambda_2} - W = 13.6 \left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\text{Solving, we get, } W = 2.25 \text{ eV}$$

$$13. (6) \text{ According to Moseley's law, } \sqrt{f} = a(Z - b) \Rightarrow f = a^2(Z - b)^2$$

$$\Rightarrow \frac{c}{\lambda} = a^2(Z - b)^2 \quad \dots(i)$$

$$\text{For } K_\alpha \text{ line, } b = 1$$

$$\text{From (i), } \frac{\lambda_2}{\lambda_1} = \frac{(Z_1 - 1)^2}{(Z_2 - 1)^2} \Rightarrow \frac{4\lambda}{\lambda} = \frac{(11 - 1)^2}{(Z_2 - 1)^2}$$

$$\Rightarrow Z_2 - 1 = \frac{10}{2} \Rightarrow Z_2 = 6$$

$$14. (15) v = v_0 \frac{Z^2}{n^3} \Rightarrow \frac{Z^2}{n^3} = 2 \quad \dots(i)$$

$$E = E_0 \frac{Z^2}{n^2} \Rightarrow \frac{Z^2}{n^2} = 4 \quad \dots(ii)$$

Solving (i) and (ii), $n = 2$, $Z = 4$

$$L = mvr = \frac{nh}{2\pi}, \Delta L = \tau \Delta t = \Delta n \frac{h}{2\pi}$$

$$\tau = \frac{\Delta n}{\Delta t} \times \frac{h}{2\pi} = \frac{1}{7 \times 10^{-9}} \times \frac{1}{2} \times 2.1 \times 10^{-34} = \frac{2.1}{14} \times 10^{-25}$$

$$\tau \times 10^{27} = \frac{2.1}{14} \times 10^{-25} \times 10^{27} = 15$$

$$15. (2) \frac{1}{\lambda} = RZ \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

$$\frac{\lambda_2}{\lambda_1} = \frac{\left(\frac{1}{9} - \frac{1}{n^2} \right)}{\frac{8}{9}},$$

where n is the principal quantum number of the initial excited state.

$$\text{Angular width } \theta = \frac{2\lambda}{d}$$

$$\frac{\theta_2}{\theta_1} = \frac{\lambda_2}{\lambda_1} = \frac{2}{25} = \frac{n^2 - 9}{8n^2} \Rightarrow n = 5$$

$$\frac{2\lambda_1}{d} = \theta_1, \frac{1}{\lambda_1} = \frac{2}{d \times \theta_1} = \frac{2}{6.4} \times 10^7 \text{ m}^{-1}$$

$$\frac{1}{\lambda_1} = RZ^2 \left(\frac{1}{9} - \frac{1}{n^2} \right)$$

$$\frac{2}{6.4} \times 10^7 = 1.097 \times 10^7 Z^2 \left(\frac{1}{9} - \frac{1}{25} \right)$$

$$Z^2 = \frac{2}{6.4} \times \frac{225}{16 \times 1.097} \Rightarrow Z = 2$$

$$16. (2) E = -(13.6 \text{ eV}) Z^2 \left(\frac{1}{(Z+2)^2} - \frac{1}{Z^2} \right)$$

$$= -13.6 \times \left(\frac{Z^2 - (Z+2)^2}{(Z+2)^2} \right) = \frac{4(Z+1) \times 13.6}{(Z+2)^2} \text{ eV} \quad \dots(i)$$

Now energy of electron is $K = \frac{h^2}{2\lambda^2 m}$

Solving $K = 6 \text{ eV}$

$$\text{So, } \frac{4(Z+1) \times 13.6}{(Z+2)^2} = 6 + 4.2 = 10.2 \text{ eV}$$

$$\frac{Z+1}{(Z+2)^2} = \frac{3}{16} \Rightarrow (Z-2)(3Z+2) = 0$$

So, the value of $Z = 2$ (neglecting the negative/fractional value)

$$17. (2) \text{ Using COM: } mu = 2mv$$

$$v = \frac{u}{2}, \Delta E = \frac{1}{2} mu^2 - 2 \times \frac{1}{2} m \left(\frac{u}{2} \right)^2$$

$$\Rightarrow \Delta E = \frac{1}{4} mu^2 = \frac{1}{4} \times 1.66 \times 10^{-27} \times (6.24 \times 10^4)^2$$

$$13.6 \left(\frac{1}{1^2} - \frac{1}{n^2} \right) = \frac{1.66 \times 10^{-27} \times (6.24 \times 10^4)^2}{4 \times 1.6 \times 10^{-19}}$$

$$\Rightarrow n = 2$$

18. (3) We have,

$$10.2 = W + K_{\max,1} \quad \dots(i)$$

$$\text{and } 10.2 Z^2 = W + K_{\max,2} \quad \dots(ii)$$

$$\therefore \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{K_2}{K_1}} = 2.3 \Rightarrow K_2 = 5.25 K_1 \quad \dots(iii)$$

Also, $10.2 Z^2 = \text{energy corresponding to longest wavelength of the Lyman series} = 3 \times 13.6$

$$\Rightarrow Z = 2$$

From Eqs. (i), (ii) and (iii), $W = 3 \text{ eV}$

$$19. (30) \lambda_{C_1} = \frac{h}{eV} = \frac{12375}{10 \times 10^3} \text{ \AA} = 1.2375 \text{ \AA} \quad \dots(i)$$

$$\lambda_{C_2} = 0.61875 \text{ \AA} \quad \dots(ii)$$

$$\frac{1}{\lambda_{K_\alpha}} = (Z-1)^2 \left(\frac{1}{1} - \frac{1}{4} \right) \times 10^7 \quad \dots(iii)$$

$$\text{It is given, } 3 \times (\lambda_{K_\alpha} - 1.2375) = \lambda_{K_\alpha} - 0.61875$$

$$\Rightarrow \lambda_{K_\alpha} = 1.54338 \text{ \AA} \quad \dots(iv)$$

$$\text{Putting this value in (iii), } Z - 1 = 29 \Rightarrow Z = 30$$

Archives

JEE Advanced

Single Correct Answer Type

$$1. (1) \frac{1}{\lambda_{H_2}} = RZ_H^2 \left[\frac{1}{4} - \frac{1}{9} \right] = R(1)^2 \left[\frac{5}{36} \right]$$

$$\frac{1}{\lambda_{He}} = RZ_{He}^2 \left[\frac{1}{4} - \frac{1}{16} \right] = R(4)^2 \left[\frac{3}{16} \right]$$

$$\frac{\lambda_{H_2}}{\lambda_{He}} = \frac{1}{4} \left[\frac{16}{3} \times \frac{5}{36} \right] = \frac{5}{27}$$

$$\lambda_{He} = \frac{5}{27} \times 6561 = 1215 \text{ \AA}$$

Multiple Correct Answers Type

1. (1), (2), (4)

$$\text{As radius } r \propto \frac{n^2}{Z}$$

$$\Rightarrow \frac{\Delta r}{r} = \frac{\left(\frac{n+1}{Z} \right)^2 - \left(\frac{n}{Z} \right)^2}{\left(\frac{n}{Z} \right)^2} = \frac{2n+1}{n^2} \approx \frac{2}{n} \propto \frac{1}{n}$$

$$\text{As energy } E \propto \frac{Z^2}{n^2}$$

$$\Rightarrow \frac{\Delta E}{E} = \frac{\frac{Z^2}{n^2} - \frac{Z^2}{(n+1)^2}}{\frac{Z^2}{(n+1)^2}} = \frac{(n+1)^2 - n^2}{n^2 \cdot (n+1)^2} \cdot (n+1)^2$$

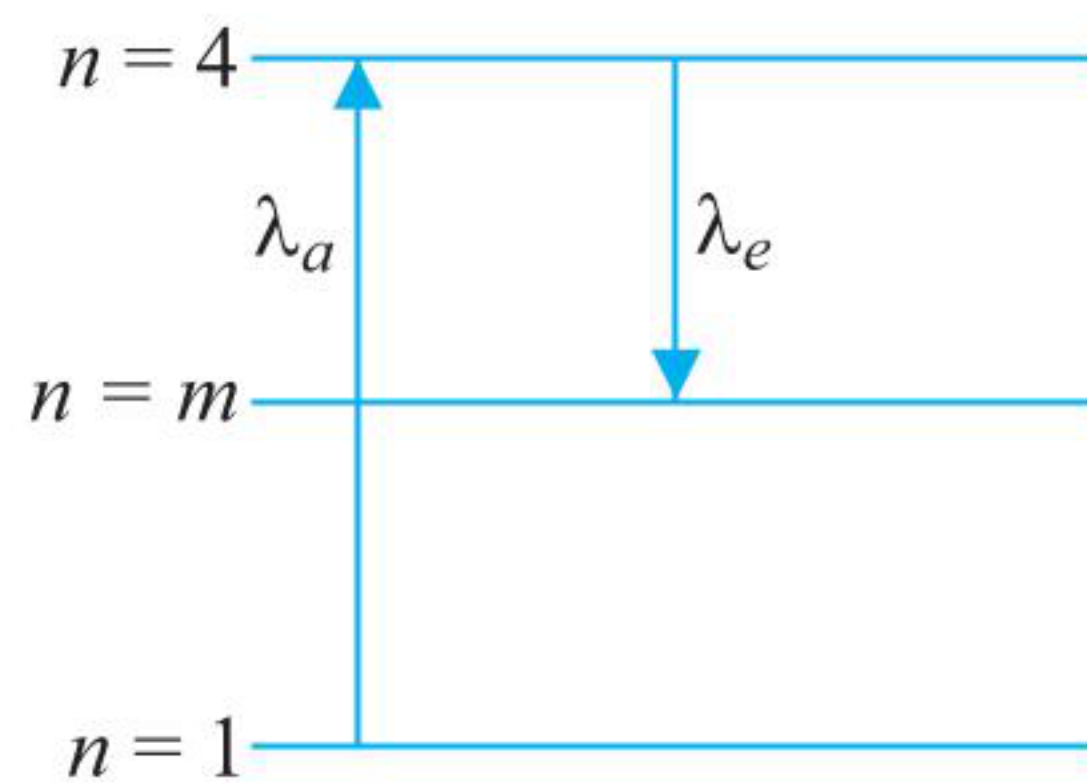
$$\Rightarrow \frac{\Delta E}{E} = \frac{2n+1}{n^2} \approx \frac{2n}{n^2} \propto \frac{1}{n}$$

As angular momentum $L = \frac{nh}{2\pi}$

$$\Rightarrow \frac{\Delta L}{L} = \frac{\frac{(n+1)h}{2\pi} - \frac{nh}{2\pi}}{\frac{nh}{2\pi}} = \frac{1}{n} \propto \frac{1}{n}$$

2. (2),(3)

The energy level diagram is shown in the figure



$$\frac{hc}{\lambda_a} = 13.6 \left[\frac{1}{1} - \frac{1}{4^2} \right] \quad \dots(i)$$

$$\frac{hc}{\lambda_e} = 13.6 \left[\frac{1}{m^2} - \frac{1}{4^2} \right] \quad \dots(ii)$$

(ii)/(i), we get

$$\frac{\lambda_a}{\lambda_e} = \frac{\left[\frac{1}{m^2} - \frac{1}{16} \right]}{\left[1 - \frac{1}{16} \right]} = \frac{1}{5}$$

$$\Rightarrow \frac{1}{m^2} - \frac{1}{16} = \frac{15}{16} \times \frac{1}{5}$$

$$\Rightarrow \frac{1}{m^2} - \frac{1}{16} = \frac{3}{16}$$

$$\Rightarrow \frac{1}{m^2} = \frac{3}{16} + \frac{1}{16}$$

$$\Rightarrow m = 2$$

Hence option (3) is correct.

$$\text{From (ii), } \frac{hc}{\lambda_e} = 13.6 \left[\frac{1}{2^2} - \frac{1}{4^2} \right] = 13.6 \times \frac{3}{16} \text{ eV}$$

$$\Rightarrow \lambda_e = \frac{1242 \times 16}{13.6 \times 3} \text{ nm}$$

$$\Rightarrow \lambda_e \approx 486 \text{ nm}$$

Hence option (1) is wrong.

We have $\text{KE}_n \propto \frac{z^2}{n^2}$

$$\Rightarrow \frac{(\text{KE})_m}{(\text{KE})_1} = \frac{1^2}{m^2} = \frac{1}{4}$$

Hence option (2) is correct.

$$\text{As } \Delta p_a = \frac{h}{\lambda_a} \text{ and } \Delta p_e = \frac{h}{\lambda_e}$$

$$\Rightarrow \frac{\Delta p_a}{\Delta p_e} = \frac{\lambda_e}{\lambda_a} = 5$$

Hence option (4) is wrong.

3. (2),(3)

Given, $V = Fr$

$$F_{\text{Conservative}} = \frac{-dV}{dr} = -F$$

It means the magnitude of force = Constant = F

$$\text{Also, } F = \frac{mv^2}{R} \quad \dots(i)$$

$$\text{And, } mvR = \frac{nh}{2\pi} \quad \dots(ii)$$

$$\text{From (i) and (ii), } F = \frac{m}{R} \times \frac{n^2 h^2}{4\pi^2} \times \frac{1}{m^2 R^2}$$

$$\Rightarrow R = \left(\frac{n^2 h^2}{4\pi^2 m F} \right)^{1/3} \quad \dots(iii)$$

$$\text{From (ii), } v = \frac{nh}{2\pi m R} \quad \dots(iv)$$

$$\text{From (iii) and (iv), } v = \frac{nh}{2\pi m} \left(\frac{4\pi^2 m F}{n^2 h^2} \right)^{1/3}$$

$$\Rightarrow v = \frac{n^{1/3} h^{1/3} F^{1/3}}{2^{1/3} \pi^{1/3} m^{2/3}}$$

Option (2) is correct.

Also, total energy,

$$E = \frac{1}{2} mv^2 + V = \frac{1}{2} mv^2 + FR$$

$$E = \frac{1}{2} m \left(\frac{n^{2/3} h^{2/3} F^{2/3}}{2^{2/3} \pi^{2/3} m^{4/3}} \right) + F \times \left(\frac{n^2 h^2}{4\pi^2 m F} \right)^{1/3}$$

$$\Rightarrow E = \left(\frac{n^2 h^2 F^2}{4\pi^2 m} \right)^{1/3} \left[\frac{1}{2} + 1 \right] = \frac{3}{2} \left(\frac{n^2 h^2 F^2}{4\pi^2 m} \right)^{1/3}$$

Option (3) is correct.

4. (1),(3)

$$\text{Cut-off wavelength } \lambda_{\min} = \frac{hc}{eV}$$

$$\Rightarrow \lambda_{\min} \propto \frac{1}{V} \Rightarrow (\lambda_{\min})_{\text{new}} = \frac{\lambda_2}{2}$$

Hence the cut-off wavelength will reduce to half.

If filament current is reduced, the number of ejected electrons, decreases. Hence intensity of X -rays decreases. Characteristic wave length does not depend on voltage, hence the wave length of characteristic X -rays remain constant.

5. (1),(4)

For 1

When the transition is from any level to $n = 2$, then photon emitted belong to Balmer series.

$\therefore$ For longest wavelength, transition occurs from $n = 3$ to $n = 2$.

$$\frac{hc}{\lambda_{\max}} = RCh \left[\frac{1}{2^2} - \frac{1}{3^2} \right] \text{ and for shortest wavelength transition}$$

occurs from $n = \infty$ to $n = 2$

$$\therefore \frac{hc}{\lambda_{\min}} = RCh \left[\frac{1}{2^2} - \frac{1}{\infty^2} \right]$$

$$\therefore \frac{\lambda_{\text{longest}}}{\lambda_{\text{shortest}}} = \frac{9}{5}$$

For (2)

$$\lambda_{\text{longest}} \text{ of Balmer} = \frac{36}{5R}$$

$$\lambda_{\text{shortest}} \text{ of Paschen} = \frac{9}{R}$$

Hence these wavelengths don't overlap.

For (3)

$$\text{For Lyman series, } \frac{1}{\lambda} = R \left[\frac{1}{1} - \frac{1}{m^2} \right]$$

$$\text{Also } \frac{1}{\lambda_0} = R$$

$$\therefore \frac{1}{\lambda} = \frac{1}{\lambda_0} \left[1 - \frac{1}{m^2} \right] \Rightarrow \lambda = \frac{\lambda_0}{1 - \frac{1}{m^2}}$$

For (4)

$$\lambda_{\text{longest}} \text{ of Lyman} = \frac{4}{3R}, \lambda_{\text{shortest}} \text{ of Balmer} = \frac{4}{R}$$

Hence that wavelength don't overlap.

Linked Comprehension Type

For Problem 1–3

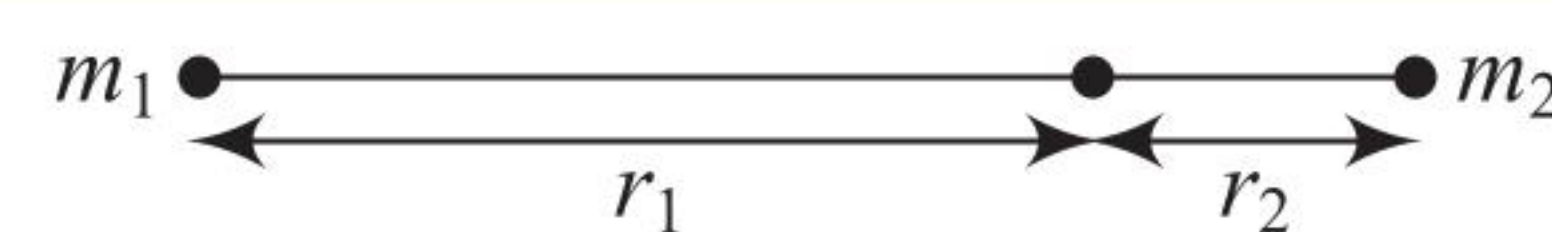
$$1. (4) L = \frac{nh}{2\pi}$$

$$KE = \frac{L^2}{2I} = \left(\frac{nh}{2\pi} \right)^2 \frac{1}{2I}$$

$$2. (2) hf = KE_{n=2} - KE_{n=1}$$

$$I = 1.87 \times 10^{-46} \text{ kg m}^2$$

$$3. (3) r_1 = \frac{m_2 d}{m_1 + m_2} \text{ and } r_2 = \frac{m_1 d}{m_1 + m_2}$$



$$I = m_1 r_1^2 + m_2 r_2^2$$

$$\therefore d = 1.3 \times 10^{-10} \text{ m}$$

Numerical Value Type

$$1. (6) E = \frac{hc}{\lambda} = \frac{12370}{970} = 12.7 \text{ eV}$$

The incident light excite electron to n quantum state

$$\Rightarrow -13.6 + 12.7 = -\frac{13.6}{n^2}$$

$$n^2 = 16 \text{ or } n = 4$$

$$\text{Number of lines} = {}^nC_2 = \frac{n(n-1)}{2} = 6$$

$$2. (5) PE = V = -27.2 \frac{Z^2}{n^2} \quad [Z = 1 \text{ for H}]$$

$$V = \frac{-27.2}{n^2} \Rightarrow V \propto \frac{1}{n^2}$$

$$\text{So, } \frac{V_i}{V_f} = \left(\frac{n_f}{n_i} \right)^2 = 6.25 \quad \therefore \frac{n_f}{n_i} = 2.5 = \frac{5}{2}$$

Minimum integral value of n_f is 5.

3. (3) In transition from $n = 2$ to $n = 1$

$$\Delta E_{2-1} = 13.6 \times Z^2 \left[1 - \frac{1}{4} \right] = 13.6 \times Z^2 \left[\frac{3}{4} \right]$$

In transition from $n = 3$ to $n = 2$

$$\Delta E_{3-2} = 13.6 \times Z^2 \left[\frac{1}{4} - \frac{1}{9} \right] = 13.6 \times Z^2 \left[\frac{5}{36} \right]$$

We are given $\Delta E_{2-1} = \Delta E_{3-2} + 74.8$

$$13.6 \times Z^2 \left[\frac{3}{4} \right] = 13.6 \times Z^2 \left[\frac{5}{36} \right] + 74.8$$

$$13.6 \times Z^2 \left[\frac{3}{4} - \frac{5}{36} \right] = 74.8$$

$$\Rightarrow Z^2 = 9 \text{ which gives } Z = +3$$